

## MHD stability and energy principle: beyond nested flux surfaces

S.Yu.Medvedev<sup>1,2</sup>, A.A. Martynov<sup>1,2</sup>, V.V. Drozdov<sup>1</sup>, A.A. Ivanov<sup>1</sup>, Yu.Yu. Poshekhonov<sup>1</sup>,  
S.V. Konovalov<sup>2</sup>, L. Villard<sup>3</sup>

<sup>1</sup>*Keldysh Institute of Applied Mathematics, Moscow, Russia*

<sup>2</sup>*National Research Centre Kurchatov Institute, Moscow, Russia*

<sup>3</sup>*EPFL, Swiss Plasma Center, Lausanne, Switzerland*

Going beyond nested flux surfaces in tokamak plasmas brings new insights both for theoretical and experimental plasma physics. Starting from the research of doublet plasmas with two magnetic axes the ideal MHD stability code KINX [1] was employed to investigate the subject using structured grids in each sub-domain with nested flux surfaces. Then, the MHD\_NX stability code on unstructured grids was developed to treat 2D equilibria with arbitrary topology of magnetic surfaces [2].

The study of equilibrium and stability of reversed current configurations with axisymmetric  $n=0$  islands related to the current hole and AC operation in tokamaks [3] brings more general questions of MHD stability of 2D equilibrium solutions with islands and other types of symmetry – chains of tearing-like islands in straight helix and cylindrically symmetric  $m=0$  islands related to the RFP configurations. For all these 2D island configurations new ideally unstable modes were discovered [4].

The energy principle formulation with MHD-compatible boundary conditions at the open field lines is needed for the treatment of tokamak plasmas with scrape off layer (SOL) region in order to self-consistently investigate the stability of diverted tokamak configurations with finite current density at the separatrix. A suite of equilibrium and stability codes for diverted plasmas with SOL has been developed to answer these questions and is ready for integration with other plasma edge modeling codes [5].

**1. Tokamaks: beyond nested flux surfaces.** Configurations with inner separatrices are usual for 3D stellarator configurations [6] including magnetic islands and other alternative confinement concepts as Galateas [7] with equilibrium configuration formed by either external or internal conductors rather than the plasma current. Doublet tokamak configurations with two magnetic axes were considered as a possibility to increase the overall plasma elongation [1]. In fact, the doublet is an example of  $n=0$  islands in an axisymmetric equilibrium configuration. Another example of  $n=0$  islands is the reversed current configuration relevant to alternating current (AC) tokamak operation, when the total plasma current passes through zero, and possibly to the current hole [3]. Finally, the reconnections due to tearing mode development are also possible with the appearance of magnetic islands. Even staying within the 2D equilibrium model (generalized Grad-Shafranov equation in

axial, helical and cylindrical symmetries) the stability of tokamak relevant configurations without nested magnetic surfaces need to be addressed with the new generation of the MHD codes [2]. Finally, the SOL currents need to be included into consideration when the edge stability properties are investigated in divertor tokamaks configurations with open field lines and finite current density at the separatrix.

**2. MHD stability formulation and open field line boundary conditions.** There are two possible approaches to the treatment of equilibrium configurations without nested flux surfaces. If the topology of magnetic surfaces is unchanged and there are several sub-domains with nested flux surfaces, as is the case with doublets, then the standard energy principle formulation based on the plasma displacement projections onto directions related to the magnetic field can be used, as in the KINX code. Using the magnetic projections is essential to explicitly reveal the anisotropic nature of the MHD equations (the derivative across the magnetic surfaces of the normal displacement only enters the perturbed potential energy) and to accurately approximate the corresponding spectrum of the elliptic non-compact operator.

