

Analytical model for dissipative shocks in pair plasmas under the combined effect of collisionality and kinematic viscosity

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Abstract

An analytical model is introduced for shock excitations in pair plasmas, taking into account collisionality and kinematic (fluid) viscosity. The description embraces pair-ion (e.g. fullerene) plasmas, in the presence of a third component (electrons, dust), but also electron-positron plasmas (disregarding annihilation, for simplicity). A hybrid Korteweg de Vries/Burgers equation (KdV-B) is derived, and the effect of relevant plasma configuration parameters, in addition to dissipation, is investigated.

Introduction. Pair plasmas (*p.p.*), comprising particles with equal masses and equal absolute charge of opposite signs [1], exist in various environments. Electron-positron (e-p) plasmas, widely occurring in astrophysical environments such as pulsar magnetosphere and neutron stars [1], but also in the laboratory [2], have been the prototypical physical system to be studied as *p.p.* Recent experimental techniques have enabled electron-positron plasma production in the laboratory, via a sophisticated laser-plasma setup [3], and lie in the motivation of our work. Interestingly, fullerene (pair-ion) plasmas have been produced in the lab [4] providing new inspiration, essentially mimicking the dynamics of e-p plasma, without recombination effects.

This work aims at providing a first analytical model for shocks and dissipative solitary waves (pulses) in *asymmetric* pair plasmas, i.e. in the presence of a third species, thus extending earlier studies of ion-acoustic type shock waves in “pure” (two-component) pair-ion plasmas [5, 6]. *Inter alia*, we aim at providing a comprehensive description of fullerene plasmas or, e.g., dusty pair-ion plasmas [7] in particular, where dissipative effects (due to collisions) may be relevant.

Theoretical model. We consider a multicomponent plasma comprising two ion populations with equal masses and opposite charge, denoted by indices + and −, viz. $q_+ = -q_- = +ze$, $m_+ = m_- = m$. The existence of a third species is taken into account, typically taken to be a neutralizing background of thermal electrons.

Interactions among the plasma components include: electrostatic interactions, taken into account via a self-consistent generated electric field, but also interparticle collisions and viscous drag. The electron inertia is neglected. At equilibrium, we have $n_{e0} = z(n_{+0} - n_{-0})$, i.e.

$\frac{n_{e0}}{zn_{+0}} = 1 - \frac{n_{-0}}{n_{+0}} = 1 - \delta$, where n_{j0} is the unperturbed number density of the particle species j ($j = (1, 2) = (+, -)$ for the two ions, respectively, or $j = 3$ for the third species, i.e. electrons) and $\delta = n_{-0}/n_{+0}$ is the positive-to-negative component number density ratio. Considering electrons as 3rd species, we assume: $n_3 = n_e = n_{e0} \exp(e\phi/k_B T_e) \approx n_{e0}(1 + c_1\phi + c_2\phi + \dots)$ where $c_1 = 2c_2 = 1$, e is the magnitude of the electron charge, ϕ the electrostatic (ES) potential, k_B is the Boltzmann constant and T_e the electron temperature.

In one-dimensional planar geometry, the normalized fluid equations read:

$$\frac{\partial n_+}{\partial t} + \frac{\partial}{\partial x}(n_+ u_+) = 0, \quad (1)$$

$$\frac{\partial u_+}{\partial t} + u_+ \frac{\partial u_+}{\partial x} = -\frac{\partial \phi}{\partial x} - v(u_+ - u_-) + \eta \frac{\partial^2 u_+}{\partial x^2}, \quad (2)$$

$$\frac{\partial n_-}{\partial t} + \frac{\partial}{\partial x}(n_- u_-) = 0, \quad (3)$$

$$\frac{\partial u_-}{\partial t} + u_- \frac{\partial u_-}{\partial x} = \frac{\partial \phi}{\partial x} - v(u_- - u_+) + \eta \frac{\partial^2 u_-}{\partial x^2}, \quad (4)$$

$$\frac{\partial^2 \phi}{\partial x^2} = -n_+ + \delta n_- + (1 - \delta) n_3, \quad (5)$$

