Sensitivity of Thomson Scattering measurements on electron distribution modelling in low density RFP Plasmas

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Background

Thomson Scattering proved to be one of the most reliable diagnostics for electron temperature measurements in fusion plasmas. In RFX-mod RFP plasmas the collisionality value is generally sufficient to ensure that electrons population is well described by a Maxwell Boltzmann (MB) distribution; on the other hand, features in Electron Temperature T_e profiles suggest that non thermal population can be detectable in low density datasets. The work presented hereafter describes these signatures can be identified and quantified, by using Information Geometry methods and data analysis based on Bayesyan Inference.

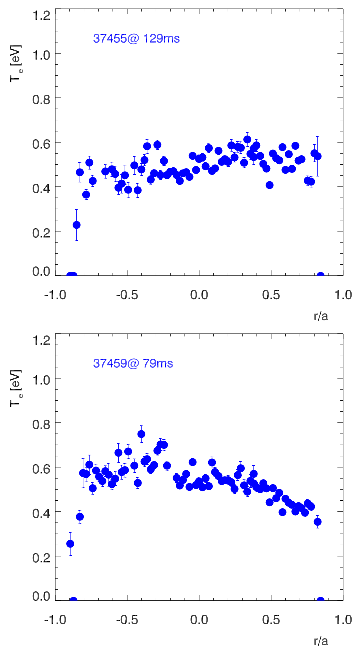

Figure 1: *Example of Two TS profiles obtained in low collisionality regime, and opposite B_t directions; the profile sloping cannot be related to the presence of tearing modes thermal structures*

Datasets are provided by the RFX-mod Main Thomson Scattering [1], which samples T_e profile once every 10 ms on the machine equator, with a spatial resolution of $\approx$ 11mm. As an example, two T_e profiles are shown in fig 1, obtained in low density conditions and with opposed direction of poloidal current. The two profiles features opposite slopes in the core, due likely to asymmetries in the electron population; the asymmetry is sistematical and hence not related to the presence of QSH-like magnetic structures [2]. In this regard, it is worth to note that the TS diagnostic collects only the blue half of the scattered spectrum, i.e. the diagnostics samples only the half distribution of electrons moving *towards* the TS collection optics. At the same time, the Doppler shifting due to bulk electron current is too low to produce such effect, being the current-dependent electron group velocity about 3 orders of magnitude lower than their thermal velocity. It is worth to note that the Dreicer electric field $E_D = n_e^2 ln\Lambda/4\pi\varepsilon_0 T_e$ [3, 4] in these cases is around 4 V/m, about twice the toroidal electric field. The poloidal field is not measured but it is of the same order of magnitude of the toroidal one; runaway electron generation in these conditions is marginal but not negligible.

Distribution modeling

Non-thermal contributions to eletron distribution function f_e are not *a priori* known; in line of principle one can assume f_e to be:

a) A parametric distribution $f_e(T_e, k)$, assuming the form of standard MB distribution for a given value of the parameter. A generalized Lorenz-Cauchy distribution is a good candidate in this distribution class, also because a semi analytical expression of its scattered spectrum has yet been evaluated [6], and it converges to MB distribution for given parameter values[1].

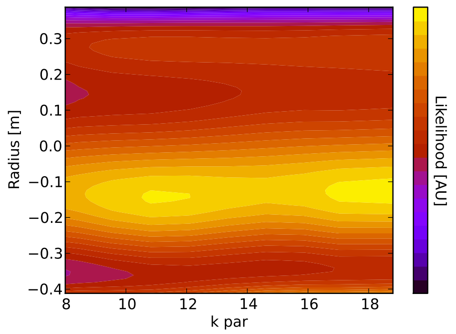

Figure 2: *Example of smoothed contour plot of likelihood as a function of radial position in the profile and κ value for a generalized Lorentz-Cauchy distribution 1; the peak at low κ values is an indication of preference for non-MB distributions*

b) The sum of a standard MB population and and a suprathermal population $f_e(k_{1,..,n})$, the latter parametrized by different group velocity, temperature, "shape" factor and so on. Such approach requires the full numerical evaluation on scatterd spectrum, following [5].

Bayes Probability Theory (BPT) is the best framework when trying to fit a parametric model $M(k)$ on a set of experimental data D. The classical expression of Bayes Probability Theorem express the probability P_o for the system to be properly described as:

$$P_o\left(M(k)|D\right) = \frac{L(D|M(k)) \cdot P_i(M(k))}{P_i(D)}$$

in terms of the *a priori* likelihood L of a model $M(k)$ *conditioned* the existence od the dataset D.

In the case of Thomson Scattering analysis, $M(k)$ can be described by the expected signal $\Gamma_j(k,T)$, in the j spectral channel for a given electron distribution $f_e(k,T)$. The ratio k of Γ_j/s_j over a signal set $\{s_j\}$ should be a constant l for a given temperature: the overall likelihood $L(M(k,T))$ over temperatures becomes the product of the ratios over channel index j: $L =$

1

It can be written in the form:

$$f_L(p) = \left[B\left(\frac{5}{2}, \kappa-3\right) {}_2F_1\left(\frac{5}{2}, -\frac{3}{2}; \kappa-\frac{1}{2}; \frac{4\alpha-\kappa}{4\alpha}\right)\right]^{-1} \cdot \frac{3\alpha^{3/2}}{4\pi(m_e c)^3 \kappa^{5/2}} \left(1+\frac{2\alpha(\gamma-1)}{\kappa}\right)^{-\kappa} \quad (1)$$

where the parameter $\alpha = m_e c^2/(2T_e)$ is related to the bulk electron temperature, B is the β funtions and F the Gauss hypergeometric function. γ is the Lorentz factor and κ a free parameter, upon which the analysis is iterated. In the limit $\kappa \to \infty$ the function becomes a MB distribution.

