Positive column discharge model with consideration of resonance radiation transport

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Introduction

Contraction of gas discharge is one of the most well-known phenomenon in gas discharge physics. The fundamental problem of contraction is actively discussed in literature recently. The biggest progress in this problem may be achieved considering an inert gas model because of the energy term simplicity. A complete description of experimental and theoretical knowledge of contraction in inert gases has been made in review [1]. A big number of full-scale contraction models were developed in the last decades. However, all these models use traditional approach of effective lifetime by Holstein for the description of resonant atoms. This approximation corresponds to a local balance of resonant atoms and does not consider resonance radiation trapping. In this work a previously proposed method [2] of accurate description of the resonance radiation transport for solving self - consistent problems is applied to a model of contracted discharge in an inert gas and the main goal of the paper is to reveal the role of resonance radiation transport in formation of different plasma parameters.

Semianalytical contraction model

A simple qualitative three - level semianalytical model of discharge contraction, based on joint solution of Boltzmann kinetic equation for an electron energy distribution function (EEDF), differential balance equation of charged particles, integral balance equation of resonant atoms and an equation for a maintaining conditions of discharge current, may be considered. The main mechanism of contraction belongs to the ionization non-linearity as a function of the electron concentration, which is related to the competition of the electron-atom and the electron-electron collisions. Boltzmann kinetic equation, accounting elastic electron – atom, electron-electron collisions and inelastic electron – atom collisions, can be written as:

$$\frac{1}{\upsilon^2}\frac{\partial}{\partial\upsilon}\upsilon^2\left[\frac{(eE)^2}{3m^2}+\nu_e(\upsilon)\frac{kT_e}{m}\right]\frac{\partial f_0(\upsilon)}{\partial\upsilon}+\frac{1}{\upsilon^2}\frac{\partial}{\partial\upsilon}\upsilon^3\left[\frac{m}{M}\nu_a(\upsilon)+\nu_e(\upsilon)\right]f_0(\upsilon)=S^*(f_0), \tag{1}$$

where e - electron charge, E - electric intensity, υ - electron velocity, m - electron mass, M - atomic mass, $\nu_a(\upsilon)$ - elastic electron – atom collisions frequency, $\nu_e(\upsilon)$ - elastic electron –

electron collisions frequency, $f_0(\upsilon)$ - isotropic component of EEDF, $S^*(f_0)$ - operator of inelastic collision, kT_e - electron temperature.

Since the operator $S^*(f_0)$ contains a total excitation cross section, by multiplying both sides of (1) by $4\pi\int_{\varepsilon_{ex}}^{\infty}(2\varepsilon)^{1/2}m^{-3/2}d\varepsilon$ integrating it from excitation energy ε_{ex} to infinity in energy scale ε, one will obtain a frequency of total inelastic electron – atom collisions:

$$\nu_r = 4\pi\cdot m^{-3/2}\int_{\varepsilon_{ex}}^{\infty} S^*(f_0)(2\varepsilon)^{1/2}d\varepsilon \qquad (2)$$

Given that the excitation cross sections of lower metastable and resonant states make the major contribution to the total excitation cross section in inert gases, it can be supposed that equation (2) could be used to describe the process of population of these lower states. In case of low average electron energy in comparison with the excitation energy ε_{ex}, an approximation of black wall, assuming a quick exponential decline of the EEDF near the ε_{ex}, can be used to find a solution of the Boltzmann equation for the isotropic component of EEDF $f_0(\upsilon)$. Using the solution for the isotropic component of EEDF in the area of elastic collisions, which dominate in the formation of EEDF in atomic gases, equation (2) will transform to a simple analytical expression and will be determined as a particles flux through the excitation threshold:

$$\nu_r = 4\pi c\left(2\varepsilon_{ex}/m\right)^{3/2}\left[\nu_a(\varepsilon_{ex})\,m/M + \nu_e(\varepsilon_{ex})\right]\exp\left[-\int_0^{\varepsilon_{ex}}\frac{\nu_a(\varepsilon)\,m/M + \nu_e(\varepsilon)}{(eE)^2\left(3m\nu_a(\varepsilon)\right)^{-1} + \nu_e(\varepsilon)kT_e}d\varepsilon\right], \qquad (3)$$

where the frequency ν_a can be approximated as $\nu_a \sim \left(\varepsilon/\varepsilon_{ex}\right)^{3/2}$ for a heavy Argon atom.

A simple three – level energy model consisting of the ground state, a resonant state and an ionization state is considered. The resonant state is populated by an excitation from the ground state with frequency ν_r and by a recombination Γ from the ionization state and depopulated by a stepwise ionization with frequency W_{si} and by spontaneous decay with an output of resonance radiation with probability A. The ionization state is depopulated by an ambipolar diffusion of charged particles and the recombination to the resonance level. An integral balance equation for resonant atoms and a differential balance equation with boundary conditions for charged particles can be written in the form:

$$\begin{cases} n_e\nu_r(T_e,E,n_e) + \Gamma(T_e,n_e) = N_r(r)W_{si}(T_e,E,n_e) + G(r,N_r) & (4) \\ N_r(r)W_i(T_e,E,n_e) = \Gamma(T_e,n_e) - D_a(T_e)\Delta n_e(r) & , \quad (5) \\ \nabla n_e\big|_{r=0} = 0, \quad n_e\big|_{r=R} = 0 & \end{cases}$$

where n_e is an electron concentration, R - discharge tube radius, D_a - ambipolar diffusion coefficient, $G(r, N_r)$- integral resonance radiation transport operator:

$$G(r,N_r) = AN_r(r) - \int_V AN_r(r')K(|r-r'|)d^3r', \tag{6}$$

and $K(|r-r'|)$ is a kernel of the integral operator $G(r,N_r)$.

