Benchmarking the generalized Rutherford equation with reduced MHD simulations

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Introduction

The nonlinear growth of neoclassical tearing modes (NTMs) in tokamaks is commonly discussed in the framework of the generalized Rutherford equation (GRE) [1, 2]. We perform a theoretical / numerical validation of the GRE by means of numerical simulations implementing the set of 2D reduced MHD equations for the helical magnetic flux ψ and the potential φ [3]. The code uses finite differences in the radial direction and a Fourier decomposition in the periodic poloidal direction. This choice of numerical method allows radial boundary conditions for the flux to be set by the step in the logarithmic derivatives over the simulated radial domain $[-L:+L]$ of each Fourier component k in accordance with the tearing stability parameters $\Delta'_{k,\mathrm{BC}}$: i.e. $\psi'_k(\pm L)/\psi_k(\pm L) = \pm 0.5\Delta'_{k,\mathrm{BC}}$. The corresponding boundary condition for the dominant Fourier harmonic of the potential is obtained in accordance with linear ideal MHD, which should be valid outside the island region. The code focusses on the nonlinear dynamics in the narrow layer in the poloidal plane of a tokamak around the resonant surface r_s including the magnetic island. In this layer the dynamics are expected to be well approximated by the 2D reduced MHD equations (se Chapter 2.4 of [4]). The equilibrium helical flux is represented by its Taylor series around the resonant surface, $\psi_\mathrm{eq}(x) = \sum_{n\geq 2}(x^n/n!)\psi_\mathrm{eq}^{(n)}$, where $x = r - r_\mathrm{s}$. When only the leading order $n = 2$ term is taken into account, the code reproduces both the linear and the nonlinear Rutherford phase in close correspondence to the theoretical expectations [3]. In this contribution we analyze the nonlinear saturation of a classical tearing mode, and the growth and suppression by electron cyclotron current drive (ECCD) of a neoclassical tearing mode (NTM).

Saturation of a classical tearing mode

When a fourth order term is included in the Taylor expansion of the equilibrium helical flux, $\psi_\mathrm{eq}(x) = \frac{1}{2}x^2\psi_\mathrm{eq}^{(2)} + \frac{1}{24}x^4\psi_\mathrm{eq}^{(4)}$, the linear tearing stability index Δ'_0 is no-longer determined solely by the boundary condition, but obtains a contribution from the fourth order term:

$$\Delta'_{k,0} = \Delta'_{k,\mathrm{BC}} - 2L\frac{\psi_\mathrm{eq}^{(4)}}{\psi_\mathrm{eq}^{(2)}} \qquad (1)$$

where L is the radial half width of the simulation box. As a result, the mode can be unstable even when the boundary condition specifies $\Delta'_{k,\mathrm{BC}} < 0$. This allows to study the nonlinear saturation of a classical tearing mode. Escande and Ottaviani have shown that in this case the Rutherford equation becomes

$$g_1 \frac{\mathrm{d}w}{\mathrm{d}t} = \eta \left(\Delta'_0 + \alpha w\right) \tag{2}$$

where $g_1 \equiv 0.82$, and $\alpha \equiv 0.41 \psi_{\mathrm{eq}}^{(4)}/\psi_{\mathrm{eq}}^{(2)}$.

Figure 1 shows the results of a calculation with our 2D reduced MHD code for the following parameters: equilibrium helical flux $\psi_{\mathrm{eq}}^{(2)} = -5 \times 10^5\ \mathrm{s}^{-1}$ and $\psi_{\mathrm{eq}}^{(4)} = 1.2 \times 10^8\ \mathrm{m}^{-2}\mathrm{s}^{-1}$, resistivity $\eta = 0.01\ \mathrm{m}^2/\mathrm{s}$, viscosity $\nu = 5 \times 10^{-8}\ \mathrm{m}^2/\mathrm{s}$, poloidal scale length $k_y = 1\ \mathrm{m}^{-1}$, and radial half width of the simulation box $L = 0.02$ m. The boundary condition for the dominant mode is given by $\Delta'_{k=1,\mathrm{BC}} = -6.6\,\mathrm{m}^{-1}$. With these parameters the Alvén time $\tau_\mathrm{A} = 1/|\psi_{\mathrm{eq}}^{(2)}| = 2 \times 10^{-6}$ s and the resistive time $\tau_\mathrm{r} = 1/k_y^2\eta = 10^2$ s, giving a Lundquist number of $S \equiv \tau_\mathrm{r}/\tau_\mathrm{A} = 5 \times 10^7$. The linear growth of the mode in the simulations is measured at 14.3 s^{-1}, which is consistent with the theoretical value of $\gamma = 0.55(\Delta'_0)^{4/5}\eta^{3/5}(k\psi_{\mathrm{eq}}^{(2)})^{2/5} = 15.7\ \mathrm{s}^{-1}$. The nonlinear growth and saturation of the mode are described well by Eq. (2). In particular the saturated island size of 3.1 cm obtained from the code corresponds well to the predicted saturated island size of $w_{\mathrm{sat}} = -\Delta'_0/\alpha = 3.0$ cm.