For more general stability analysis, the potential and kinetic energy functionals assuming the time dependence  $\exp(i\omega t)$  can be expressed in terms of the electric field  $\vec{E} = i\omega\vec{e}$ ,  $\vec{e} = -\vec{\xi} \times \vec{B}$ , where  $\vec{\xi}$  is the vector of plasma displacement:

$$W_p = \frac{1}{2} \int \left\{ |\nabla \times \vec{e}|^2 - \frac{\vec{j} \cdot \vec{B}}{B^2} \vec{e} \cdot \nabla \times \vec{e} + \frac{\vec{j} \cdot \vec{e}}{B^2} \left[ 2\vec{B} \cdot \nabla \times \vec{e} - \vec{t} \cdot \vec{e} \right] \right\} d^3r, \quad (1)$$

$$K_p = \frac{1}{2} \int \rho |\vec{e}|^2 / B^2 d^3r, \quad \vec{t} = \vec{j} + B^2 \nabla \left( \frac{1}{B^2} \right) \times \vec{B}.$$

The variational formulation  $\delta(W_p - \omega^2 K_p) = 0$  in terms of electric field in general projections (i.e. not connected with the equilibrium magnetic field) should be supplied with the ideal MHD condition  $\vec{e} \cdot \vec{B} = 0$ . The ideal MHD boundary conditions at the open field lines are applied by setting

$$\vec{\xi}_\perp \cdot \vec{n} = 0, \quad \vec{\xi}_\parallel \cdot \vec{n} = 0 \quad (2)$$

at the divertor plates, where  $\vec{n}$  is the normal to divertor plates. This is a consequence of the following sheath-compatible boundary conditions [8]:  $\vec{v} = \vec{v}_\perp + v_\parallel \vec{B} / B$ ,  $\vec{v}_\perp \cdot \vec{n} = 0$ ,  $v_\parallel = c_s$ , noting that in low beta plasmas the Alfvén velocity is much larger than the sound velocity  $v_A \gg c_s$ . The formulation with the sheath-compatible boundary conditions (2) is almost identical to the standard variational principle, with the exception of only one non-symmetric term

$$\delta W_d = \frac{1}{2} \int_{S_d} \vec{e} \cdot \delta \vec{j}_s dS, \quad (3)$$

where  $\delta \vec{j}_s = \vec{n} \times \langle \delta \vec{B} \rangle$  is a perturbed surface current at the divertor plates and the integral is taken over divertor plate surface. Assuming the absence of surface currents on divertor plates (e.g. due to toroidal gaps) we end up with a variational formulation with the only difference due to a special treatment of the boundary conditions at the divertor plates, where  $\vec{B} \cdot \vec{n} \neq 0$ . The sensitivity to the way in which the conditions (2) are implemented can be checked by setting even more stringent line-tying condition  $\vec{\xi} = 0$  at the divertor plates. It turns out that the difference between ideal MHD growth rates corresponding to the sheath-compatible boundary condition or the line-tying boundary condition (the latter leading also to  $\delta W_d = 0$ ) for medium- $n$  kink-balloon modes localized at the outboard side of the plasma is rather weak. In the unstructured grid stability code MHD\_NX the divertor plate boundary condition treatment is quite natural: the ideal MHD stability condition  $\vec{e} \cdot \vec{B} = 0$  is just replaced by the condition  $\vec{e} \cdot \vec{B} \times \vec{n} = 0$  there. The vacuum treatment is automatically consistent because of the tangential electric field continuity  $\langle \vec{e} \times \vec{n} \rangle = 0$  condition at the plasma-vacuum boundary which is natural for the vector Whitney basis functions used in the approximation [2].

**3. Divertor tokamak: pedestal stability with currents in SOL.** New versions of the CAXE and KINX equilibrium and stability codes were applied to the analysis of the peeling-balloon modes (PB) stability of ITER plasmas. The edge stability limits in terms of ideally conducting plasma width in the SOL were investigated for the 15MA plasma, assuming  $\psi_b / \psi_{sx} = 0.995$  cutoff to get the profiles with finite parallel current density at the separatrix, as described in [5]. Figure 1a shows the limiting values of the conducting plasma width in the SOL (measured at the equatorial plane) as a function of toroidal mode number and for various profiles. The original profiles of the pressure gradient and parallel current density in the pedestal (marginally stable against peeling-balloon modes) were scaled by a factor of 1.25 (in the legend: PJC1.25). So figure 1a demonstrates that about 1 cm of conducting plasma in the SOL can lead to 25% increase of the pedestal height due to the stabilization of external PB modes. Very high pressure gradient in the SOL (3 times the separatrix value with exponential falloff in  $\sim 1$  mm: x3PSOL) will give approximately the same pedestal height limit as rescaled pedestal profiles within the separatrix (with zero parallel current in SOL: SOL0) for  $n < 20$ . For higher  $n$ 's the localized pressure gradient in the SOL is more unstable due to the lack of second stability access. Having a finite parallel current in the SOL does not

