New Possibilities of Probe Registration of Anisotropic Distribution Functions of Charged Particles in Plasmas with Arbitrary Symmetry

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There are analyzed the accuracy and the systematic errors of the known method of a flat one-sided probe for measuring the anisotropic distribution functions of electrons and ions in a plasma. It is shown that, with the exception of strong fields, ten members of the series are usually enough for an adequate description of the ion distribution function. For highly anisotropic distribution functions of charged particles, a technique of spline interpolation of experimental data is proposed and tested, which significantly reduces the systematic error for a given number of terms of the series. For frequently encountered real plasma objects with "mirror symmetry" it is shown that the number of necessary measurements is substantially reduced in comparison with the most general case of absence of any symmetry.

In the works [1, 2] it was proposed the probe method for determining the anisotropic electron velocity distribution functions (EVDF) as a finite series of Legendre polynomials and later this method was developed in [3, 4]. The number of terms of the series depends on the type of symmetry of the problem and on the number of orientations of the flat one-sided or cylindrical probe, for which the dependences of the second derivative of the current on the probe by the potential on the probe is registered. A similar method applied to the ion distribution function (IDF) using a flat one-sided probe was proposed in [5, 6].

When plasma is under certain conditions [1, 2, 5], the second derivative of the probe current $I''(eU,\alpha)$ by the probe potential and the DF $f_\varepsilon(E,\theta)$ are related the following way:

$$I''(eU,\alpha) = A\left[f_\varepsilon(eU,\alpha) - \frac{1}{2\pi}\int_0^{2\pi} d\varphi' \int_{eU}^{\infty} \frac{\partial f_\varepsilon(E,\theta')}{\partial eU} dE\right], \tag{1}$$

To determine $f_\varepsilon(E,\theta)$ first one must measure the dependence of the second derivative of the probe current by the probe potential U at different angles α. Then the second derivative and DF are represented as a finite series in terms of Legendre polynomials whose number of terms, for example, for an axial-symmetric case is equal to the number of orientations of the probe N [1, 2]:

$$I''(eU,\alpha_i) = A\sum_{k=0}^{N} D_k(eU)L_k(cos\alpha_i), \tag{2}$$

$$f_\varepsilon(E,\theta) = \sum_{k=0}^{N} C_k(E) L_k(\mu). \tag{3}$$

Then, using the experimental data and the expansion (1), the coefficients of the expansion of the second derivative $D_k(eU)$ are found. Finally, from these coefficients, the coefficients in the analogous expansion of the DF $C_k(eU)$ are calculated using the known relationship [4]:

$$C_k(eU) = D_k(eU) + \frac{1}{2eU}\int_{eU}^{\infty} D_k(\varepsilon)\frac{dL_k(x)}{dx}_{x=\sqrt{\frac{\varepsilon}{eU}}} d\varepsilon. \tag{4}$$

1. Dependence of the necessary number of orientations of the probe on the plasma parameters when measuring the ion distribution function.

Consider the situation when the IDF has an axial symmetry. Studies show that in the case when the only cause of anisotropy is the presence of an electric field in the plasma, the IDF calculated only taking into account the charge exchange is the most anisotropic one of all possible. So the number of terms of the series (3) necessary for an adequate description of the IDF, which corresponds to the process of resonant charge exchange and does not take into account other processes, is an upper value for the number N of necessary terms of the Legendre series for a plasma under any conditions. Using the results of [5, 6], the following relation was found for such value $\varepsilon_{max}(N,\varepsilon_0,T_a,\mu)$, that for all such ion energies $\varepsilon < \varepsilon_{max}(N,\varepsilon_0,T_a,\mu)$ the following relationship is satisfied:

$$\Delta(E,\mu) = \frac{|R_N(E,\mu)|}{\bar{f}_\varepsilon(E,\mu)} < \delta, \tag{5}$$

where $\mu = cos\theta$; $\bar{f}_\varepsilon(E,\mu) = f_\varepsilon(E,\theta)$ is the energy DF; $R_N(E,\mu)$ is the remainder of the series in the approximation of the DF as a finite series of Legendre polynomials with exact coefficients $C_k(eU)$; $\varepsilon_0 = \frac{k_0 T_a}{eE_0\lambda_{ex}}$; E_0 is the electric field; λ_{ex} is the mean free path of the ion with respect to the charge exchange process, T_a is the temperature of the neutral particles and k_0 is the Boltzmann constant. For example, Fig. 1 shows the dependence $\varepsilon_{max}(N,\varepsilon_0,T_a,1)$ on the parameter ε_0 for various N and $T_a = 600\ K$.

