The static electrical conductivity of the fully ionized plasma in the first Born approximation of the linear response theory

V.S.Karakhtanov

Institute of Problems of Chemical Physics RAS, Chernogolovka, Russia

Abstract

The applicability of the first Born approximation to the description of plasma transport properties is discussed. The influence of the electronic degeneration on the electrical conductivity of the fully ionized plasma is investigated with the linear response theory in the formulation of Zubarev, Lenard-Balescu-type collision integrals with dynamical screening of the Coulomb interaction, the first Born approximation in the thermodynamic Green's functions technique. The values for correlation functions for electron-ion and electron-electron scattering as functions of a degeneracy parameter $\Theta = k_B T/E_F$ in the region from $\Theta \gg 1$ to $\Theta = 1$ are obtained. The data for the electron-electron scattering contribution to the conductivity are obtained through the Chebyshev polynomial expansion of the Fermi distribution functions.

Motivation

It was found [1] that the electrical conductivity, calculated both with the Boltzmann-statistic long-wavelength asymptotic first Born approximation and the multiparameter interpolation formula [2], describe experimental data equally well for plasma with parameters out of formal applicability of the Born approximation (Figure 1). It is well-known that the strong collisions approximation (the Spitzer formula and its later modifications) describes experimental data reasonably only with model values of the Debye screening radius. There are no good preliminary arguments for the choice of these values.

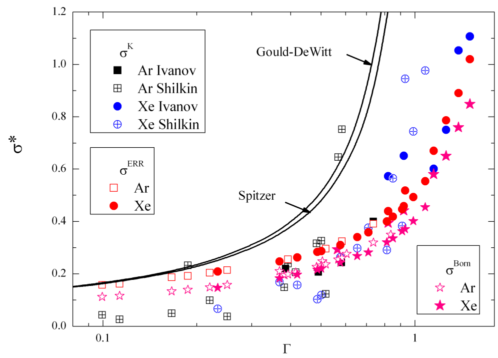

Figure 1: The Coulomb part of the reduced plasma electrical conductivity from [1]. σ^{ERR} - the interpolation formula, σ^{Born} - the first Born approximation, σ^{K} - experimental data, $\Gamma = (e^2/4\pi\varepsilon_0 k_B T)(4\pi n/3)^{1/3}$ - the coupling constant

Some ambiguities of the theory

The Spitzer formula appears as a low-Γ asymptotic limit of the ladder diagram summation (T-matrix approximation) in the perturbation theory.

So far as similar summation should be done simultaneously in the thermodynamic functions, we have to start with the initial system of non-interacting free charged particles and hope to obtain real observable densities of all particles in the system (including appeared bound states). So we have to guess the right parameters of the Hamiltonian of non-interacting particles.

The second problem is the ladder diagram summation with a frequency dependent potential (dynamical screening in electron-electron scattering). Usually it is replaced by the model static (Debye) potential with the free parameter (Debye radius), which is assorted from the condition of the coincidence of results of dynamical screening and the model static one in the first Born approximation. After that we hope to have the similar compensation in the higher orders of the perturbation theory.

The basic idea

- Consider a system of non-interacting electrons and a set of ions and atoms. Suppose that as a result of the inclusion of interaction all particle densities are unchanged.
- So all the densities in the initial Hamiltonian have to be taken from the experimental data and calculations have to be restricted with the only the lowest order terms of the perturbation theory. The criterion is the conservation of particles densities.