Holstein approximation and precise description of resonance radiation transport

Traditional approach in the description of resonance radiation transport is related to an assumption that the resonant atoms density $N_r(r)$ is close to a fundamental radiation mode and decreases much slower than the kernel $K(|r-r'|)$ of the integral operator (6). Such assumption makes possible to take $N_r(r)$ as a constant in (6) within the fall of the kernel. It leads to an expression $G(r) = N_r(r)A_{eff}$, where A_{eff} is an inverse effective lifetime of the resonant state given by Holstein. In this case initial equations (4), (5) will be rewritten as an algebraic equation for resonant atoms and a differential equation for electrons:

$$N_r(r) = \frac{Z_r(T_e,E,n_e) + \Gamma(T_e,n_e)}{W_{si}(T_e,E,n_e) + A_{eff}}, \tag{7}$$

$$\begin{cases} D_a(T_e)\Delta n_e(r) + Z_r(T_e,E,n_e)\dfrac{W_{si}(T_e,E,n_e)}{W_{si}(T_e,E,n_e)+A_{eff}} - \Gamma(T_e,n_e)\dfrac{A_{eff}}{W_{si}(T_e,E,n_e)+A_{eff}} = 0 \\ \nabla n_e|_{r=0} = 0, \quad n_e|_{r=R} = 0 \end{cases}. \tag{8}$$

If the radial distribution of resonant atoms drastically differs from the fundamental radiation mode, Holstein effective lifetime approximation goes beyond its applicability. In this case resonant radiation transport should be taken into account precisely. In paper [2] a precise method of joint solution of integral equation (4) and differential equation (5) with an accurate consideration of resonance radiation transport is described. It is proposed to divide the whole plasma volume *V* into a large number of parts ΔV_j which are small compared to *V*, and, within this volumes, the concentration of resonant atoms can be considered constant. Then, equations (4), (5) takes the form of a system of linear algebraic equations:

$$\begin{cases} D_a(E)\sum_j \left[a_{kj}n_e(r_j)\right] + N_r(r_k)W_{si}(T_e,E,n_e(r_k)) - \Gamma(T_e,n_e(r_k)) = 0 & (9) \\ \sum_j \left[A_{eff}b_{kj}N_r(r_j) + N_r(r_j)W_{si}(T_e,E,n_e(r_j))\delta_{kj}\right] - Z_r(T_e,E,n_e(r_k)) - \Gamma(T_e,n_e(r_k)) = 0, & (10) \end{cases}$$

where matrix with elements b_{kj} takes into account appearance or the resonance atoms outside the excitation zone due to radiation transport and the differential operator in (5) is replaced by a matrix with elements a_{kj} using finite differences.

Results and conclusion

Comparison of solutions, obtained using the traditional approximation of the effective lifetime (7), (8) and with the correct calculation of resonance radiation transfer (9), (10), will reveal the influence of resonance radiation transport on the formation of plasma parameters. Results are presented in figure 1 in case of $PR = 100$ Torr*cm, where P is pressure.

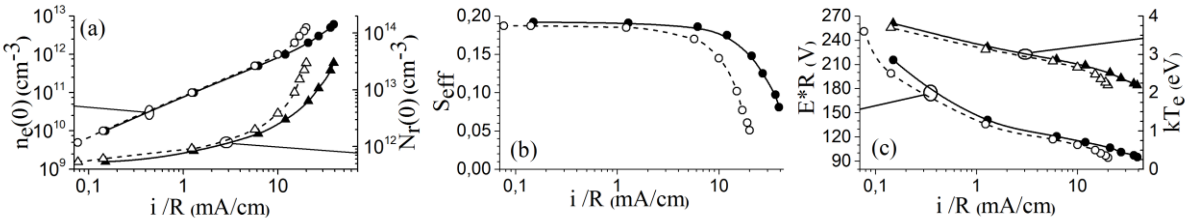

Fig. 1. Dependence of the electron concentration and resonant atoms concentration (a) at the discharge axis, effective cross section of the current filament (b) and electric intensity with electron temperature (c) on current, calculated in two approximations: solid dots- exact solution, open dots- approximation of the effective lifetime.

As can be seen in fig. 1 at low currents, when the radial distributions are close to the fundamental modes of diffusion and radiation problems, the solutions within the frameworks of effective lifetimes yield sufficiently accurate results. With a growth of current, when the radial distributions start to differ from the fundamental modes, shortcomings of the local solution become evident. This fact illustrates the effect of higher diffusion and radiation modes in formation of plasma parameters. Radiation transport leads to a broadening of an effective cross section of discharge current $S_{eff} = \int_o^R (n_e(r)rdr)/(n_e(0)R^2)$ (fig 1b) and reduces electron concentration at the axis (fig 1a) as well as resonant atoms concentration at the axis (fig 1a). As a result, a critical discharge current is shifted towards larger values. The influence of radiation transport on electric intensity and electron temperature is insignificant (fig. 1c). The discharge current equation can be written as $i = 2\pi R^2 e b_e E n_e(0) \cdot S_{eff}$, where b_e - electron mobility.

As a result a simple three - level semianalytical model shows a good applicability of Holstein approximation while the radial distributions of different plasma components are close to fundamental diffusion and radiation modes. In contracted discharge a notable difference between two approaches is demonstrated. These facts show the influence of higher diffusion and radiation modes.

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References

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