Influence of toroidal ripple on peeling-ballooning stability in the pedestal of H-mode plasmas

T. Weyens[1], A. Loarte[1], G. Huijsmans[2,4], R. Sánchez[3], L. García[3]

[1] *ITER Organization, Route de Vinon sur Verdon, 13067 Saint Paul Lez Durance, France* — [2] *CEA, IRFM, F-13108 Saint-Paul-Lez-Durance, France* — [3] *Departamento de Física, Universidad Carlos III de Madrid, Spain* — [4] *Department of Applied Physics, Eindhoven University of Technology*

Introduction

Toroidal field (TF) coils generate a large toroidal magnetic field that forms the backbone of the tokamak concept for magnetic toroidal confinement schemes. Though the number of TF coils employed is generally large, for example 32 in the case of JET and 18 for ITER, the construction is inherently discrete in nature. This causes the toroidal magnetic field to have a small ripple: It is about 0.08% in the case of JET and it is projected to be about 0.3% for ITER, with the help of ferritic insets. The reason why the ripple should be low is that experience with tokamaks shows that a large ripple degrades confinement in the H-mode regime substantially, which is characterized by a steep gradient in plasma pressure near the edge of the plasma, called the pedestal. Experiments verified this by varying the ripple in the JET and JT-60U tokamaks and observing the changes occuring to the pedestal properties [Sai+07].

The aim of this work is to address 3-D topics such as the TF ripple using the new PB3D code [Wey+17] based on the high-n edge MHD stability theory created in [Wey+14]. PB3D (Peeling-Ballooning in 3-D) is capable of assessing the edge stability of fluted modes such as peeling-ballooning modes, of general 3-D equilibria. An important hypothesis that is to be tested here is, therefore, that the mechanism of degradation of pedestal properties by larger TF coil ripple is through these high-n modes, which are thought to be unstable only for these equilibria with larger ripples, and that prevent the higher pedestals of equilibria with smaller ripples to be reached.

In the next section, the PB3D code and the theory behind it is are discussed concisely. Following this, in the next section, the axisymmetric equilibria and their stability are discussed. Subsequently, the results on the stability analysis for ripple cases are stated. And finally a discussion of the results, conclusions and suggestions for further research are given.

PB3D

The PB3D code is capable of assessing MHD stability of high-n modes that are allowed to perturb the last closed flux surface of the plasma. High-n modes, also called fluted modes, are characterized by having fast variations in the directions accross the magnetic field lines, while varying slowly along them, as sketched in figure 1. This is the structure of peeling-ballooning modes, which are a class of modes that are observed in tokamaks and that are commonly associated with the appearance of Edge Localized Modes (ELMs) that occur in the H-mode regime and that periodically expell hot matter to the plasma, hereby limiting the maximum height pedestals can achieve.

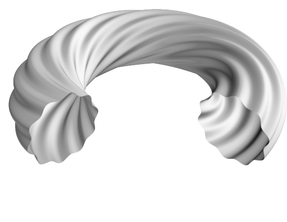

Fig. 1: fluted mode

The theory behind PB3D is based on ideal MHD, considering small (linear) perturbations around equilibriums configurations through a perturbation vector $\boldsymbol{\xi}$, the Eulerian derivative of which is the velocity vector of the perturbation. Normal modes $\sim e^{i\omega t}$ are then employed in the time dimension, introducing a complex frequency ω that indicates whether the mode is stable or unstable. Furthermore, Fourier series with mode numbers n and m are chosen to represent the toroidal and poloidal angular dimensions θ and ζ. The high-n assumption is then introduced through the magnetic field label $\alpha = \zeta - q\theta$ on which the (α, ψ, θ) coordinate system is based: The α coordinate determines on which magnetic field line a point is situated, the ψ coordinate can be a general flux function, but in this work is chosen to be the poloidal magnetic flux divided by 2π, and the θ coordinate now corresponds to the direction along the magnetic field lines. Using this, the perturbation vector becomes

$$\boldsymbol{\xi}(\boldsymbol{r},t) = \sum_{n,m} \hat{\xi}_{n,m}(\psi)\, e^{i\omega t} e^{in\alpha} e^{i(nq-m)\theta} \ , \tag{1}$$

where the exponent in α now varies much faster than the exponent in θ. An approximation is therefore made by assuming the modes pertaining to different field lines α are not coupled, so the system can be solved for each field-line individually, which results in a single summation only in m.

This form for the perturbation is then plugged into the generalized energy principle [GP04, sec. 6.6], that treats the system of a plasma surrounded by an infinite vacuum, in order to allow for possible edge perturbations of the plasma. The extremal values of the Rayleigh quotient are then sought, which is defined as the ratio of the perturbed potential energy of the system divided by the kinetic energy of the pertubation: $\Lambda = \delta W_{\mathrm{p}}/K$.