In the above set of equations we have used the time scale t and space scale x are normalized in units of the ion plasma frequency $\omega_{p,+} = (4\pi z^2 e^2 n_{+0}/m)^{1/2}$ and the ion Debye length $\lambda_{D,i,+} = (4\pi z e^2 n_{+0}/k_B T_e)^{-1/2}$. The number density n_j and fluid velocity u_j variables ($j = +, -, 3$) are scaled by the unperturbed number density n_{j0} and the ion-acoustic speed $c_s = (z K_B T_e/m)^{1/2}$, respectively. The ES potential ϕ , η and v are scaled by $k_B T_e/e$, $\omega_{p,+}$ and $\lambda_{D,+}^2 \omega_{p,+}$, respectively. Thermal effects have been neglected in the pair components, for simplicity.

Linear wave analysis. Linearizing and Fourier transforming ($\sim e^{i(kx - \omega t)}$), one finds the dispersion relation: $\Gamma_3 \omega^3 + i\Gamma_2 \omega^2 + \Gamma_1 \omega + i\Gamma_0 = 0$, where k is the wavenumber, ω is the frequency, and: $\Gamma_0 = -k^4 \eta (1 + \delta)$, $\Gamma_1 = -k^2 [1 + \delta + c_1 \eta (1 + k^2 - \delta) (k^2 \eta + 2v)]$, $\Gamma_2 = 2c_1 (k^2 \eta + v) (1 + k^2 - \delta)$ and $\Gamma_3 = c_1 (1 + k^2 - \delta)$. Omitting dissipation for a minute, this leads to $\omega^2 = \frac{k^2(1+\delta)/c_1}{1-\delta+k^2}$. This is an acoustic mode, sustained by the +/- component asymmetry (see that $\omega \rightarrow \infty$ if $\delta \rightarrow 1$).

Nonlinear wave analysis. In order to study the small amplitude nonlinear ion acoustic wave in dissipative multicomponent media, the reductive perturbation technique has been employed [8] and the following stretched coordinates has been introduced: $\xi = \varepsilon^{1/2} (x - Vt)$ and $\tau = \varepsilon^{3/2} t$, where V is the phase velocity (to be determined) and ε ($\ll 1$) is a small (real) expansion parameter. The state variables appearing in Eqs. (1-5) are expanded around their equilibrium values as $n_j = 1 + \varepsilon n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \varepsilon^3 n_j^{(3)} + \dots$, $u_j = \varepsilon u_j^{(1)} + \varepsilon^2 u_j^{(2)} + \varepsilon^3 u_j^{(3)} + \dots$ and $\phi = \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \varepsilon^3 \phi^{(3)} + \dots$. For the sake of analytical tractability, we shall also assume that $v = \varepsilon^{3/2} v_0$ and $\eta = \varepsilon^{1/2} \eta_0$ – where v_0 and η_0 are $\simeq O(1)$ – suggesting that collisionality

and viscosity related timescales are much slower than the plasma period. Combining into Eqs. (1)-(5) and considering different powers of ϵ , a tedious but straightforward algebraic procedure leads to a *hybrid Korteweg - de Vries/Burgers* (KdV-B) equation in the form:

$$\frac{\partial \psi}{\partial \tau} + A\psi \frac{\partial \psi}{\partial \xi} + B \frac{\partial^3 \psi}{\partial \xi^3} = C \frac{\partial^2 \psi}{\partial \xi^2} - D\psi, \quad (6)$$

in terms of the leading potential disturbance $\phi^{(1)} = \psi$. The coefficients in this equation read:

$$A = \frac{(2c_2V^4 - 3)(-1 + \delta)}{2V(1 + \delta)}, \quad B = \frac{V^3}{2(1 + \delta)}, \quad C = \frac{\eta}{2}, \quad D = v, \quad V^2 = \frac{(1 + \delta)}{c_1(1 - \delta)}.$$