change this behavior for  $n>20$ , but can result in low- $n$  mode destabilization once its amplitude is large enough. Note that having large SOL parallel current density at the separatrix leads to a strong X-point angle decrease, and only by shifting the maximum of  $j_{\parallel}$  from the separatrix the values of the poloidal current in the SOL larger than 100kA can be attained for ITER plasma.

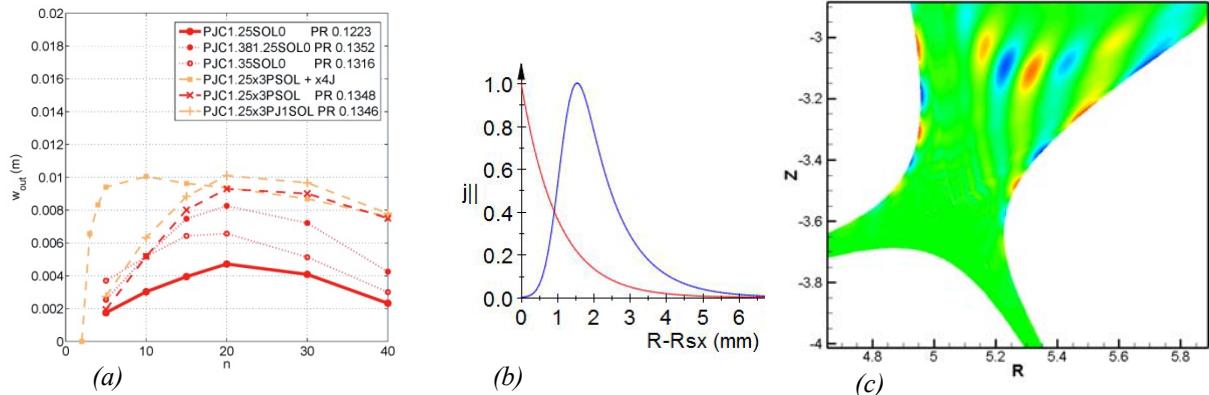


Figure 1. a) Critical widths of conducting plasma outside the separatrix for stability of PB modes with different toroidal wave numbers. The case with current-less mantle (red solid line) is compared to the cases with high pressure gradient in SOL (dashed) and to cases with rescaled pedestal pressure (dotted). The orange dashed lines correspond to cases with finite parallel current density in SOL: crosses are for the continuous  $j_{\parallel}$  across the separatrix, filled squares are for 4 times higher amplitude of  $j_{\parallel}$  shifted from the separatrix b) normalized profiles of  $j_{\parallel}$ : exponential with 1mm falloff length (red), shifted from the separatrix (blue) c) level lines of normal plasma displacement for the  $n=5$  mode for the case with large  $j_{\parallel}$  in SOL .

**Conclusions.** A large variety of tokamak relevant 2D equilibria without nested magnetic surfaces are of theoretical and experimental interest. MHD equilibrium and stability codes have been developed to study the stability for configurations with different topologies of magnetic surfaces using the unstructured grid approach applicable to arbitrary flux surface topology. The suite of equilibrium and stability codes for diverted plasmas with SOL is ready for use in edge stability calculations and integration with other plasma edge modeling codes. The role of SOL currents in equilibrium limits, edge stability and ELM triggering is under investigation. Also, 2D models of magnetic islands are to be further studied.

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