Fast-electron dynamics in the presence of weakly ionized impurities

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Introduction Runaway electrons constitute a significant threat to tokamak experiments. To minimize the risk of damage, it is crucial to understand the runaway-electron dynamics, which during runaway mitigation can be strongly influenced by the interaction with partially ionized atoms. It is therefore important for runaway mitigation to have accurate models of the interaction between fast electrons and partially screened nuclei of heavy ions. Fast electrons are not simply deflected by the Coulomb interaction with the net charge of the ion, but also probe its internal electron structure, so that the nuclear charge is not completely screened. They can therefore be expected to experience higher collision rates against impurities, leading to a more efficient damping.

To model the interaction between fast electrons and partially screened impurities, we have derived a generalized collision operator from first principles, resulting in analytic expressions for the collision frequencies [1]. We model elastic electron-ion collisions quantum-mechanically in the Born approximation, using density functional theory (DFT) to obtain the electron-density distribution of the impurity ions. To describe inelastic collisions with bound electrons, we employ Bethe's theory for the collisional stopping power [2]. We find that the deflection and slowing-down frequencies are increased significantly compared to standard collisional theory, already at sub-relativistic electron energies [1]. Furthermore, we derive an analytical expression for the effective critical field for runaway generation and decay that takes into account the presence of partially screened impurities. In this contribution, we detail the derivation of this effective critical field, and present a formula that takes arbitrary ion species into account.

Generalized collision operator The Fokker-Planck collision operator between species a and b can be simplified to $C^{ab} = \nu_D^{ab}\mathscr{L}(f_a) + \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^3\left(\nu_S^{ab} f_a + \frac{1}{2}\nu_\parallel^{ab} p \frac{\partial f_a}{\partial p}\right)\right]$, in the limit when species b has a Maxwellian distribution. Here, f_a is the distribution function of species a, $\mathbf{p} = \gamma\mathbf{v}/c$ is the normalized momentum (with γ the Lorentz factor, $\mathscr{L}$ represents scattering at constant energy, and ν_D^{ab}, ν_S^{ab} and $\nu_\parallel^{ab}$ are the deflection, slowing-down and parallel-diffusion frequencies, which are well known in the limits of *complete screening* (i.e. the electron interacts only with the net ion charge) and *no screening* (the electron experiences the full nuclear charge). Accounting for partial screening, we obtain generalized expressions for ν_D^{ei} and ν_S^{ee} [1]. Focusing on the effective critical field $E_c^{\rm eff}$, the following equations are specialized to the superthermal momentum region, in which the critical momentum p_c corresponding to $E_c^{\rm eff}$ is found. Thus all of the following expressions are given for superthermal electrons.

In units of relativistic collision times $\tau_c = (4\pi n_e c r_0^2 \ln\Lambda_0)^{-1}$ (where r_0 is the classical electron radius), the generalized deflection frequency is given by

$$\nu_D = \frac{\sqrt{p^2+1}}{p^3 \ln\Lambda_0}\left(\ln\Lambda^{ee} + \ln\Lambda^{ei} Z_{\rm eff} + \sum_j \frac{n_j}{n_e}\left[(Z_j^2 - Z_{0,j}^2)\ln\left(\frac{2a_j p}{\alpha}\right) - \frac{2}{3}(Z_j - Z_{0,j})^2\right]\right), \quad (1)$$

where $Z_{0,j}$ is the ionization state and Z_j is the charge number of the nucleus for species j, $\alpha \approx 1/137$ is the fine-structure constant, $Z_{\rm eff} = \sum_j n_j Z_{0,j}^2/n_e$, where n_j is the density of species j and n_e represents the density of free electrons. The parameter a_j was determined from DFT calculations, and is an effective ion size which depends on the ion species j. For example, we obtain the following for the first ionization states of argon: $a_{\rm Ar^{+}} = 0.329$, $a_{\rm Ar^{2+}} = 0.306$, $a_{\rm Ar^{3+}} = 0.283$, $a_{\rm Ar^{4+}} = 0.260$, and $a_{\rm Ar^{5+}} = 0.238$. For the Coulomb logarithms, in the superthermal limit we use $\ln\Lambda^{ee} = \ln\Lambda_0 + \ln[\sqrt{2(\gamma-1)}/p_{\rm Te}]$ and $\ln\Lambda^{ei} = \ln\Lambda_0 + \ln(2p/p_{\rm Te})$, where $p_{\rm Te}$ is the thermal momentum and $\ln\Lambda_0 = 14.9 - 0.5\ln(n_e/10^{20}\,{\rm m}^{-3}) + \ln(T_e/{\rm keV})$. For the superthermal slowing-down frequency, we obtain

$$\nu_S = \frac{p^2+1}{p^3 \ln\Lambda_0}\left(\ln\Lambda^{ee} + \sum_j \frac{n_j}{n_e} N_{e,j}\left(\ln h_j - \beta^2\right)\right). \quad (2)$$

Here $h_j = p\sqrt{\gamma-1}/I_j$ and I_j is the mean excitation energy of the ion, normalized to the electron rest energy.