$\Pi_i P(\Gamma_j(k,T)/s_j = l)$. Since the Γ_j set describes probability envelopes rather than exact values, Γ_j/s_j are treated as 2-dimensional maps.

Parameter space definition

To be fully consistent, parametrization of models $M(k) \rightarrow f_e(T,k)$ should cover *uniformely* the functional spaces F of distributions $f_e(T,k)$ rathter than the numeric parameter space. This uniformity requires the definition of a distance in the space F, which can be identified with the Fisher-Rao metric [8, 7]:

$$ds_{k,k'}(f) = \int_v d(log(f(v)))/dk \cdot d(log(f(v)))/dk' \cdot f(v)dv$$

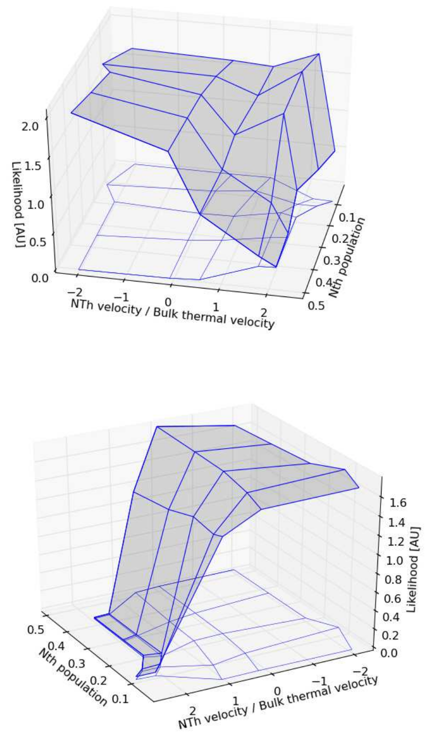

Figure 3: *Up, likelihood of a parametric model consisting in a sum of two MB distributions; Down, standard profile showing a convergence to a single MB contribution*

Grid relaxation in the Fisher distance is obtained numerically [9]. Geodesics equations are applied iteratively between grid boundary points: grid relaxation should follow geodesics path if $ds_{k,k'}$ is non diagonal or in any case if $Dim_{ds_{k,k'}} \leq 2$.

Incrementing the number of parameters is non trivial, being often associated to divergences developping in the Fisher-Rao grid steps lenghts; this problem can be reduced with a proper choiche of parameter starting range. Last, the measurement degrees of freedom (DOF) correspond to the TS spectral channels and are limited to four; the reasonable number of parameters is thus limited to 2 plus T_e and n_e

Application to RFX-mod T_e measurements

As a first step, the distribution has been assumed in the form 1 to exploit the availability of a semi-analitical expression for the scatterd spectrum. The calculated likelihood of the model, once averaged on the T_e profile, displays often (fig 2) a maxium also for low κ values, which is an indication of the presence of non MB e^- populations.

This qualitative approach can be refined once the spectrum from arbitrary parametric distribution is calculated; fig. 3 shows, as an example, the likelihood of a two MB sum as a function of the two distributions relative weight and average energy. The upper plot refers to a core point in a low density profile, while the lower one is obtained at higher collisionality. In the limits due to low grid resolution, likelihood is higher when also hot e^- populations are included: a denser grid should be evaluated in order to highlight the effect.

The variable-distribution method proved to be quite robust returnign T_e values also for low signal-to-noise points; it generally tends to return sligthly higher bulk T_e values and to suppress the T_e profile sloping in the core (depending also on the symmetry/aymmetry of distribution class which are modeled). Nonetheless it introduces a higher point dispersion (fig. 4) and higher errors, which could be expected as a result of the increased DOF of the analysis; in this sense it is not recommended for standard profile processing.

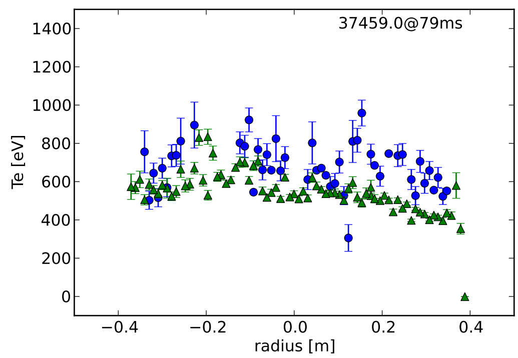

Figure 4: *Example of processed profile; in green, standard data processing. Blue: parametric distribution analysis. The T_e value for blue points refers to the temperature of bulk electrons only*

Conclusions

RFX-mod TS data display little but non-negligible evidences of non-thermal e^- populations. In particular, sloping of temperature profiles in plasma core (where they are expected to flatten) can be definitely related to electron populations emerging in low-collisionality plasmas; such sloping is lowered once the profile is processed with the method here discussed. The method is quite robust, but requires high computation time in the calculation of scattered spectra for generalized distributions.

Further work is required, testing different parametrizati on, increasing datasets and and improving the parameter space coverage

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