This is done by decomposing the perturbation in three orthogonal components, of which the two corresponding to the stable sound waves and fast magnetosonic waves can be minimized individually, resulting in an expression solely in terms of the Fourier amplitudes X_m, the components normal to the flux surfaces. After minimizing the resulting functional for each of these X_m through Euler miminzation, he system of equations takes on the shape of a system of second order linear differential equations containing an eigenvalue ω^2:

$$\sum_{m}^{M} \left(P_{k,m} - \omega^2 K_{k,m}\right)[X_m] = 0 \qquad k = 1 \ldots M \ . \tag{2}$$

In PB3D this system of equations is discretized using central finite differences of arbitrary order and the resulting generalized eigenvalue problem is solved to obtain the eigenvalues ω^2 and their corresponding eigenvectors. The code is written with speed and parameter scans in mind through parallelization and extensive use of fast external libraries. PB3D has been verified using MISHKA and COBRA [Wey+17].

Axisymmetric results

The 3-D rippled equilbria are created starting from a stylized D-shape axisymmetric configuration with major radius $R_0 = 2.96m$, aspect ratio $\varepsilon = 0.3$ and ellipticity $\kappa = 1.7$, with multiple pedestal pressure heights but constant total pressures, as shown in figure 2. Furthermore, the pedestal width is about 4.75cm, the total toroidal current $I_{\mathrm{tor}} = 2.6MA$, the vacuum magnetic field at the geometric axis $2.1T$ and the poloidal beta 1.0.

The high-n stability of these axisymmetric configurations is investigated for various primary mode numbers n Figure 3 shows the growth rate for these different mode numbers n and different pedestal heights, normalized with respect to the geometrical major radius and the vacuum toroidal magnetic field at the geometrical axis. As expected, the main instability of these configurations is the ballooning instability, of which the growth rate increases towards the ideal infinite-n limit. This is verified using the

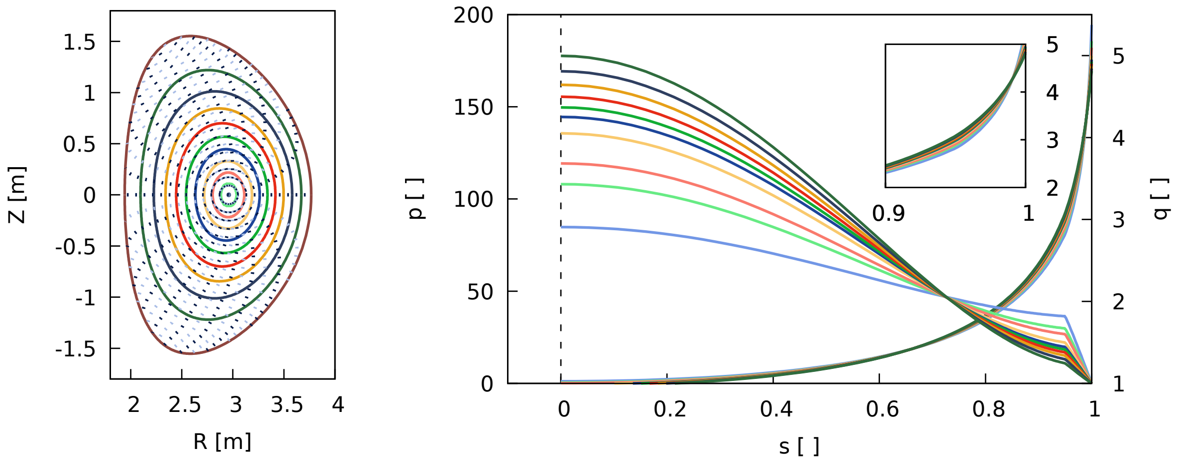

Fig. 2: (left) Cross-section. (right) Pressure and safety factor for different pedestal heights.

COBRA numerical code [San+00] that returns the infinite-n ideal ballooning limit. The COBRA results are always more unstable than the ones calculated by PB3D, but within 9%.

For the purpose of the TF ripple, the most relevant information that can be extracted from these results is the lowest primary mode number n at which the plasma is still unstable, called the marginal mode number. Figure 3 shows this as a function of the pedestal pressure gradient, as well as results for simulations where the pedestal width is varied for constant height. Note that below about 3.5kPa/cm everything is stable.