Figure 1 shows the enhancement of the deflection and slowing down frequencies compared to the completely screened limit (CS) for singly ionized argon. Both are enhanced significantly already at sub-relativistic energies. A widely used rule of thumb that is mentioned in passing by Rosenbluth and Putvinski [3], suggests that inelastic collisions with bound electrons can be taken into account by adding half the number of bound electrons to the number of free electrons. As shown in Fig. 1, this model (RP) overestimates the slowing-down frequency at low energies and is a significant underestimation at high runaway energies.

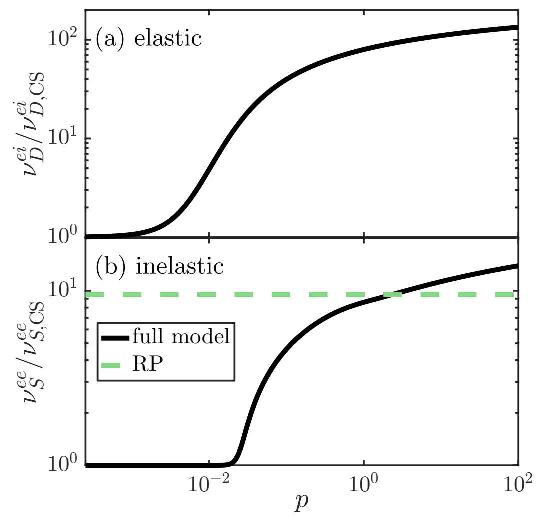

Figure 1: (a) The deflection frequency and (b) the slowing-down frequency as a function of the incoming-electron momentum, normalized to the completely screened collision frequencies. The black lines denote our model, and the approximate Rosenbluth-Putvinski (RP) model of ν_S^{ee} is shown in dashed green. Parameters: $T = 10\,{\rm eV}$ and $\rm Ar^+$, with density $n_{\rm Ar} = 10^{20}\,{\rm m}^{-3}$.

Effective critical electric field The critical electric field is a central parameter for both generation of a runaway current and for its decay rate in a highly inductive tokamak; in the latter case it is predicted that the induced electric field will be close to the critical electric field so that the current decays according to $dI/dt = 2\pi R E_c^{\rm eff}/L$ [4], where $L \sim \mu_0 R$ is the self-inductance and R is the major radius. The drag due to synchrotron radiation reaction increases the critical field [5] but the effect is small for high density and low temperature characteristic of disruptions. However, the minimum electric field required to accelerate a runaway beam is strongly increased by collisions with partially stripped ions, due to both collisional friction and pitch-angle diffusion.

We calculate the effective electrical field due to collisions with partially screened ions by considering the pitch-angle averaged force-balance equation $\langle eE_c^{\rm eff}\rangle = \min_p p\nu_S$, assuming fast pitch-angle dynamics compared to the timescale in the momentum variable p [6, 7]. In the Fokker–Planck equation, this amounts to requiring that the pitch-angle flux vanishes (here written in the relativistic limit and neglecting synchrotron radiation reaction):

$$\frac{\partial \bar f}{\partial t} = \frac{\partial}{\partial p}\left[(p\nu_S - eE\xi)\bar f\right] + \frac{\partial}{\partial \xi}\left[(1-\xi^2)\underbrace{\left(\frac{eE}{pmc}\bar f + \frac{1}{2}\nu_D\frac{\partial \bar f}{\partial \xi}\right)}_{=0}\right], \qquad (3)$$

where $\bar f = p^2 f$, and E is normalized to the critical field $E_c = n_e e^3 \ln\Lambda_0/4\pi\varepsilon_0^2 m_e c^2$. The slowing-down frequency ν_s and the deflection frequency ν_D are enhanced compared to their classical values due to the presence of partially ionized atoms as well as the energy-dependence of the Coulomb logarithm $\ln\Lambda$ according to Eqs. (1) and (2).

