Quasi-static electromagnetic instabilities in relativistic plasmas heated by an electromagnetic wave

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I. Introduction

In laser-plasma interaction, magnetic fields play a crucial role as they could affect the transport processes and the energy absorption of the electromagnetic (ELM) by electrons. It has been shown [1] that anisotropy in the average kinetic energy of electrons (temperature anisotropy) provides a free energy that can drive quasi-static electromagnetic (QSELM) instabilities. The temperature anisotropy can be produced by the laser energy deposition [2] via inverse bremsstrahlung absorption mechanism. In current and near future experiments of inertial confinement fusion [3], the plasma temperature can reach values of about $10 - 15\ keV$ which corresponds to mildly relativistic plasmas. In this work, we presented an analysis of the QSELM instability in homogeneous relativistic plasmas heated by ELM waves. The relativistic effects are due to the high electron thermal energy which is no longer negligible with respect to the electron rest energy. In Section II we presented the kinetic model used in this work. Section III is devoted to the dispersion relation of QSELM waves and we give in a last section the discussion of the numerical results.

II Kinetic Model

Let us consider an homogeneous, unmagnetized relativistic plasma in presence of a high frequency (*hf*) electric field linearly polarized along *Ox*,

$$\vec{E}_h = Re(E_0 exp\, i\omega_0 t)\vec{e_x}. \tag{1}$$

The electron distribution function (EDF) is governed by the Fokker-Planck (FP) equation in the Lorentz approximation

$$\frac{\partial f}{\partial t} - e\vec{E}_h \cdot \frac{\partial f}{\partial \vec{p}} = C_{ei}(f) \tag{2}$$

where $f(\vec{p}, t)$ is the EDF, $\vec{p}$ is the relativistic momentum vector, e is the electron charge and C_{ei} the electron-ion collision operator given by [5]

$$C_{ei} = \frac{\varepsilon}{c^2} \frac{\nu}{p^3} \frac{\partial}{\partial p_i} \left(p_i p_j - p^2 \delta_{ij} \right) \frac{\partial f}{\partial p_j} \tag{3}$$

where $\varepsilon = \gamma m_e c^2$, $\gamma = \sqrt{1 + (p/m_e c)^2}$, $\nu = \frac{Z_i n_e e^4 L_c}{8\pi \varepsilon_0^2}$, m_e is the rest mass of the electron, Z_i is the ion charge number, n_e, is the electron density, L_c is the Coulomb logarithm and c is the speed of light in vacuum. We should note that the Coulomb logarithm deviates from the

classical expression, and typically its values ranges from 6 to 20. To solve Eq. (2), we use the high- and low-frequency splitting, i.e; $f = f_s + f_h$, where the subscripts s and h stand for low and high frequency. It readily results

$$\frac{\partial f_s}{\partial t} - e < \vec{E}_h \cdot \frac{\partial f_h}{\partial \vec{p}} > = C_{ei}(f_s) \tag{4}$$

$$\frac{\partial f_h}{\partial t} - e\vec{E}_h \cdot \frac{\partial f_s}{\partial \vec{p}} = C_{ei}(f_h). \tag{5}$$

The brackets in (4) indicate the average over the high-frequency field period $T = 2\pi/\omega_0$. The hf part f_h of the EDF oscillates at the electric field frequency ω_0 and it is expressed as $f_h = Re[f_h(p,\mu)\exp(i\omega_0 t)]$, where $\mu = p_x/p$. In addition for solving Eqs. (4) and (5), we expand f_s and f_h in the Legendre polynomials basis $P_l(\mu)$, i.e.; $f_s(p,\mu) = \sum_{l=0}^{\infty} P_l(\mu)f_{sl}(p)$ and $f_h(p,\mu) = \sum_{l=0}^{\infty} P_l(\mu)f_{hl}(p)$. To compute the components f_{sl} of the secular distribution function, first, we express from Eq. (4), f_h as a function of f_s with the use of the high-frequency approximation $\omega_0 \gg \nu_{ei}$ where $\nu_{ei} = \frac{\nu}{m_e^2 c^3}$ is the electron-ion collision frequency. Then, the secular FP equation (4) could be deduced by substituting the components f_{hl} into Eq. (5). We just give here the expression of the second anisotropic distribution function $f_{s,2}$ enough for the stability analysis of the electromagnetic modes

