Antihydrogen Synthesis Via Magnetobound States of Protonium Within Proton-Positron-Antiproton Plasmas

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Through classical trajectory simulation, the possibility that antihydrogen can be synthesized via three body recombination involving magnetobound protonium is studied. It has been previously reported that proton-antiproton collisions can result in a correlated drift of the particles perpendicular to a magnetic field. While the two particles are in their correlated drift, they are referred to as a magnetobound protonium system. Possible three body recombination resulting in bound state antihydrogen is studied when a magnetobound protonium system encounters a positron. The simulation incorporates a uniform magnetic field. A visual search and an energy analysis was done in an attempt to find parameters for which the positron is captured into a bound state with an antiproton, resulting in the formation of antihydrogen.

Recently, simulations have shown that within a low temperature plasma containing protons and antiprotons, binary collisons invloving proton-antiproton pairs can cause them to become bound temporarily and experience giant cross-magnetic field drifts [1, 2]. These particle pairs have been referred to as being in a magnetobound state, which could serve as a useful intermediate step in the production of neutral antimatter. Magnetobound states occur in low temperatures and strong magnetic fields such as those found inside Penning traps that produce antihydrogen. The possibility of three body recombination resulting in bound state antihydrogen is studied when a magnetobound protonium system encounters a positron.

The interactions between particles throughout the simulation are treated classically. Coulomb's law states that the electric force on the proton (particle 1) by the antiproton (particle 2) is given by $\mathbf{F}_{on1by2} = k_c q_1 q_2 \mathbf{r}_{12}/r_{12}^3$. Here, $k_c = 1/(4\pi\varepsilon_0)$, where ε_0 is the permittivity of free space, is the Coulomb force constant, q_1 and q_2 are the charges of the proton and antiproton, $r_{12} = |\mathbf{r_{12}}|$ is the distance between particles, and $\mathbf{r_{12}} = \mathbf{r_1} - \mathbf{r_2}$, is the separation vector between the particles. Similarly, the rest of the electric forces acting on the proton, antiproton, and positron (particle 3) are found to be $\mathbf{F}_{on1by3} = k_c q_1 q_3 \mathbf{r}_{13}/r_{13}^3$, $\mathbf{F}_{on2by1} = k_c q_1 q_2 \mathbf{r}_{21}/r_{21}^3$, $\mathbf{F}_{on2by3} = k_c q_2 q_3 \mathbf{r}_{23}/r_{23}^3$, $\mathbf{F}_{on3by1} = k_c q_1 q_3 \mathbf{r}_{31}/r_{31}^3$, and $\mathbf{F}_{on3by2} = k_c q_2 q_3 \mathbf{r}_{32}/r_{32}^3$. The magnetic force acting on the proton is $\mathbf{F}_{on1byB} = k_L q_1 B(v_{1y}\hat{\mathbf{i}} - v_{1x}\hat{\mathbf{j}})$, where $(\hat{\mathbf{i}},\hat{\mathbf{j}},\hat{\mathbf{k}})$ are unit vectors in a Cartesian coordinate system, v_{1x}, v_{1y}, v_{1z} are the velocity components of the proton,$k_L = 1$ is the Lorentz force constant in SI units, q_1 is the charge of the proton, and B is the magnitude of a uniform magnetic field paralled to the z-axis. For the antiproton, $\mathbf{F}_{on2byB} = k_L q_2 B(v_{2y}\hat{\mathbf{i}} - v_{2x}\hat{\mathbf{j}})$, where v_{2x}, v_{2y}, v_{2z} are its velocity components, and q_2 is its charge. Lastly, for the positron, $\mathbf{F}_{on3byB} = k_L q_3 B(v_{3y}\hat{\mathbf{i}} - v_{3x}\hat{\mathbf{j}})$, where v_{3x}, v_{3y}, v_{3z} are its velocity components, and q_3 is its charge. By Newton's second law, the motion of the proton, antiproton, and postitron is governed by $\Sigma\mathbf{F} = m\mathbf{a}$. For the proton, $\mathbf{F}_{on1by2} + \mathbf{F}_{on1by3} + \mathbf{F}_{on1byB} = m_1\mathbf{a}_1$, where m_1 is the mass of the proton and $\mathbf{a}_1$ is its acceleration. For the antiproton, $\mathbf{F}_{on2by1} + \mathbf{F}_{on2by3} + \mathbf{F}_{on2byB} = m_2\mathbf{a}_2$, where m_2 is the mass of the antiproton and $\mathbf{a}_2$ is its acceleration. For the positron, $\mathbf{F}_{on3by1} + \mathbf{F}_{on3by2} + \mathbf{F}_{on3byB} = m_3\mathbf{a}_3$, where m_3 is the mass of the positron and $\mathbf{a}_3$ is its acceeration. The position and velocity of the particles are functions of time that are written as $\mathbf{r}_i(t) = x_i(t)\hat{\mathbf{i}} + y_i(t)\hat{\mathbf{j}} + z_i(t)\hat{\mathbf{k}}$, and $\mathbf{r}'_i(t) = x'_i(t)\hat{\mathbf{i}} + y'_i(t)\hat{\mathbf{j}} + z'_i(t)\hat{\mathbf{k}}$. Let