Growth and suppression of a neoclassical tearing mode [6]

Noninductive current perturbations lead to modification of Ohm's law and thereby affect the dynamics of tearing modes. One such current perturbation is the annihilation of the neoclassical bootstrap current density J_{bs} inside the magnetic island, which is responsible for destabilization of neoclassical tearing modes (NTMs). Another contribution comes from the ECCD that is applied for the suppression of NTMs. In this case the Rutherford equation is generalized to [7]

$$g_1 \frac{\mathrm{d}w}{\mathrm{d}t} = \eta \left(\Delta'_0 + \Delta'_{\mathrm{bs}} + \Delta'_{\mathrm{ECCD}}\right), \tag{3}$$

where the last two terms on the right hand side represent the effect due to the missing bootstrap current and the ECCD, respectively.

We performed simulations of NTM growth and suppression by ECCD, which were reported previously in [6]. The plasma parameters were identical to those given above except that $\psi_{\mathrm{eq}}^{(4)} = 0$ and $\Delta'_{k=1,\mathrm{BC}} = -1\ \mathrm{m}^{-1}$. An NTM is triggered at a finite island size of 0.5 cm. The perturbation to the bootstrap current inside the island is taken to be $\delta j_{\mathrm{bs}} = -6630\ \mathrm{s}^{-1}$. Note that the model assumes that the bootstrap current is annihilated over the entire island, and consequently does not model the partial annihilation expected for small island sizes [8]. When the

NTM reaches an island size of 3 cm, ECCD is switched on with a maximum driven current density of $J_{\rm cd} = 15000\ {\rm s}^{-1}$ and centered exactly at the resonant surface $x_{\rm cd} = 0$ with a Gaussian profile width of $w_{\rm cd} = 1$ cm. Two cases are simulated: one for CW ECCD and the other for modulated ECCD with a duty cycle of 50% centered around the O-point phase of the magnetic island. With Δ'_0 given by the boundary condition, the initial growth phase of the mode allows us to benchmark the bootstrap term $\Delta'_{\rm bs}$ in the GRE, while to second phase with ECCD is then used to benchmark the $\Delta'_{\rm ECCD}$ term. In figure 2 we compare the results of the simulations with analytical expressions for these terms given by [9, 10]. Excellent agreement is found between the 2D code simulations and the analytical predictions of the GRE. In the literature sometimes an additional term $\delta\Delta'(J_{\rm ECCD})$ is added to the Rutherford equation in order to describe the effect of the ECCD on the equilibrium current density and thereby on the mode stability. As shown by [6] this effect is already encompassed in $\Delta'_{\rm ECCD}$. Figure 2 also shows the results of calculations in which only the $k = 0$ or $k = 1$ components of the current density perturbations are taken into account. Whereas $\Delta'_{\rm bs}$ and $\Delta'_{\rm ECCD}$ are generally believed to represent the effect of only the helical $k = 1$ component of the current perturbation, these results show that $\Delta'_{\rm bs}$ and $\Delta'_{\rm ECCD}$ also include the effect of the poloidally averaged $k = 0$ component of the current perturbation. Except for the modulated ECCD case, the $k = 0$ component is even seen to be responsible for the dominant effect.

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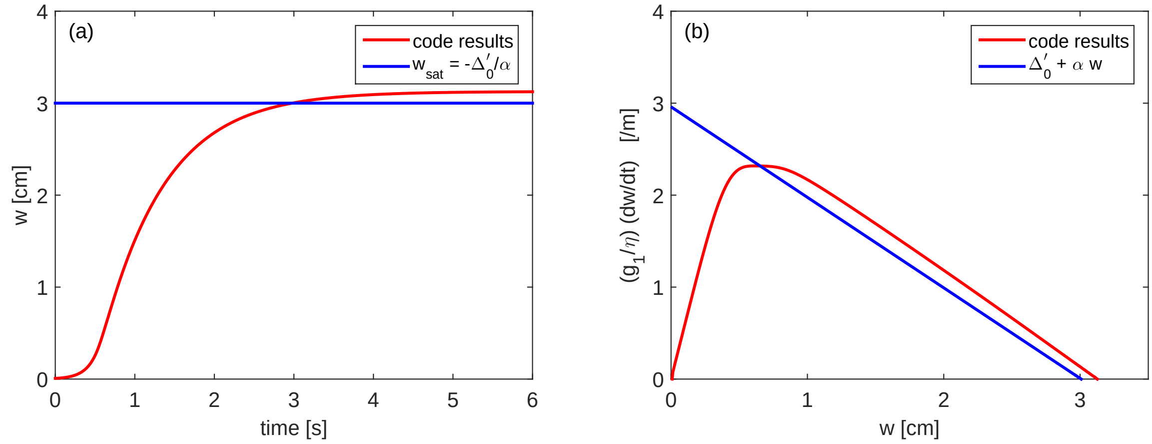