$$f_{s,2} = \frac{1}{9\sqrt{5}} \frac{v_0^2}{c^2} \frac{(\gamma^2-1)^{5/2}}{\gamma^2} \frac{\partial}{\partial\gamma} \frac{1}{(\gamma^2-1)^{3/2}} \frac{\partial f_{s,0}}{\partial\gamma} \tag{6}$$

where the isotropic EDf $f_{s,0}$ is the Maxwell-Boltzmann-Jüttner function [5] and $v_0 = \frac{eE_0}{m_e\omega_0}$ is the peak velocity of oscillation in the high-frequency electric field. To derive Eq. (6) we used the ordering $\frac{v_0^2}{c^2} \ll 1$.

II Dispersion relation

Let us now consider the stability analysis of small amplitude QSELM modes described by a wave vector $\vec{k}$ along the z-direction (polar axis), an electric field $\delta\vec{E}$ along x-direction and magnetic field $\delta\vec{B}$ along y-direction. The EDF is the sum of a perturbed EDF δf and a background EDF f_s, It results the following perturbed kinetic equation

$$\frac{\partial\delta f}{\partial t} + c^2 \frac{\vec{p}}{\varepsilon} \frac{\partial\delta f}{\partial\vec{r}} - e(\delta\vec{E} + \delta\vec{B}) \cdot \frac{\partial f_s}{\partial\vec{p}} = C_{ei}(\delta f). \tag{7}$$

Equation (7) have to be coupled to Faraday's equation, $\delta\vec{E} = -\omega\frac{\vec{k}}{k^2} \wedge \delta\vec{B}$ and to Ampere's equation $\vec{k} \wedge \delta\vec{B} = i\mu_e \int c^2 \frac{\vec{p}}{\varepsilon} \delta f d\vec{p}$. The derivation of dispersion relation is similar to that obtained in a previous work [6]. Following the same procedure, first we expand

the EDF in the spherical functions basis $\delta f = \sum_{l,m} \delta f_{l,m}(p) Y_{l,m}(\theta, \varphi)$ where θ and φ are the polar and the azimuthal angles of the momentum vector $\vec{p}$ such as $\cos\theta = p_x/p$. After some algebra we obtain the following expression for the growth rate of the QSELM,

$$\Gamma = -\frac{3K_2(z)}{z^2}\frac{k^2c^2}{\omega_p^2}\frac{\nu_{ei}}{\int_1^\infty \frac{(\gamma^2-1)^3}{\gamma^2} F_1 exp(-z\gamma)d\gamma} + \frac{1}{15}\frac{v_0^2}{c^2}\frac{k^2c^2}{\nu_{ei}}\frac{\int_1^\infty \left[3\frac{(\gamma^2-1)^{\frac{9}{2}}}{\gamma^5}+z\frac{(\gamma^2-1)^{\frac{11}{2}}}{\gamma^6}\right]F_1 F_2 exp(-z\gamma)d\gamma}{\int_1^\infty \frac{(\gamma^2-1)^3}{\gamma^2} F_1 exp(-z\gamma)d\gamma}.$$

(8)

We have used the notations, $z = m_e c^2/T$, and T is the electron temperature expressed in energy units, $K_2(z)$ is the modified Bessel function of the second kind, ω_p is the plasma frequency and F_l the continued fraction defined by the following recursive formula:

$$F_l = \left[l(l+1) + \frac{k^2c^2}{\nu_{ei}^2}\frac{(\gamma^2-1)^4}{\gamma^4}\frac{(l+1)^2-1}{4(l+1)^2-1}F_{l+1}\right]^{-1}.$$

(9)

We have solved numerically Eqs. (8) and (9) with standard numerical methods to calculate the integrals and the continued fractions and we summarize the results obtained in Figs. 1 and 2.
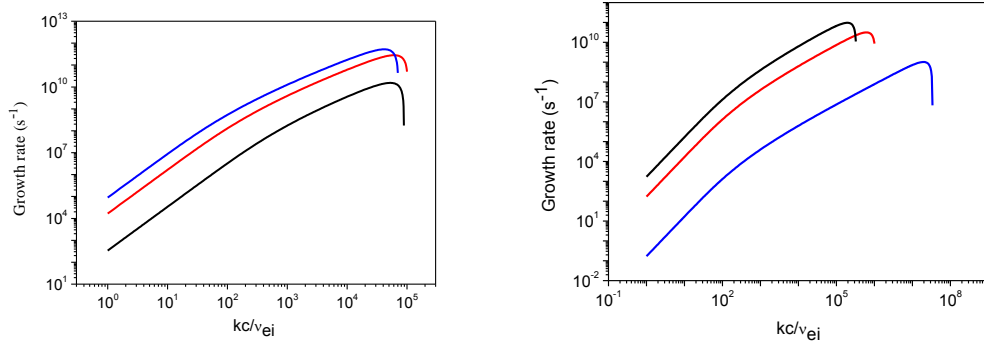

