# Scattering by spherical blobs in plasma: a discrete eigenfunction approach (*)

G. P. Zouros[1], G. C. Kokkorakis[1] K. Hizanidis[1], S.-I. Valvis[1], J. A. Roumeliotis[1]
[1]*National Technical University of Athens, Athens, Greece*
A. K. Ram[2]
[2]*Plasma Science and Fusion Center, MIT, Cambridge MA, USA*

The scattering of radio frequency electromagnetic waves, propagating inside a background cold plasma, by a spherical blob, is investigated. A different approach based on the expansion of the electric field and the displacement vector in terms of discrete sets of eigenvectors of different values of wavevector, is used [1]. The expansion functions and the related eigenvalues are calculated through an eigenvalue equation determined by the permittivity tensor. Such different sets of vectors are used to describe the induced field inside the blob, as well as the scattered field in the ambient plasma. The polarization of the incident field should be consistent with the constitutive relations of the plasma. This method is an alternative to Fourier expansion technique, which is a continuous spectrum approach, that has been developed by some of the authors recently [2]. This different method provides a new way of describing the various phenomena and fluctuations that affect the uniformity of power flow into the plasma.

*(a) Introduction*

In fusion plasmas, radio frequency waves propagate from the excitation structures to the core of the plasma in a region characterized by large density fluctuations and blobs. So, the traditional geometric optics approximation for the scattering process—used for small fluctuations of up to 10% of the background density—is not valid. A full wave treatment is necessary, based on the solution of the vector Helmholtz equation (VHS) both inside the blob and for the scattered fields outside the blob.

Although the background plasma space is finite in extent, it is practical to consider this subpart of the space as infinite, as far as its interaction with the blob is concerned. Such a procedure, valid for static fields only, has been developed in the literature [3]. Now, in the context of VHS, this simplifying assumption implies the calculation of wave solutions in an infinite anisotropic space. The scattered fields should also fulfill the radiation condition at infinity for the magnetoplasma medium [4].

In the past, and recently by some of the authors [2], such a method has been constructed via the use of the vector Fourier transform. The unknown fields are expanded in vector Fourier integrals. The expansion variable spans in a continuous manner all vector Fourier space. So, from the dispersion relation turns out a variety of waves, both propagating and evanescent, that have to be taken into account to satisfy the boundary conditions. Despite its mathematical complexity, the physical context of the approach is imminent.

Here, an alternative approach stemming from a discrete eigenvalue expansion of the solution of the VHS, in an infinite anisotropic space, is presented. The eigenvalue equation does not involve in any way the radial part of the vector wave functions, so its validity for constructing solutions for the outgoing (radiative) function is unquestionable. Although this new method possesses less physical insight, the calculation of the discrete sums is more easily performed in the computer, thus allowing accurate and stabilized results, as well as the study of fluctuations that affect the uniformity of power flow into the plasma.

*(b) The method*

Let us define the blob tensor as

$$\overleftrightarrow{\mathbf{K}}^B = \begin{pmatrix} K_\perp^B & -jK_\times^B & 0 \\ jK_\times^B & K_\perp^B & 0 \\ 0 & 0 & K_\parallel^B \end{pmatrix} \tag{1}$$

and the ambient space tensor as

$$\overleftrightarrow{\mathbf{K}}^P = \begin{pmatrix} K_\perp^P & -jK_\times^P & 0 \\ jK_\times^P & K_\perp^P & 0 \\ 0 & 0 & K_\parallel^P \end{pmatrix} \tag{2}$$

Inside the spherical blob, the electric field is expressed as [1]

$$\mathbf{E}^B = -j\sum_{n,m}\bar{E}_{mn}\sum_{l}\alpha_l\left[c_{mnl}^B\mathbf{M}_{mn}^{(1)}(k_l^B,\mathbf{r}) + d_{mnl}^B\mathbf{N}_{mn}^{(1)}(k_l^B,\mathbf{r}) + \frac{\bar{w}_{mnl}^B}{\lambda_l^B}\mathbf{L}_{mn}^{(1)}(k_l^B,\mathbf{r})\right] \tag{3}$$

and the magnetic field as [1]

$$\mathbf{H}^B = -\sum_{n,m} \bar{E}_{mn} \sum_l \alpha_l \frac{k_l^B}{\mu_s^B \omega} \left[ d_{mnl}^B \mathbf{M}_{mn}^{(1)}(k_l^B, \mathbf{r}) + c_{mnl}^B \mathbf{N}_{mn}^{(1)}(k_l^B, \mathbf{r}) \right] \tag{4}$$

In (3) and (4), $\lambda_l^B$ are the eigenvalues and $\left[ d_{mnl}^B \quad c_{mnl}^B \right]^T$ are the eigenvectors of the eigenvalue problem

$$\begin{bmatrix} \bar{E} & \tilde{E} \\ \bar{G} & \tilde{G} \end{bmatrix} \begin{bmatrix} d_{mnl}^B \\ c_{mnl}^B \end{bmatrix} = \lambda_l^B \begin{bmatrix} d_{mnl}^B \\ c_{mnl}^B \end{bmatrix},$$

$$\bar{E} = \frac{\bar{E}_{uv}}{\bar{E}_{mn}} \bar{e}_{mn}^{uv}, \quad \tilde{E} = \frac{\bar{E}_{uv}}{\bar{E}_{mn}} \tilde{e}_{mn}^{uv}, \quad \bar{G} = \frac{\bar{E}_{uv}}{\bar{E}_{mn}} \bar{g}_{mn}^{uv}, \quad \tilde{G} = \frac{\bar{E}_{uv}}{\bar{E}_{mn}} \tilde{g}_{mn}^{uv} \tag{5}$$

where $\bar{e}_{mn}^{uv}$, $\tilde{e}_{mn}^{uv}$, $\bar{g}_{mn}^{uv}$, $\tilde{g}_{mn}^{uv}$ are given by (10) of [1] and relate the elements of the permittivity tensor of the blob, i.e., are functions of $K_\perp^B$, $K_\times^B$, $K_\parallel^B$.