The condition that the pitch-angle flux vanishes yields the following form for the angular distribution: $\bar f = G(t,p)A\exp(A\xi)/2\sinh A$, where $A(p) = 2E/p\nu_D$. Then, Eq. (3) integrated over pitch-angle yields

$$\frac{\partial G}{\partial \tau} + \frac{\partial}{\partial p}\left[U(p)G\right] = 0, \quad U(p) = \frac{E}{\tanh A} - p\nu_S - \frac{E}{A}. \qquad (4)$$

The effective critical field is now the minimum field for which force balance is possible:

$$E_c^{\rm eff} = \min_p\left[E\,\middle|\,U(p,E)=0\right] \approx \min_p\left[D + H\ln p + \frac{1}{2p}(B + C\ln p)\right], \qquad (5)$$

where we took the limit $\tanh A \to 1$ and assumed $p \gg 1$, which is consistent with the full solution of Eq. (5). In this limit, $\nu_D \approx p^{-2}(B + C\ln p)$, where $B = (1+Z_{\rm eff})\left[1 + \frac{1}{\ln\Lambda_0}\ln\left(\frac{2}{p_T}\right)\right] + \frac{1}{\ln\Lambda_0}\sum_j \frac{n_j}{n_e}\left[(Z_j^2 - Z_{0,j}^2)\ln\left(\frac{2a_j}{\alpha}\right) - \frac{2}{3}(Z_j - Z_{0,j})^2\right]$ and $C = \frac{1}{\ln\Lambda_0}\sum_j \frac{n_j}{n_e}Z_j^2$, if terms of order $\mathscr{O}(1/\ln\Lambda_0)$ are neglected. Similarly, $\nu_S \approx \frac{1}{p}(D + H\ln p)$ where $D = 1 + \frac{1}{\ln\Lambda_0}\left[\ln(\sqrt{2}/p_T) + \sum_j \frac{n_j}{n_e}N_{e,j}\ln(1/eI_j)\right]$

and $H = \frac{3}{2}\frac{1}{\ln\Lambda_0}\sum_j \frac{n_j}{n_e} N_{e,j}$. The effective critical field is then approximated by

$$E_c^{\text{eff}} = D + H\left[1+\ln\left(\frac{B+C\ln(B/H)}{2H}\right)\right] \quad (6)$$

$$\approx 1 + \frac{\ln(\sqrt{2}/p_T)}{\ln\Lambda_0} + \frac{N_{e,Z}}{\ln\Lambda_0}\frac{n_Z}{n_e}\left(\ln(1/I_j)+\frac{1}{2}+\frac{3}{2}\ln[X(Y+\ln XY)]\right), \quad (7)$$

where $X = Z^2/3N_e$ and $Y = \ln(2a_j/\alpha) - 2/3$. The last approximation is valid if a few low ionization states of a single element dominate. For example, for singly ionized argon we obtain $E_c^{\text{eff}} \approx 1 + \frac{1}{\ln\Lambda_0}\left(7 - \ln\sqrt{T_{\text{eV}}} + 240\frac{n_{\text{Ar}}}{n_e}\right)$. This formula can also be used for higher argon ionization states $Z_{0,j} \leq 3$ with an error of less than 10%. By numerically solving Eq. (3) when synchrotron radiation losses are included, we find that the formula for the effective critical field (6) is accurate to within 10% for magnetic fields in the range $B[\text{T}]^2 \lesssim n_{\text{Ar}}[10^{18}\,\text{m}^{-3}]$.

Figure 2 shows the effective critical electric field normalized to E_c. The full model (solid black line for the solution to Eq. (5) and dashed red for the approximate analytical formula for Ar^+) is compared to the Rosenbluth-Putvinski model in dashed green. The RP model can be seen to underestimate the effective critical field, which is a result of both neglecting the enhancement of elastic collisions and approximating the inelastic collision rate.

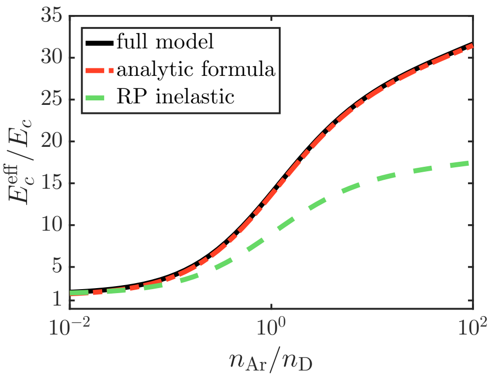

Figure 2: Effective critical field as function of $n_{\text{Ar}}/n_{\text{D}}$, where n_{Ar} is the density of Ar^+, $n_{\text{D}} = 10^{20}\,\text{m}^{-3}$ and $T = 10\,\text{eV}$.

Conclusion We have derived an expression for the collision operator between fast electrons and partially ionized atoms. With kinetic simulations using CODE [8] we have shown that the modifications to the deflection and slowing down frequencies are of equal importance in describing the runaway current evolution [1]. Here we apply the generalized formulas to calculate the effective critical electric field, which can be expressed with an analytical formula if we assume fast pitch-angle dynamics. The effective critical field is significantly enhanced compared to previous models. This is relevant for the efficacy of mitigation strategies for runaway electrons in tokamak devices.

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