Suppression of parametric instabilities induced by lower hybrid waves

C. Castaldo[1], F. Napoli[1]

[1]ENEA, Fusion and Nuclear Safety Department, C. R. Frascati, Via E. Fermi 45, 00044 Frascati (Roma), Italy

Abstract

In the framework of the study of parametric instabilities involving lower hybrid wave propagation in a magnetized plasma, this work presents a new nonlinear parametric dispersion equation, based on a kinetic model, taking into account the collisional effects, useful to analyze the instabilities emerging in the outer layers of a tokamak plasma. For typical parameters of present day LHCD experiments, we compare the numerical solutions of the full parametric dispersion equation in collisionless plasma with the numerical solutions obtained in both collisional and collisionless case, considering only the particle dynamics parallel to the equilibrium magnetic field. The role of the electron temperature and the ion composition are also investigated in order to find outer plasma conditions useful to suppress the parametric instabilities in future fusion reactor scenarios.

Introduction

Microwave power coupled to tokamak plasmas produces high frequency (HF) density perturbations which may nonlinearly couple with low frequency (LF) plasma density fluctuations, driving them unstable [1,2]. These instabilities may grow in space and in time, broadening the incident pump wave spectrum versus the parallel wavenumber and causing the lower hybrid (LH) absorption near the surface of the plasma column, preventing its penetration to the interior [3,4]. Thus it is of great importance to know for present day LHCD experiments under which circumstances parametric instabilities occur.

In this paper we improve our previous modeling of parametric instabilities [5,6] deriving a full kinetic description of the LH wave propagation, taking into account the collisional effects, along with a more accurate modeling of the nonlinear mode coupling of the pump wave with ion-sound modes. This was achieved performing a more accurate perturbative analysis of the kinetic equations retaining terms up to the third order and taking into account the self-consistent electric field with the charge density perturbation. As in previous works [5,6,7], in order to describe the nonlinearity in the outer layers of a tokamak plasma, we consider dominant the particle dynamics parallel to the equilibrium magnetic field. Then, we evaluate the growth rates of the parametric instabilities in different edge plasma conditions concerning the electron temperature and the ion composition.

Collisional parametric dispersion equation

In order to obtain a full kinetic description of the nonlinear mode coupling of the HF lower hybrid waves with the LF plasma fluctuations, in the presence of collisional effects, the general Maxwell-Boltzmann system of equations is solved by means of a perturbative method. The Boltzmann equation is reduced to a 1D kinetic equation in velocity space to take into account only the particle dynamics parallel to the confinement magnetic field. We use a particle conserving BGK collision operator [8] and assume that the velocity distribution function for both ion and electron populations is isotropic in the perpendicular direction. The analysis is limited to fluctuations at frequencies much smaller than the ion cyclotron frequency ($\omega_{LF} \ll \omega_{ci}$) and characteristic lengths in directions perpendicular to the static magnetic field much larger than the ion Larmor radius ($k_\perp \rho_i \ll 1$). We use a slab geometry with the static magnetic field $\boldsymbol{B}_o = B_o\hat{\boldsymbol{z}}$ and with the inward radial direction oriented as $\hat{\boldsymbol{x}}$. We consider steady-state solutions for homogenous plasma. These assumptions allow us to solve the Maxwell-Boltzmann system of equations by means of a spectral method. Following a perturbative analysis up to the third order, we obtain the following hierarchy of kinetic equations. At the first order we have the HF linear electron response:

$$-j(\omega - k_z v_z)\,\tilde{g}_e^{(1)} + \frac{q_e}{m_e}\tilde{E}_z^{(1)}\partial_{v_z} g_e^{(0)} = -\nu_e\left\{\tilde{g}_e^{(1)} - g_e^{(0)}\left(\tilde{n}_e^{(1)}/n_e^{(0)}\right)\right\} \quad (1)$$

The ion species, due to their large inertial mass, cannot follow the high frequency oscillations. At the second order, we have the LF nonlinear electron response and the LF ion linear response:

$$-j(\omega - k_z v_z)\tilde{g}_e^{(2)} + \frac{q_e}{m_e}\tilde{E}_z^{(2)}\partial_{v_z} g_e^{(0)} + \frac{q_e}{m_e}\mathrm{C}\left[\tilde{E}_z^{(1)}, \partial_{v_z}\tilde{g}_e^{(1)}\right] = -\nu_e\left\{\tilde{g}_e^{(2)} - g_e^{(0)}\left(\tilde{n}_e^{(2)}/n_e^{(0)}\right)\right\} \quad (2)$$

$$-j(\omega - k_z v_z)\tilde{g}_i^{(2)} + \frac{q_i}{m_i}\tilde{E}_z^{(2)}\partial_{v_z} g_i^{(0)} = -\nu_i\left\{\tilde{g}_i^{(2)} - g_i^{(0)}\left(\tilde{n}_i^{(2)}/n_i^{(0)}\right)\right\} \quad (3)$$