We note that, in the limit $\delta \rightarrow 1$ (for *symmetric p.p.*, i.e., in the absence of 3rd species), the phase speed $V \sim (1 - \delta)^{-1}$ diverges, while A and B diverge like $A \sim \sqrt{2}c_2c_1^{-3/2}(1 - \delta)^{-1/2}$ and $B \sim (2c_1^3)^{-1/2}(1 - \delta)^{-3/2}$. The model therefore collapses for symmetric (“pure”) pair plasmas, and no excitations may then be sustained, as predicted earlier by Verheest [9]. We note that, as expected, $V = \lim_{k \rightarrow 0}(\omega/k)$ essentially denotes the true sound speed in asymmetric *p.p.* ($\delta \neq 1$).

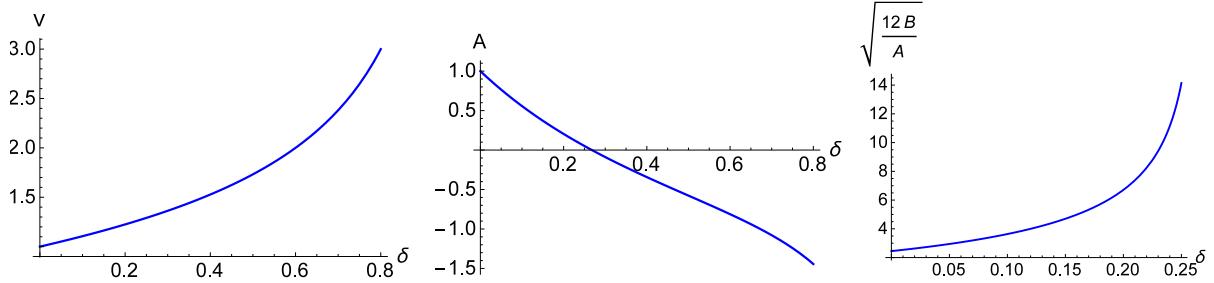


Figure 1: Left panel: the phase speed (scaled by $(zk_B T_e/m)^{1/2}$) is depicted. Middle panel: The nonlinearity coefficient A in Eq. (6) is shown. Right panel: the factor $L(0)\psi(0)^{1/2} = (12B/A)^{1/2}$, representing the pulse width $L(0)$ for fixed amplitude $\psi(0)$ (cf. the analytical pulse solution below) is depicted. The x -axis represents the density ratio $\delta = n_{-0}/n_{+0}$ in all 3 plots.

A number of comments are in row, considering special cases. First of all, *in the absence of dissipation* ($v = \eta = 0$, or $C = D = 0$), one recovers the KdV (pulse) soliton:

$$\psi(\xi, \tau) = \psi_0(0) \operatorname{sech}^2 \left(\frac{\xi - U(0)\tau}{L(0)} \right) \quad (7)$$

where the pulse velocity and width are given by $U(0) = A\psi_0(0)/3$ and $L(0) = \{12B/[A\psi_0(0)]\}^{1/2}$. We note that, for a higher negative-ion concentration (i.e., increasing δ), solitary waves are faster and wider (see Fig. 1a, c), for fixed maximum amplitude $\phi(0)$. Importantly, A (and thus $\psi(0)$) changes sign (suggesting polarity reversal) at a critical value of $\delta \approx 0.268$ (see Fig. 1b). *In the absence of viscosity* ($\eta = 0$, i.e. $C = 0, D \neq 0$), simple perturbative analysis leads to a

(time-dependent) damped soliton solution of the dissipative KdV Eq. (6) in the form [10]

$$\psi(\xi, \tau) = \psi_0(\tau) \operatorname{sech}^2 \sqrt{\frac{A\psi_0(\tau)}{12B}} \left(\xi - \frac{A}{3} \int_0^\tau \psi_0(\tau') d\tau' \right) \quad (8)$$

where the (decaying) amplitude is $\psi_0(\tau) = \psi_0(0) \exp(-\frac{4D}{3}\tau)$, the (decreasing) velocity is $U(\tau) = \frac{A\psi_0(0)}{3} \exp(-\frac{4D}{3}\tau)$, and the width of the (spreading) pulse is $L(\tau) = \sqrt{\frac{12B}{A\psi_0(0)}} \exp(\frac{2D}{3}\tau)$.