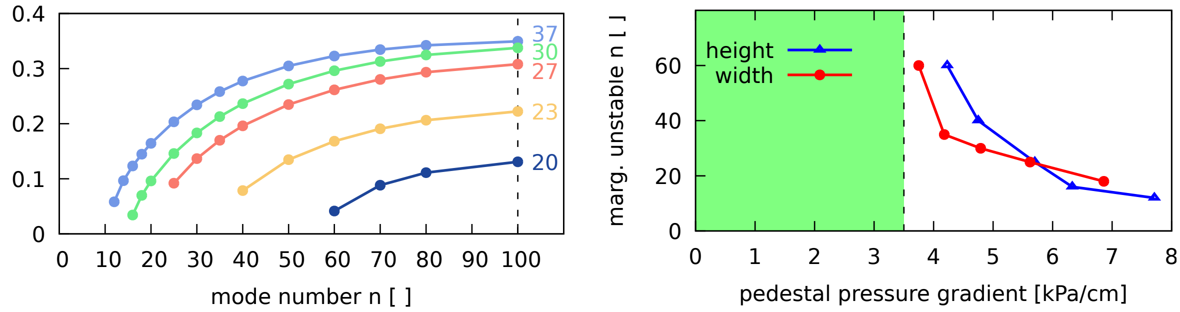

Fig. 3: (left) Normalized growth rate for unstable pedestal heights. Numbers on the right indicate pedestal pressure in kPa. (right) Marginal n for different pedestal pressure gradients obtained through variation of the height with constant width or the width with constant height.

From this point of view it can therefore be reasoned that the maximum achievable pressure gradient is about 3.5kPa/cm The question is now: *What happens when ripples are introduced?*

TF coil ripple

To investigate this, ripples $\delta_r(\theta,\zeta)$ in the position of the last flux surface are added to the equilibrium with a sinusoidal toroidal dependency with period 16 and a poloidal profile that is mostly situated on the outboard side, through the formula $\delta_r \sim \left[\frac{1}{2}(1+\cos(\theta))\right]^3$ The amplitude of the ripple is varied from 0cm to 6cm through steps of 1cm and the high-n stability of the resulting equilibria is analyzed using PB3D, as well as COBRA.

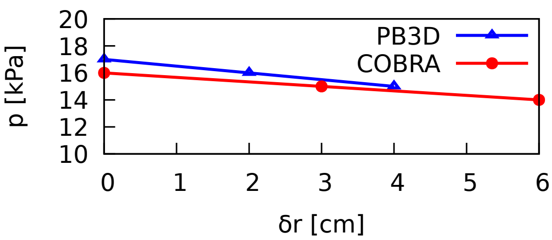

Fig. 4: Marg. stable pedestal pressure gradient.

The marginal pressure heights are shown in figure 4, for various position ripple amplitudes δ_r. Note that indeed, the marginal pressure height decreases for larger ripples. This is due to the toroidal modification of the curvature, which has a stabilizing effect in the toroidal positions of the coils (where the plasma is at it narrowest) but this is overcompensated by the destabilization in the positions in between the coils.

Discussion, conclusions and future work

Using the high-n edge MHD stability results from the PB3D code the stability of a variety of rippled equilibrium cases were studied. A decrease in marginal pedestal height was found for higher ripple amplitudes.

The experimental results from [Sai+07] showed a similar trend, but for ripple strengths that were much smaller. From analysis of the ripple results with POST, the accompanying post-processing program to PB3D, it is found that the maximum toroidal field ripple of about 1% used commonly in literature, as well as in this reference, is equivalent to a position ripple δ_r of only about 2.5mm. These results, obtained from VMEC equilibria with ripple, show that pressure gradient considerations alone clearly cannot explain the pedestal degradation through the TF ripple.

Work is well on its way now, to include the bootstrap current in the equilibria, which might be an important ingredient of these ripple effects, as the same experiments have failed to come up with a noticeable ripple influence for plasmas with high-collisionality and therefore low bootstrap current. Furthermore, work is also being done on the topic of shaping, which might play a role too, with more accurate X-point approximations possible through adaptation of EFIT equilbirium reconstruction results. Also, the pedestal structure might have to be described more accurately. In particular, the pedestals found in JET are generally less wide and less high than the ones used here. Finally, the simple sinusoidal toroidal dependency of the position ripple should be scrutizined with more accurate free-boundary equilibrium simulations through VMEC. On the other hand, it is also possible that other effects play a role, such as thermal ion losses. For the analysis of these effects more complete and also expensive numerical codes will have to be used.

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References

[GP04] J. P. Hans Goedbloed and Stefaan Poedts. *Principles of Magnetohydrodynamics*. Cambridge University Press, 2004.

[Sai+07] G. Saibene et al. "The H-mode pedestal, ELMs and TF ripple effects in JT-60U/JET dimensionless identity experiments". In: *Nuclear Fusion* 47.8 (2007), pp. 969–983.

[San+00] R. Sanchez et al. "COBRA: An Optimized Code for Fast Analysis of Ideal Ballooning Stability of Three-Dimensional Magnetic Equilibria". In: *Journal of Computational Physics* 161.2 (2000), pp. 576–588.

[Wey+14] T. Weyens et al. "Three-dimensional linear peeling-ballooning theory in magnetic fusion devices". In: *Physics of Plasmas* 21.4 (2014).

[Wey+17] T. Weyens et al. "PB3D: A new code for edge 3-D ideal linear peeling-ballooning stability". In: *Journal of Computational Physics* 330 (2017), pp. 997–1009.