Nonlinear self-consistent kinetic simulations of the anomalous Doppler instability of suprathermal electron populations in fusion plasmas

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1. Introduction

The anomalous Doppler instability (ADI) is a key relaxation mechanism for suprathermal electron populations in magnetic confinement fusion (MCF) plasmas. The underlying physics of the ADI [1–12] involves a shift of energy from parallel to perpendicular electron motion, accompanied by the excitation of waves at frequency and wavenumber $(\omega, \boldsymbol{k})$ satisfying the anomalous Doppler resonance condition $\omega - \boldsymbol{k} \cdot \boldsymbol{v} = -\Omega_{ce}$, where Ω_{ce} is the electron cyclotron frequency for the magnetic field $\mathbf{B}$, and $\boldsymbol{v}$ is electron velocity. The approximation $\omega \approx \omega_{pe} \cos\theta$ holds for the electrostatic waves excited in the limit $\omega_{pe} \ll \Omega_{ce}$, where θ is the angle between $\boldsymbol{k}$ and $\mathbf{B}$ and ω_{pe} is the electron plasma frequency. Importantly, the ADI can take place even for monotonically decreasing extended tails in the probability distribution of the component $v_{\|}$ of $\boldsymbol{v}$ parallel to $\mathbf{B}$. The ADI has been observed under a broad range of plasma conditions in tokamaks [13–16]. Its significance is becoming increasingly apparent as diagnostics with very high time resolution are used in combination with first principles numerical simulations that exploit high performance computation. For example, the ADI has recently been found to play a role in the fast timescale electron physics of ELMs in MAST [16]. In the present paper we use 1D3V particle-in-cell code (PIC) simulations to extend recent computational studies of the ADI [11, 12] by testing, for the first time, two long-standing analytically-based conjectures. First, we investigate the role of the additional normal modes that are supported by the non-Maxwellian electron distribution. As noted by Nezlin [3], these additional modes may include a negative-energy branch. This led to the 1984 analytically-based conjecture by Dendy and Lashmore-Davies [8], who noted that this effect may persist even for monotonically decreasing electron tail distributions, so that a wave-wave resonant variant of the ADI may operate in parallel with – and potentially counteract – the wave-particle resonant ADI under tokamak plasma conditions. Second, we consider the 1977 analytically-based conjecture by Molvig, Tekula and Bers [5] that the ADI-driven diffusion of electrons in velocity space may give rise to a distribution with a small positive slope at high energies, such that waves with $\omega \approx \omega_{pe}$ are consequently excited, in addition to the waves with $\omega \approx \omega_{pe} \cos\theta$ that are excited by anomalous Doppler wave-particle resonance. Testing the 1984 [8] and 1977 [5] conjectures has had to await the development of first principles, fully self-consistent simulation codes [11,12] that can follow the spatiotemporal evolution of particles and fields deep into the nonlinear regime of the ADI.

2. Computational model and simulation results

In the 1D3V PIC simulations reported here, the full gyro-orbit electron dynamics, together with all three vector components of $\mathbf{E}(\mathbf{x}, \mathbf{t})$ and $\mathbf{B}(\mathbf{x}, \mathbf{t})$, evolve self-consistently and nonlinearly under the relativistic Lorentz force law and Maxwell equations [17]. The methodology is described in detail in Refs. [11, 12]. Figure 1 shows snapshots in time of the evolving probability density function (pdf) of electron velocity in the (v_x, v_y) plane, $f(v_x, v_y)$, calculated from macroparticle dynamics, for a distribution initialised with a 6% flat suprathermal tail oriented parallel to the magnetic field direction and extending up to $15v_{th}$, where v_{th} is the thermal velocity of the bulk Maxwellian population. The simulation domain is aligned at $\theta = 35°$ from the magnetic field hence components of $\boldsymbol{k}$ at angle θ from the magnetic field are resolved. Electron number density and magnetic field are set to $n_e = 4 \times 10^{19} m^{-3}$ and $B = 2$T, so that $\omega_{pe}/\Omega_{ce} = 0.9$ in this simulation. The ADI manifests as a "fan instability": the most energetic electrons are transported, in a broadly diffusive [12] manner, to higher $v_\perp$ and lower $v_\parallel$ under the action of the wave-particle resonant waves that they excite. These waves contribute to the intense feature,