The scattered fields inside the ambient plasma region are again given by (3), (4), where "*B*" is replaced by "*P*", while $\alpha_l$ is replaced by $\beta_l$ and the wavefunctions of the third kind are employed in place of the wavefunctions of the first kind. Therefore, another eigenvalue problem should be solved which depends on the elements of the permittivity tensor of the ambient space. This means that now $\bar{e}_{mn}^{uv}$, $\tilde{e}_{mn}^{uv}$, $\bar{g}_{mn}^{uv}$, $\tilde{g}_{mn}^{uv}$ in (5) are functions of $K_\perp^P$, $K_\times^P$, $K_\parallel^P$.

Once the incoming plane wave is set -see (17) of [1]- the boundary conditions are satisfied and a linear system of equations is formed from which the expansion coefficients $\alpha_l$ and $\beta_l$ are calculated. This procedure concludes our solver, and the time-average Poynting vector can be easily computed by (3), (4), and the corresponding applicable ones in the background plasma space.

*(c) Results*

Following the above method, we present some results for the distribution of power flux in the near region outside the blob. We set the ambient space parameters as $K_\perp^P = 0.9481\varepsilon_0$, $K_\times^P = -0.0353\varepsilon_0$, $K_\parallel^P = 0.9721\varepsilon_0$, where $\varepsilon_0$ is the free space permittivity, and the blob parameters as $K_\perp^B = 0.9377\varepsilon_0$, $K_\times^B = -0.0423\varepsilon_0$, $K_\parallel^B = 0.9665\varepsilon_0$. Using the ordinary wave as stimulation, the corresponding incident electric field is impinging at an angle $\theta_0 = 88.9°$ with respect to the $z$-axis, and at an angle $\varphi_0 = 0$ with respect to the $x$-axis. The operating frequency

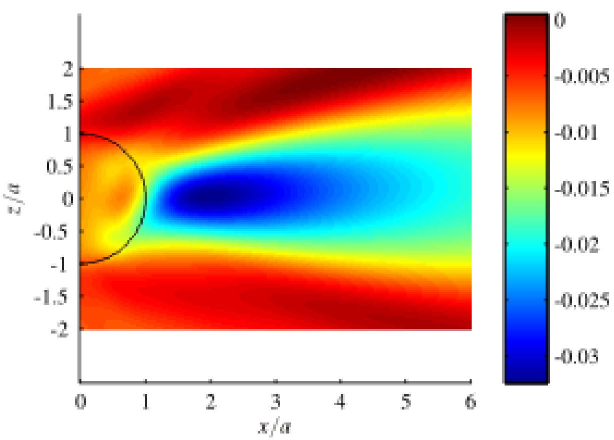


Figure 1: Normalized time-average power flux due to scattering by a blob with $a = 0.2$ cm.

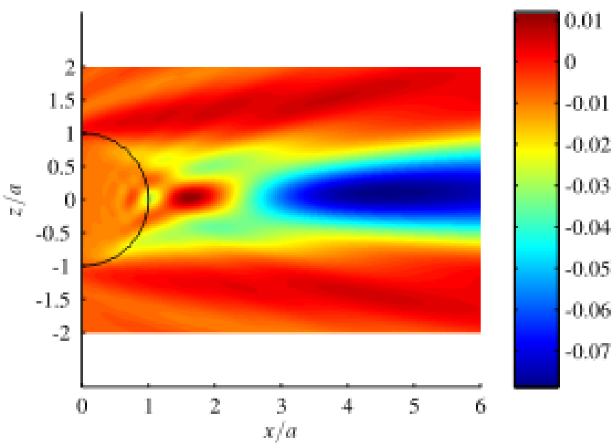


Figure 2: Normalized time-average power flux due to scattering by a blob with $a = 0.5$ cm.

is set at $f = 170$ GHz. In Figs. 1 and 2 we plot the absolute value of the normalized time-average power flux $\log\left|\langle \mathbf{S}_N \rangle\right|$ on the $xz$-plane, for two different blob radii, i.e., for $a = 0.2$ cm and for $a = 0.5$ cm. The logarithm is used to reveal the fluctuations around the blob. The plots are confined in the region of positive $x$, while the backscattered region is not plotted. The effect of the blob is to induce a coupling of the incoming ordinary wave to the ordinary and extraordinary waves travelling in all directions. Obviously, the scattering is more pronounced for bigger blobs. In both cases depicted, a shadow area appears in the forward scattering region.

(*)Supported in part by the Hellenic National Programme on Controlled Thermonuclear Fusion associated with the EUROfusion Consortium

[1] Z. Lin and S. T. Chui, Phys. Rev. E 69, 056614, 2004
[2] A. K. Ram, K. Hizanidis and Y. Kominis, Phys. Plasmas 20, 056110, 2013
[3] A. Sihvola, Electromagn. 17, 269-286, 1997
[4] S. R. Seshadri and T. T. Wu, Quart. J. Mech. Appl. Math. XXIII, 285-313, 1970