Technical Description

Within the linear response theory in the formulation of Zubarev [3], transport properties are expressed via force-force correlation functions [4, 5, 6]. The resulting expression for the conductivity is

$$\sigma = -\frac{e^2}{\Omega \det(d)} \begin{vmatrix} 0 & N_0 \\ \overline{N_0} & d \end{vmatrix}, \tag{1}$$

$$N_n = \begin{pmatrix} N_{n0} & N_{n1} & \dots & N_{nl} \end{pmatrix}, \tag{2}$$

$$\overline{N_n} = \begin{pmatrix} N_{n0} \\ N_{n1} \\ \vdots \\ N_{nl} \end{pmatrix}, d = \begin{pmatrix} d_{00} & d_{01} & \dots & d_{0l} \\ d_{10} & d_{11} & \dots & d_{1l} \\ \vdots & \vdots & \ddots & \vdots \\ d_{l0} & d_{l1} & \dots & d_{ll} \end{pmatrix}. \tag{3}$$

In (1) - (3) Ω - the system volume, N_{mn}, d_{mn} are correlation functions for the thermodynamic equilibrium, N_e - the number of electrons and $\beta = (k_B T)^{-1}$. The dimension of the matrix d coincides with the number of moments in the corresponding relevant statistical operator (see [3]). In the adiabatic limit we can omit the ion flux and obtain for Eq.(3)

$$d_{mn} = d_{mn}^{ei} + d_{mn}^{ee} + d_{mn}^{ea}, \tag{4}$$

$$N_{mn} = N_e \frac{\Gamma(m+n+5/2)}{\Gamma(5/2)} \frac{I_{m+n+1/2}(\beta\mu_e^{id})}{I_{1/2}(\beta\mu_e^{id})}, \tag{5}$$

with $I_\nu(y) = \frac{1}{\Gamma(\nu+1)} \int\limits_0^\infty \frac{x^\nu dx}{e^{x-y}+1}$ - Fermi integrals, μ_e^{id}- the ideal part of the electronic chemical potential.

Correlation functions d_{mn} (for electron-ion, electron-electron and electron-atom collisions) are evaluated using thermodynamic Green's functions. The diagram technique in the lowest order of perturbation (the first Born approximation) gives for the Coulomb interaction $V(q) = e^2(q^2\Omega\varepsilon_0)^{-1}$, screened due to the medium polarization, Lenard-Balescu collision integrals [7].

Treatment of collision integrals

Let $d_{mn}^{ea} = 0$ (fully ionized plasma), $Z_{ion} = 1$, $l = 1$ in (3).

In the adiabatic limit all d_{mn}^{ei} reduce to one-dimensional Zyman-type integrals with a static dielectric function and a static ion-ion structure factor.

In general, d_{mn}^{ee} reduce to 4-dimensional integral (for Boltzmann distribution functions in Lenard-Balescu collision integrals - to 2-dimensional one). After using the Chebyshev polynomial expansion of Fermi distribution functions we reduce d_{mn}^{ee} to 2-dimensional integral in the range $\mu_e^{id} < 0$ (approximately $\Theta > 1$). Here we use 3-order polynomial expansion. It gives approximately 0.01 accuracy in the representation of the Fermi distribution function.

Results and discussion

As a consequence of a momentum conservation law, all $d_{0n}^{ee} = d_{n0}^{ee} = 0$. The first non-zero electron-electron collision integral is d_{11}^{ee}. In the high-temperature low-density limit $d_{11}^{ee}/d_{00}^{ei} = \sqrt{2}$, in the high-degeneracy limit $d_{11}^{ee}/d_{00}^{ei} = 0$, and it is the case of Lorentz plasma. It is interesting to retrace the transition between these states.

On Figure 2 the dependence of the ratio mentioned above on the degenerate parameter Θ is shown.

As it seen from Figure 2, the contribution of electron-electron scattering to the electrical conductivity decreases sharply near $\Theta = 3$, depending on Γ.

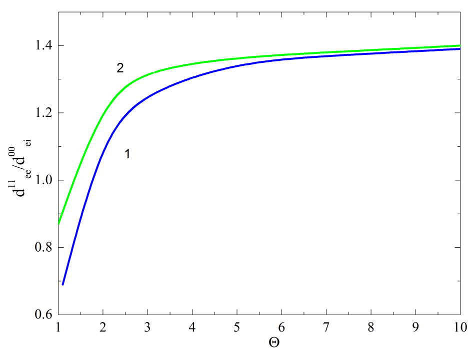

Figure 2: The d_{11}^{ee}/d_{00}^{ei} ratio as the function of the degeneracy parameter. 1 - $r_s = 1.842(\Gamma\Theta = 5)$, 2 - $r_s = 9.21(\Gamma\Theta = 1)$.