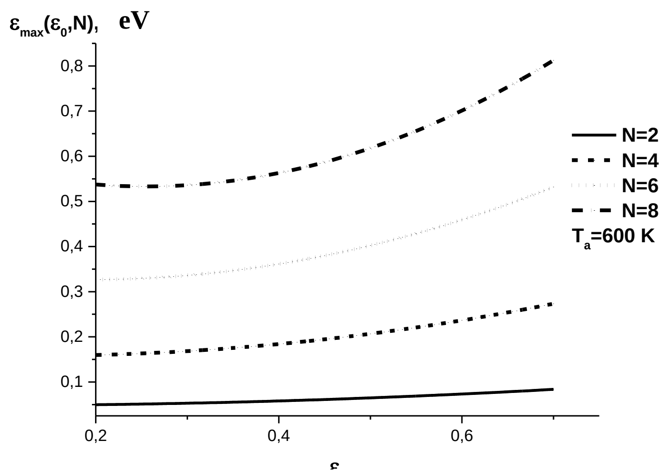

Fig. 1. Dependence of $\varepsilon_{max}(N, \varepsilon_0, T_a)$ on various N at a temperature of the atoms $T_a = 600$ K.

2. Application of the theory of splines to reduce the number of orientations of the probe while maintaining the accuracy of the determination of the DF in a gas-discharge plasma with "mirror symmetry".

Let us consider some possibilities to reduce the systematic errors in the probe registration of anisotropic DF of charged particles in the form of finite series of the Legendre polynomials. As before, we consider the axial-symmetric case. We define the remainder of the series $R_{NI}(eU,\alpha)$ during the approximation of the second derivative of the probe current $I''(eU,\alpha)$ as a finite series with exact coefficients $D_k(eU)$ by the formula:

$$R_{NI}(eU,\alpha) = A\sum_{k=N}^{\infty} D_k(eU)L_k(cos\alpha)\,; D_k(eU) = (k+0.5)\int_{-1}^{1} I''(eU,\alpha)L_k(cos\alpha)dcos\alpha \quad (6)$$

Using the definition of $L_k(\mu)$ [20], we get:

$$R_N(E,\mu) = \sum_{k=N}^{\infty} \frac{(k+0.5)}{2^k k!} L_k(\mu) \int_{-1}^{1} \overline{f_\varepsilon}(E,\mu)^{(k)}(1-\mu^2)^k d\mu. \quad (7)$$

It is seen from the relation (7) that the larger the value of the maximum of the derivative of the function with respect to the argument μ, the larger the remainder of the series at a fixed value N, and the more terms of the series are required for an adequate description of the function $\overline{f_\varepsilon}(E,\mu)$ with a finite series. In the representation of the quantity $I''(eU,\alpha)$ as a finite series, an error is introduced a priori in the determination of the coefficients of the series (3), since in the equation (2) with the sum on the right-hand side up to the maximum $k_{max} = N$, on the left one have the exact experimentally found value of the second derivate $I''(eU,\alpha)$, which is itself a sum of an infinite number of terms, so the equality with $k_{max} = N$ is an approximation and is satisfied with the coefficients ${D_k}^N(eU) = D_k(eU) + \Delta {D_k}^N(eU)$ different from $D_k(eU)$ when expanding it into an infinite Legendre series. It is possible to obtain an expression for the error in the determination of the DF caused by these factors:

$$\Delta\bar{f}_{\varepsilon ex}(E,cos\alpha_i) = \frac{1}{2eU}\left\{\sum_{k=0}^{N} L_k(cos\alpha_i)\int_{eU}^{\infty} \Delta {D_k}^N(\varepsilon)\frac{dL_k(x)}{dx}_{x=\frac{\varepsilon}{eU}} d\varepsilon - \sum_{k=N}^{\infty} L_k(cos\alpha_i)\int_{eU}^{\infty} D_k(\varepsilon)\frac{dL_k(x)}{dx}_{x=\frac{\varepsilon}{eU}} d\varepsilon\right\}, \quad (8)$$

where $\Delta\bar{f}_{\varepsilon ex}(E,cos\alpha) = \bar{f}_{\varepsilon ex}(E,cos\alpha) - \bar{f}_\varepsilon(E,cos\alpha)$.

Thus, we see that there is a systematic error in determining the angular dependence of the DF when representing the second derivative as a finite series of Legendre polynomials even for angles $\alpha = \alpha_i$, for which the second derivative is determined exactly (that is, without systematic error).

There is an alternative way of solving the problem. Namely, using N measurements of the probe current, it is possible to approximate the dependence on the angle $I''(eU,\alpha)$ for different values of the potential U by using splines (e.g. of third order) and then calculate a number of Legendre coefficients (as necessary for an accurate description of the interpolation results) $M > N$ for the second derivative, using the resulting smooth curves. To illustrate the above relations calculations were carried out for the model DF.

In Fig. 2 is illustrated the significant decrease in the error of the description of experimental data of the second derivate of the probe current due to the use of splines.

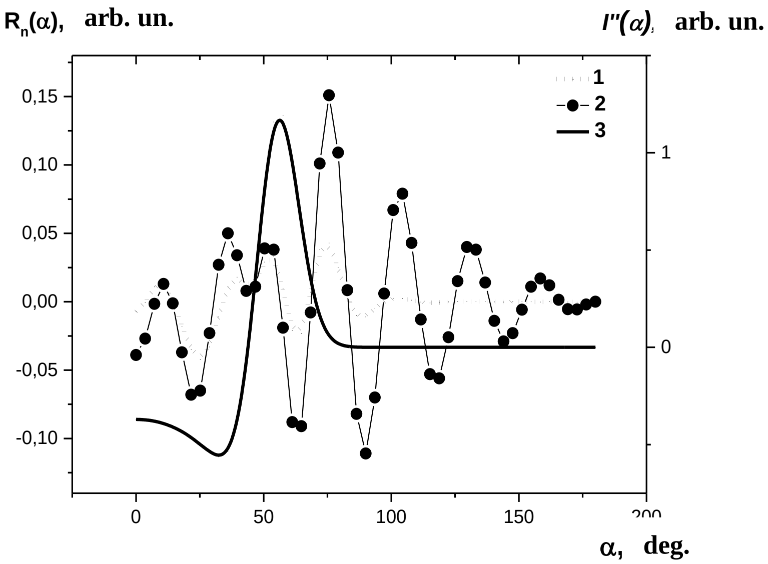

Fig. 2. Difference between the exact value of $I''(eU,\alpha)$ and the second derivative of the probe current, calculated as a series of 14 Legendre polynomials and as a series of 25 Legendre polynomials with spline interpolation over 14 points for $\Delta_T E = 1$ эВ; $eU = 0.1$ эВ; $\beta_1 = 0.02$ with a imposed random error of 10%; 1 - result of using splines (Y axis on the left); 2 - calculation in the form of a series of 14 Legendre polynomials (Y axis on the left); 3 - $I''(eU,\alpha)$, calculated by the formula (1) (the Y axis on the right).

Now consider the plasma, which has the property that there exists a plane in respect to which the properties of the plasma symmetric. We call such symmetry a mirror symmetry. Then, choosing the coordinate system XYZ so the plane ZX coincides with the plane of symmetry, we obtain that the DF is even in respect to the azimuthal angle φ. Such a situation is realized in the practice very often, for example, when probe measurements are performed in a cylindrical discharge not in the axis (including in the region near the wall). In this case, as it is easy to show, the number of probe orientations in the measurements for determining N Legendre coefficients is $0.5N\ (N + 1)$ rather than N^2, as in the absence of any type of symmetry.

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