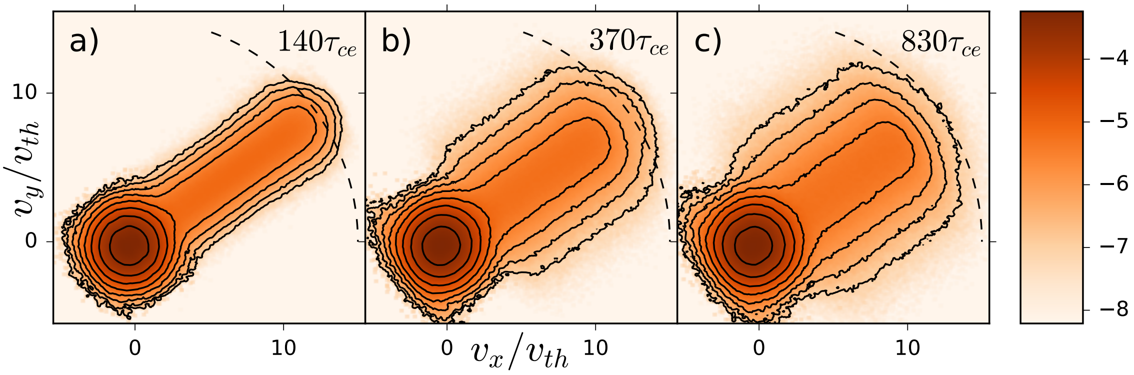

Figure 1: Snapshots in time of the evolving pdf of electron velocity in the (v_x, v_y) plane, $f(v_x, v_y)$, calculated from macroparticle dynamics, for a distribution initialised with a 6% flat suprathermal tail oriented parallel to the magnetic field direction and at 35° from the simulation domain, extending up to $15v_{th}$. Times are $t/\tau_{ce} = 140$ where $\tau_{ce} = 2\pi/\Omega_{ce}$ for panel (a); 370 (b); and 830 (c).

marked A, in Fig.2, which plots the spatiotemporal Fourier transform of the amplitude of the z component of the **E** field at times corresponding to the snapshots in Fig.1. At higher k on this bulk-supported electrostatic wave branch, the later panels in Fig.2 show a growing feature, marked B, at its intersection with the line $\omega - k\cos\theta v_b = -\Omega_{ce}$, where in general $v_b(t) = \int 2\pi v_\parallel v_\perp f_b(v_\perp, v_\parallel, t) dv_\perp dv_\parallel$ is the instantaneous mean parallel velocity of the suprathermal (i.e., initially non-Maxwellian) electron population $f_b(v_\perp, v_\parallel, t)$, computed from the simulation.

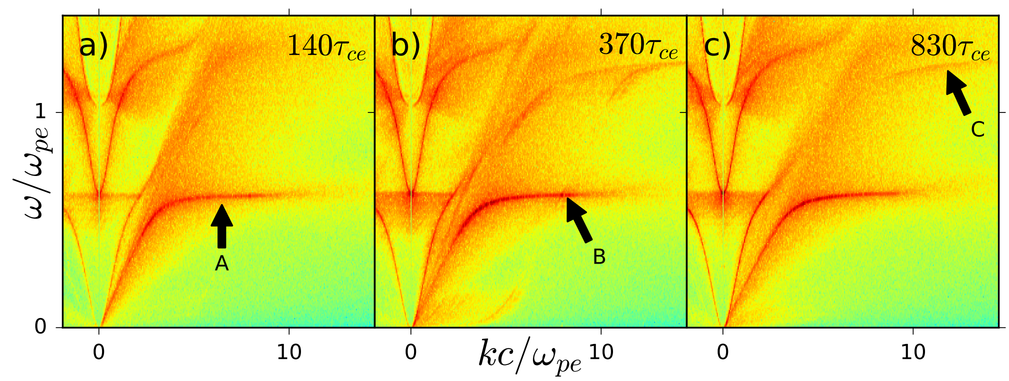

Figure 2: Snapshots in time of the spatiotemporal Fourier transform of the z component of the E field in the simulation. Panels (a) to (c) correspond in time to those of Fig.1.

This growing feature is the wave-wave resonant ADI anticipated in Ref. [8], and Fig.3 shows an expanded plot of this region of (ω,k) space. Feature B in Fig.2 thus corresponds to, and confirms by direct numerical simulation, the ADI wave-wave resonant feature in the sketch Fig.2 of Ref. [8] which, with the accompanying analysis, is a key aspect of the 1984 conjecture. The existence of the feature marked C in the central and right panels of Fig.2 was first noted

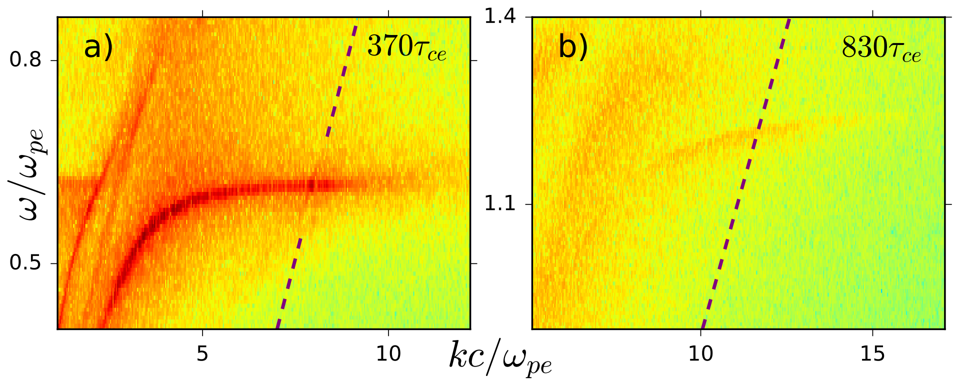

Figure 3: Evidence for (a) wave-wave resonant ADI and (b) excitation of waves with $\omega \approx \omega_{pe}$. Magnified segments of Fig.2(b) and (c) of Fig.2. Panel (a) shows the region of (ω,k) space near the intersection of the bulk-supported $\omega \approx \omega_{pe}\cos\theta$ branch with the marked line $\omega - k\cos\theta v_b = -\Omega_{ce}$, where v_b is the computed mean parallel velocity of the suprathermal electron population. Panel (b) shows the late stage evolution of of waves with $\omega \approx \omega_{pe}$.