Fig. 1 Growth rate as a function of the collisionality parameter kc/ν_{ei}.

a) The blue curve for $I = 10^{15} W/cm^2$, $\lambda_L = 1.06 \mu m$, $T = 15\ keV$ and $n = 10^{21} cm^{-3}$.

b) The red curve for $I = 10^{15} W/cm^2$, $\lambda_L = 1.06 \mu m$, $T = 10\ keV$ and $n = 10^{21} cm^{-3}$.

c) The black curve for $I = 10^{14} W/cm^2$, $\lambda_L = 1.06 \mu m$, $T = 5\ keV$ and $n = 10^{21} cm^{-3}$.

Fig. 2 Growth rate as a function of the collisionality parameter kc/ν_{ei} for different plasma densities. The laser and plasma parameters are: $I = 10^{15} W/cm^2$, $\lambda_L = 1.06 \mu m$, $T = 10\ keV$. a) black curve for $n = 10^{20} cm^{-3}$; b) red curve for $n = 10^{19} cm^{-3}$; c) blue curve for $n = 10^{16} cm^{-3}$.

III Discussion and conclusion

In Fig. 1 we presented the growth rate for the ion charge number $Z_i = 15$ at the critical densities, and for different physical parameters corresponding to $z = 30$ (blue curve), $z = 50$ (red curve) and $z = 100$ (black curve). We have checked numerically that the

relativistic effects are no longer negligible in plasmas defined by parameters $z < 100$. We can see that the growth rate of the QSELM modes increases with increasing z. In particular, for the maximum growth rate, $\Gamma_{max}(z = 30) \approx 34\,\Gamma_{max}(z = 100)$ and $\Gamma_{max}(z = 50) \approx 17\,\Gamma_{max}(z = 100)$. This is due to the high laser intensities in the relativistic range which drive higher temperature anisotropy.

For the most unstable modes the optimum wavenumbers are included in the collisionless range. This can be explained by the efficiency of the electron-ion collisions as compared to the Landau damping as stabilizing mechanisms. In Fig. 2 we consider the physical situation where the ELM wave propagates in underdense plasmas. We can see that the growth rates decrease drastically with decreasing electron densities. In particular for the critical density $n_c = 10^{21} cm^{-3}$ and the underdense density $n_c = 10^{16} cm^{-3}$, the growth rate decreases by two order of magnitude. This is due to the absorption of the laser wave by free electrons through the inverse bremsstrahlung mechanism, since the collisional absorption coefficient is proportional to the electron density.

In this work a new dispersion relation of the QSELM modes valid in the whole collisionality range is established taking into account the relativistic effects. An explicit expression of the second anisotropy (temperature anisotropy) of the electron distribution function is also derived.

The stability analysis of QSELM modes in mildly relativistic plasmas relevant in near future experiments of inertial confinement fusion is presented. It is found that high growth rates are driven by the temperature anisotropy induced by the collisional absorption in the privileged high-frequency electric field direction. These strong magnetic fields are collisionless and thus their scale lengths are smaller than the electron mean free path. These fields could prevent the heating of the inner layer of the target by inhibiting the thermal transport of the laser energy deposited near the critical surface.

References

[1] E. S. Weibel , Phys. Rev. Lett. **2**, 83 (1959).

[2] A. Bendib, K. Bendib, and A. Sid, Phys. Rev. E **55**, 7522 (1997).

[3] A. Bers, I. P. Shkarofsky, and M. Shoucri, Phys. Plasmas 16, 022104 (2009).

[4] C. F. F. Karney, Computer Physics Reports 4 183 (1986).

[5] F. Jüttner, Ann. Phys. **34**, 856 (1911).

[6] A. Bendib and J. F. Luciani, Phys. Fluids 30, 1353 (1987).