In order to ensure the smooth transition between degenerate and non-degenerate states without the knowledge of d^{ee}_{mn} in the intermediate region of Θ, some authors ([8, 9]) postulated the coefficient R_{ee} - the ratio of the real plasma electrical conductivity to those of the Lorentz plasma. Recently attempts were made to calculate d^{ee}_{11} and R_{ee} with Debye screening [10] and to obtain their asymptotic behaviour with dynamical screening [11]. On Figure 3 the corresponding results are compared with present ones.

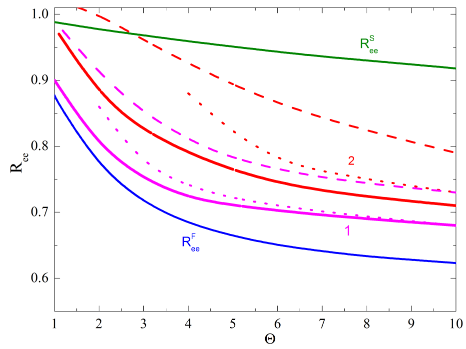

Figure 3: R_{ee}-factor in different approximations. R^S_{ee} - from [8], R^F_{ee} - from [9], 1 - $r_s = 1.842$, 2 - $r_s = 9.21$. Solid lines - present results, dotted lines - asymptotic values in 6-moment approximation [11], dashed lines - Debye screening [10].

The generalization of the method of d^{ee}_{mn} calculations for any m and n is obvious. For $\mu^{id}_e > 0$ the forth integration for the reduction of the integral dimension is impossible. In this region it is suitable to use the high-degeneracy representation of d^{ee}_{mn} [12, 13] for the construction of their continuation.

Acknowledgements

This work was supported by the RFBR Grant 14-02-91346 "Thermodynamic and optical properties of dense noble gas plasma".

References

[1] V. S. Karakhtanov, R. Redmer, H. Reinholz, and G. Röpke, Contrib. Plasma Phys. **53**, 639 (2013).

[2] A. Esser, R. Redmer, G. Röpke, Contrib. Plasma Phys. **43**, 33 (2003).

[3] D. N. Zubarev, *Nonequilibrium Statistical Mechanics* (Plenum, New York, 1974).

[4] C.-V. Meister and G. Röpke, Ann. Phys. A **494** 133 (1982).

[5] G. Röpke, Physica A **121**, 92 (1983).

[6] G. Röpke, Phys. Rev. A **38**, 3001 (1988).

[7] R. Redmer, G. Röpke, F. Morales, K. Kilimann, Phys. Fluids B **2**, 390 (1990).

[8] W. Stygar, G. Gerdin, and D. Fehl, Phys. Rev. E **66**, 046417 (2002).

[9] V. E. Fortov, V. Ya. Ternovoi, M. V. Zhernokletov, M. A. Mochalov, A. L. Mikhailov, A. S. Filimonov, A. A Pyalling, V. B. Mintsev, V. K. Gryaznov, and I. I. Iosilevski , JETP **97**, 259 (2003).

[10] H. Reinholz, G. Röpke, S. Rosmej, and R. Redmer, Phys. Rev. E **91**, 043105 (2015).

[11] V. S. Karakhtanov, Contrib. Plasma Phys. **56**, 343 (2016).

[12] V. S. Karakhtanov, J. Phys: Conf. Series **529**, 012017 (2014).

[13] V. S. Karakhtanov, The EPS Conference on Plasma Physics, Lisbon, Portugal, 2015, http://ocs.ciemat.es/EPS2015PAP/pdf/P1.406.pdf.