Figure 1: Growth and saturation of a classical tearing mode. The parameters are given in the main text. (a) The island width as a function of time. The blue line indicates the predicted saturated island size according to [5]. (b) The normalized island growth as a function of island width. The blue curve indicates the nonlinear growth according to equation (2).

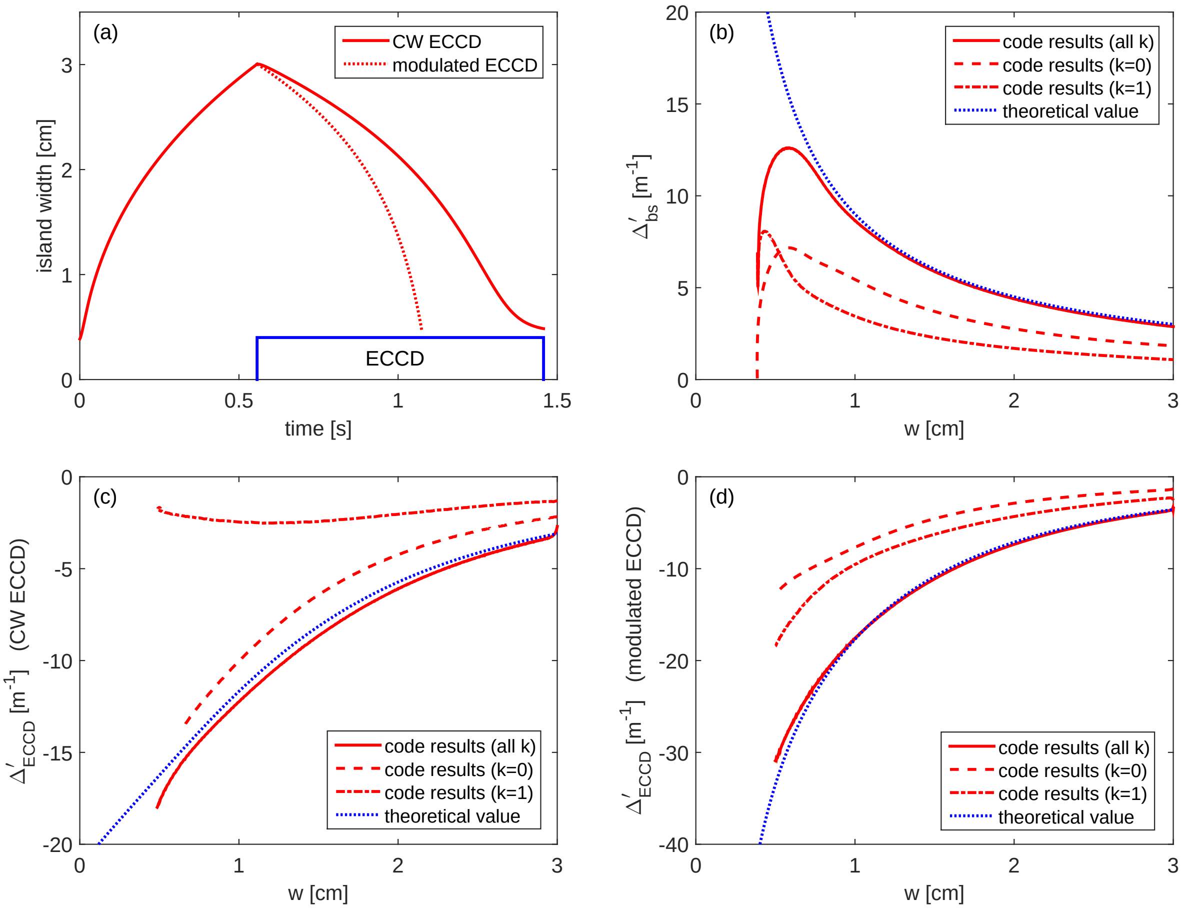

Figure 2: Growth and suppression by ECCD of an NTM. The parameters are given in the main text. (a) The island width as a function of time. (b) $\Delta'_{\rm bs}$ as a function of island width. (c) and (d) $\Delta'_{\rm ECCD}$ as a function of island width for CW and modulated ECCD, respectively. The dotted blue curves in (b,c,d) indicate the theoretical expectations according to [9, 10].