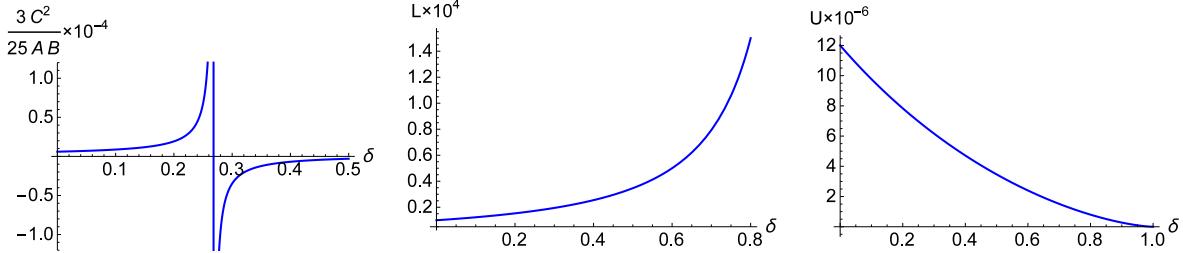


Figure 2: The structural features of the shock solution (9) are shown (left panel: the amplitude $(3C^2/(25AB))$; middle panel: the speed U ; right panel: the width L) versus the density ratio δ .

In the absence of collisions ($v = 0$, i.e. $D = 0$, $C \neq 0$), shock-type solutions exist [11]

$$\psi(\xi, \tau) = \frac{3C^2}{25AB} \left\{ 4 - \left[1 + \tanh \left(\frac{\xi - U\tau}{L} \right) \right]^2 \right\}, \quad (9)$$

with speed $U = 6C^2/(25B)$. The shock width is $L = 10B/C$. Higher negative-ion concentration (larger δ) leads to wider and slower shocks; see Figs. 2b,c. A polarity switch is also observed here, as pointed out above, at the root of A : see Fig. 2a. Interestingly, for $\delta \rightarrow 1$ (near-symmetric p.p.), the shock width reduces to nil, while its width spreads infinitely: physically speaking, charge neutrality (in the symmetric case) collapses the shock.

Our investigation should yield a richer dynamical profile in the warm (+/-) fluid model.

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References

- [1] G. P. Zank and R. G. Greaves, Phys. Rev. E **51**, 6079 (1995); N. I. Iwamoto, Phys. Rev. E **47**, 604 (1992).
- [2] H. Boehmer, M. Adams and N. Rynn, Phys. Plasmas **2**, 4369 (1995).
- [3] G. Sarri *et al*, Nat. Commun. **6**, 6747 (2015).
- [4] W. Oohara and R. Hatakeyama, Phys. Rev. Lett. **91**, 205005 (2003).
- [5] S. Ghosh, A. Adak, and M. Khan, Phys. Plasmas **21**, 012303 (2014).
- [6] A. Adak, S. Ghosh, and N. Chakrabarti, Phys. Plasmas **22**, 102307 (2015).
- [7] B. Atamaniuk and A.J. Turski, AIP Conf. Proc. **1397**, 259 (2011).
- [8] H. Washimi, and T. Taniuti, Phys. Rev. Lett. **17**, 996 (1966).
- [9] F. Verheest, Phys. Plasmas **13**, 082301 (2006).
- [10] S. Sultana and I. Kourakis, Phys. Plasmas **22**, 102302 (2015).
- [11] I. Kourakis, S. Sultana, and F. Verheest, Astrophys. Space. Sci. **338** 245 (2012).