$$F_{Cij} = \frac{k_c q_i q_j}{([x_i(t) - x_j(t)]^2 + [y_i(t) - y_j(t)]^2 + [z_i(t) - z_j(t)]^2)^{3/2}}, \tag{1}$$

where the indices i and j correspond to the proton (particle 1), antiproton (particle 2), and positron (particle 3). Then the equations of motion for the proton are

$$F_{C12}[x_1(t) - x_2(t)] + F_{C13}[x_1(t) - x_3(t)] + k_L B q_1 y'_1(t) = m_1 x''_1(t) \tag{2}$$

$$F_{C12}[y_1(t) - y_2(t)] + F_{C13}[y_1(t) - y_3(t)] - k_L B q_1 x'_1(t) = m_1 y''_1(t) \tag{3}$$

$$F_{C12}[z_1(t) - z_2(t)] + F_{C13}[z_1(t) - z_3(t)] = m_1 z''_1(t). \tag{4}$$

The equations of the antiproton are

$$F_{C12}[x_1(t) - x_2(t)] + F_{C23}[x_2(t) - x_3(t)] + k_L B q_2 y'_2(t) = m_2 x''_2(t) \tag{5}$$

$$F_{C12}[y_1(t) - y_2(t)] + F_{C23}[y_2(t) - y_3(t)] - k_L B q_2 x'_2(t) = m_2 y''_2(t) \tag{6}$$

$$F_{C12}[z_1(t) - z_2(t)] + F_{C23}[z_2(t) - z_3(t)] = m_2 z''_2(t). \tag{7}$$

The equations of motion of the positron are

$$F_{C13}[x_1(t) - x_3(t)] + F_{C23}[x_2(t) - x_3(t)] + k_L B q_3 y'_3(t) = m_3 x''_3(t) \tag{8}$$

$$F_{C13}[y_1(t) - y_3(t)] + F_{C33}[y_3(t) - y_3(t)] - k_L B q_3 x'_3(t) = m_3 y''_3(t) \tag{9}$$

$$F_{C13}[z_1(t) - z_3(t)] + F_{C23}[z_2(t) - z_3(t)] = m_3 z''_3(t). \tag{10}$$

Initially, the proton and antiproton are treated as traveling in opposite directions towards each other from an infinite distance, while the positron is at an infinite distance from both the protonium system. At these distances the electric potential energy is zero. Conservation of energy requires

$$K_{1\infty} + K_{2\infty} + K_{3\infty} = \frac{1}{2}m_1\left(v_{x10}^2 + v_{y10}^2 + v_{z10}^2\right) + \frac{1}{2}m_2\left(v_{x20}^2 + v_{y20}^2 + v_{z20}^2\right) + \frac{1}{2}m_3\left(v_{x30}^2 + v_{y30}^2 + v_{z30}^2\right)$$
$$+ \frac{k_c q_1 q_2}{r_{120}} + \frac{k_c q_1 q_3}{r_{130}} + \frac{k_c q_3 q_2}{r_{230}}, \tag{11}$$

where v_{xi0}, v_{yi0}, and v_{zi0} are the initial velocity components of the particles.