in the less highly resolved simulations in Ref. [11]. Here we are able to identify it as, most probably, a consequence of the effect proposed in 1977 on analytical grounds in Ref. [5]: the excitation of waves at $\omega \approx \omega_{pe}$ due primarily to beam-plasma instability which is enabled by restructuring of the velocity space distribution of the tail electron population consequent on ADI-driven transport. From our simulations we have calculated the perpendicular-integrated parallel velocity distribution, $F(v_{\|}) = \int 2\pi v_\perp f(v_\perp, v_{\|}) dv_\perp$. Figure 4 shows that $F(v_{\|})$ develops positive slope in the region $6 \leq v_{\|}/v_{th} \leq 13$ by time $t/\tau ce = 370$ in this simulation, and that an extensive region of positive slope of $F(v_{\|})$ persists in the evolving tail at later times. This enables beam-plasma instability, and is a direct consequence of the continuous redistribution of electron kinetic energy from parallel to perpendicular under the action of the ADI, as shown in Fig.1. The positive slope regions in Fig.4 confirm, by direct numerical simulation, the aspect of

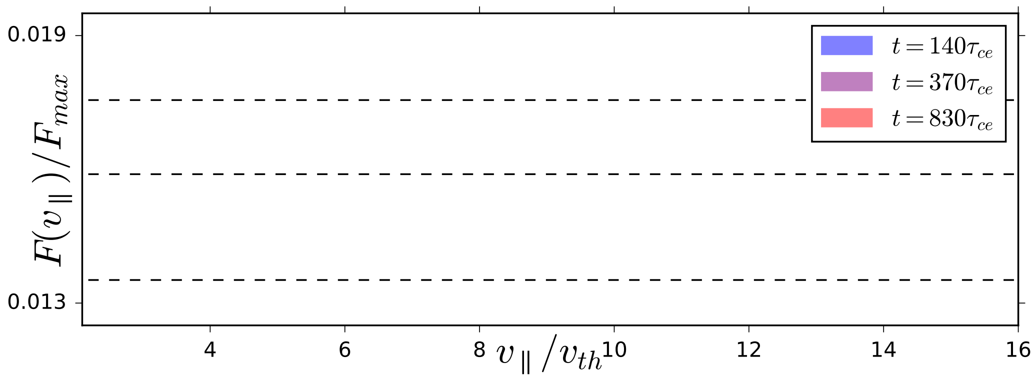

Figure 4: Snapshots in time of the perpendicular-integrated parallel velocity distribution $F(v_{\|})$ in the simulations. The three profiles (a) to (c) correspond to the times in Fig.1 and Fig.2.

the 1977 conjecture which is embodied in the sketch Fig.1(a) of Ref. [5]. The emergence of a small positive slope between $t = 140\tau_{ce}$ and $t = 370\tau_{ce}$ in Fig.4 correlates with the growth of feature C in Fig.2, as anticipated in Ref. [5].

3. Conclusions

The results presented here show that both the 1984 [8] and 1977 [5] analytically-based conjectures about the fully nonlinear physics of the ADI are, in essence, correct. (1) The electrostatic component of the dielectric tensor calculated for a monotonically decreasing tail distribution includes a cold-plasma element that corresponds to a beam-supported negative energy wave at (ω, k) values close to the anomalous Doppler resonance [8]. Our simulations show that, where this wave is resonant with the wave $\omega \approx \omega_{pe} \cos\theta$ that is supported by the bulk thermal electron population, wave-wave resonant instability can indeed occur, as conjectured [8]. This instability arises spontaneously as the electron distribution and perturbed electric and magnetic fields evolve self-consistently under the ADI beyond its purely wave-particle resonant stage. (2) The 1977 [5] conjecture rests on the potential generation and sustainment of a small local positive slope with respect to parallel velocity during the nonlinear phase of the ADI. Testing this conjecture therefore requires a self-consistent computational approach that, as here, can accommodate both the growth of positive slope and the positive-slope-driven fast-timescale physics of the mechanisms that would naturally act to eliminate this slope: inverse Landau wave-particle damping, and the unmagnetised two-stream instability. Our simulations show features in (ω, k) during the nonlinear phase of the ADI that correspond to the waves that are driven under this conjecture [5], and analysis of the contemporaneous distribution of electrons in velocity space provides further support.

This work was supported in part by the RCUK Energy Programme [grant number EP/I501045] and by Euratom. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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