The kinetic energies at infinite distances of separation of the proton, antiproton, and positron are $K_{1\infty}$, $K_{2\infty}$, and $K_{3\infty}$ respectively, and r_{ij0} is the initial separation between particles. For this simulation, $v_{x10} = v_{x20} = v_{y10} = v_{y20} = 0$, $K_{1\infty} = K_{2\infty} = K_{3\infty} = K_\infty$, $m_1 = m_2 = m$, and $v_{z10} = -v_{z20}$. The initial positions of the particle are shown in (Fig. 1). The parameter b is the impact parameter, ζb is the inital axial separation between the proton and the antiproton, and δ is the distance between the origin and the positron on the y-axis. By simplifying and rewriting Eq.(11), the initial nonzero velocities of the proton and antiproton at the start of the simulation are

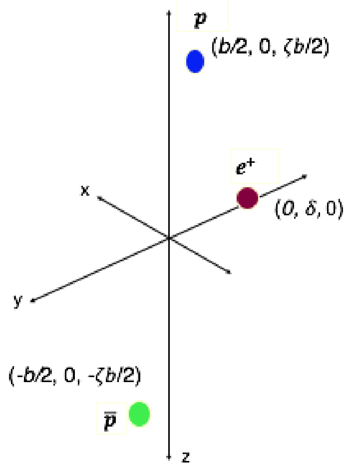

Figure 1: *Initial positions of particles*

$$v_{z10,z20} = \pm\sqrt{\frac{2K_\infty}{m} - \frac{k_c q_1 q_2}{mb\sqrt{1+\zeta^2}}}. \tag{12}$$

The parameters used in the simulation are $B = 17$ T and $K_\infty = 11\ \kappa$, where κ has the value of Boltzmann's constant in SI units with units of energy, and the impact parameter $b = 1.65 r_c$. The cyclotron radius r_c is defined as $\sqrt{2K_\infty m/(k_L^2 B^2 q^2)}$.

The trajectories of the particles were found by solving their equations of motion numerically and varying the parameters. Due to the chaotic behavior of the system of particles there was no specific range of δ and ζ values that would guarantee capture.It appeared that the motion of the positron did not stay correlated with that of the protonium system long enough to ensure capture. The trajectories of the particles projected onto the $yz-$plane are shown in (Fig. 2). The graph is normalized by r_c. The figure shows that the positron does interact with the magnetobound system and orbits around it, but does not remain bounded to the antiproton. While the magnetobound protonium system experienced a larger cross-magnetic field drift distance than the positronium system, it seemed to be more challanging for the antiproton to capture the positron. Further studies on this phenomenon can be made by varying the parameters mentioned above in smaller increments in the efforts to find a successful capturing of the positron by the antiproton.

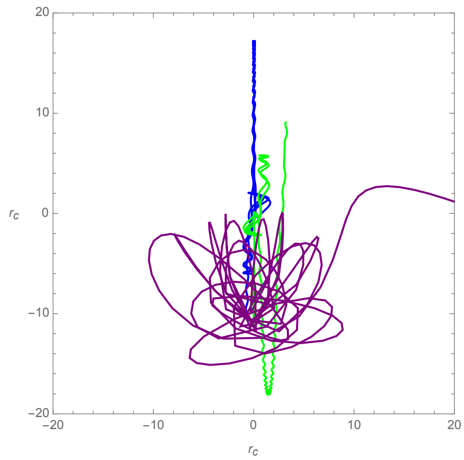

Figure 2: *Trajectory on yz-plane*

In addition to a visual analysis of the three particle system, (Fig. 3) shows the change in energy of the system throughout the simulation. If capture occured, the kinetic and potential energy would stabalize to a constant value, which does not occur for a long enough period of time in this simulation. Successful trials would show the proton carrying away the excess energy from the system, which would then allow the remaining particles to form antihydrogen.

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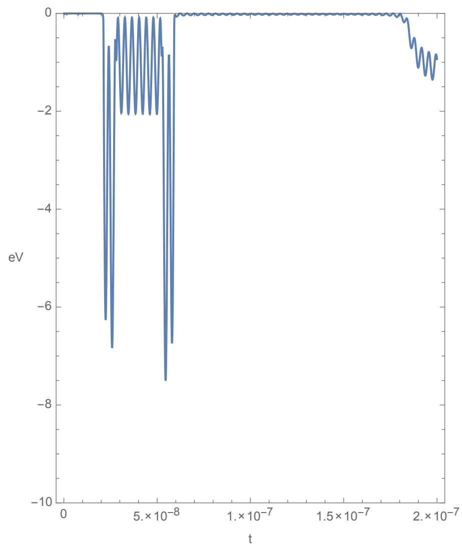

Figure 3: *Energy of the system.*

References

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