At the third order, we have the HF nonlinear electron response:

$$-j(\omega - k_z v_z)\tilde{g}_e^{(3)} + \frac{q_e}{m_e}\tilde{E}_z^{(3)}\partial_{v_z} g_e^{(0)} + \frac{q_e}{m_e}\mathrm{C}\left[\tilde{E}_z^{(1)}, \partial_{v_z}\tilde{g}_e^{(2)}\right] + \frac{q_e}{m_e}\mathrm{C}\left[\tilde{E}_z^{(2)}, \partial_{v_z}\tilde{g}_e^{(1)}\right] = -\nu_e\left\{\tilde{g}_e^{(3)} - g_e^{(0)}\left(\tilde{n}_e^{(3)}/n_e^{(0)}\right)\right\} \quad (4)$$

where in the equation above C[.] denotes the convolution operator in the spectral domain $\{\omega, k_x, k_y, k_z\}$, $\tilde{n}_\alpha^{(m)} = \int dv_z\, \tilde{g}_\alpha^{(m)}$, $g_\alpha^{(0)}$ is the local equilibrium distribution function that here we assume Maxwellian, i.e. $g_\alpha^{(0)} \equiv n_\alpha^{(0)} e^{-(v_z^2/v_{\mathrm{th},\alpha}^2)}/\sqrt{\pi}\, v_{th,\alpha}$, $n_\alpha^{(0)}$ is the unperturbed density for the α species (with mass m_α and charge q_α), $v_{th,\alpha} = \sqrt{2\,T_\alpha/m_\alpha}$ is the thermal velocity, the temperatures T_α are measured in energy units and ν_α is the collisional relaxation which can be approximated by $\nu_e = 2.9\,10^{-6}\lambda_C n_i^{(0)} T_e^{-3/2}$ $\nu_i = 4.8\,10^{-8}\lambda_C n_e^{(0)} T_i^{-3/2}\mu_i^{-1/2}$ with μ_i is the ion mass in units of the proton mass and λ_C is the Coulomb logarithm.

From Maxwell equations, taking the electrostatic limit, we obtain the following collisional nonlinear dispersion relation:

$$\varepsilon - \left(1+\sum_i \chi_i\right)\frac{e^2 k_{z0}^2 \omega_{pe}^2 |\phi_0|^2}{m_e T_e k_z v_e^2 \omega_0^2}\left[\frac{1}{\varepsilon_1}\frac{k_{z1}^2}{k_1^2}\frac{A}{(1+j\,\gamma_{e1}\,S(u_{e1}))}+\frac{1}{\varepsilon_2}\frac{k_{z2}^2}{k_2^2}\frac{B}{(1+j\,\gamma_{e2}\,S(u_{e2}))}\right]=0 \quad (5)$$

We considered a four wave interaction between a pump wave $\phi_0 e^{-j(\omega_0 t-\boldsymbol{k_0}\cdot\boldsymbol{r})}$, a lower sideband $\phi_1 e^{-j(\omega_1 t-\boldsymbol{k_1}\cdot\boldsymbol{r})}$, an upper sideband $\phi_2 e^{-j(\omega_2 t-\boldsymbol{k_2}\cdot\boldsymbol{r})}$ and a low frequency quasi-mode $\phi\, e^{-j(\omega t-\boldsymbol{k}\cdot\boldsymbol{r})}$, with selection rules $\omega=\omega_{1,2}\pm\omega_0$, $k_z=k_{z1}\pm k_{z0}$. Here ε is the low frequency dielectric function, $\varepsilon_{1,2}$ are the high frequency dielectric functions, S is the plasma dispersion function, χ_i is the ion susceptibility, and the limit of dominant particle dynamics parallel to the equilibrium magnetic field is considered. We also have:

$$A \equiv \left[\frac{F_e(u_e,u_{e0})}{k_{z0}}+\frac{F_e(u_e,u_{o,e1})}{k_{z1}}\right] \qquad B \equiv \left[\frac{F_e(u_e,u_{e0})}{k_{z0}}+\frac{F_e(u_e,u_{e2})}{k_{z2}}\right] \quad (6)$$

where $u_\alpha \equiv v_z/v_{th,\alpha}$, $u_\alpha \equiv (\omega + j\,\nu_\alpha)/k_z v_{th,\alpha}$, $\gamma_\alpha \equiv \nu_\alpha/k_z v_{th,\alpha}$, $u_{eh} \equiv (\omega_h + j\,\nu_e)/k_{zh} v_{th,e}$, $\gamma_{eh} \equiv \nu_e/k_{zh} v_{th,e}$ $(h=0,1,2)$ and $F_e(u_e,u_e')$ is the coupling function:

$$F_e(u_e,u_e') \equiv \frac{F_a(u_e,u_e')}{(1-j\,\gamma_e\,S(u_e))}+\frac{2\,j\,\gamma_e'\,C_2(u_e')\,F_b(u_e,u_e')}{(1-j\,\gamma_e\,S(u_e))(1-j\,\gamma_e'\,S(u_e'))} \quad (7)$$

$$F_a(u_e,u_e') \equiv \frac{C_1(u_e)-C_1(u_e')}{(u_e'-u_e)} - \frac{u_e\,(C_1(u_e')+S(u_e))-2\,u_e'^2\,C_2(u_e')}{(u_e'-u_e)^2} \qquad F_b(u_e,u_e') \equiv \frac{C_2(u_e)}{(u_e'-u_e)}+\frac{1}{2}\frac{S(u_e')-S(u_e)}{(u_e'-u_e)^2}$$

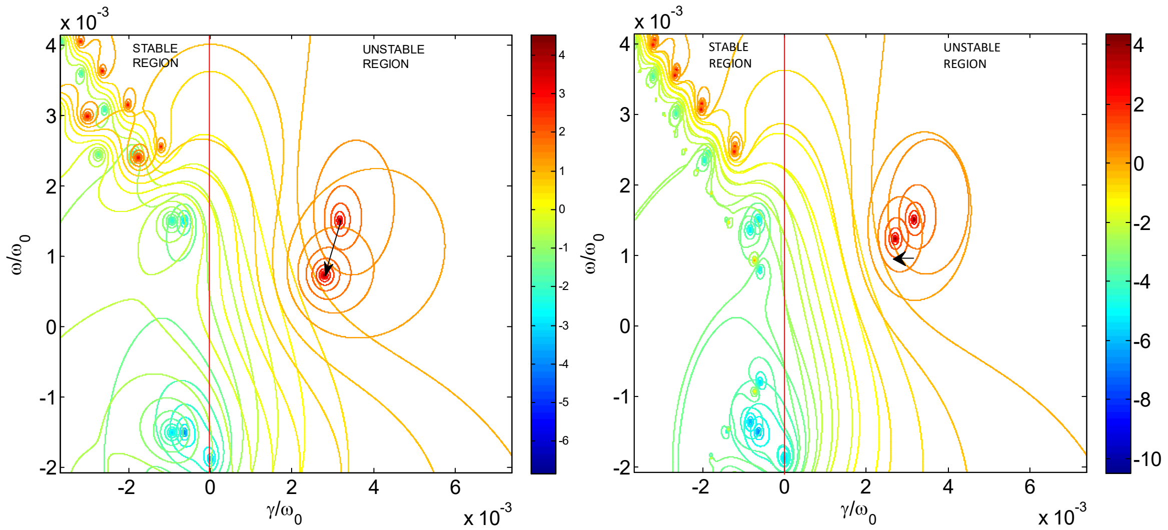

Figure 1 – Contour plot of $Log_{10}(1/|x|)$ where x is the lhs of Eq. (5). It is assumed a collisionless Deuterium plasma with $n_e = 2\,10^{12} cm^{-3}, T_e = T_i = 10\ eV, P_{LH} = 3.57\ kW/cm^2, N_{z0} = 2.0$, $f_0 = 8\ GHz$, $\Delta N_z = 10$. In (left) we show the effect of the increment of the electron temperature $\Delta T_e = 10\ eV$; in (right) we consider a plasma of 50% Deuterium and 50% Lithium (^{7}Li). The vertical line separates stable ($\gamma < 0$) from unstable ($\gamma > 0$) region. The arrow highlights the shift of the unstable zeros toward the stable region.

Numerical results and conclusions

We first solved Eq. (5) for the RF plasma parameters typical for LH experiments on the FTU tokamak [9] for a collisionless plasma. In Fig. 1 we show the contour plot of $Log_{10}(1/|x|)$

where x is the lhs of Eq. (5) for two electron temperature (left) and two plasma compositions (right). Larger electron temperature and heavier ion species reduce the parametric instabilities, in agreement with previous findings [10]. In Fig. 2 (left) we compare the numerical solutions of Eq. (5) with an approximate analytical solution [7] in the limit of parallel dynamics as well as with the numerical solutions of the full parametric dispersion relation [3]. Our preliminary results seem to overestimate growth rates though the order of magnitude is in agreement with previous results. This problem is still under investigation, mainly concerning numerical issues. In Fig. 2 (right) we show the effect of collisions. The growth rate of the instability is slightly reduced by collisions, as suggested by previous analysis [6,11].

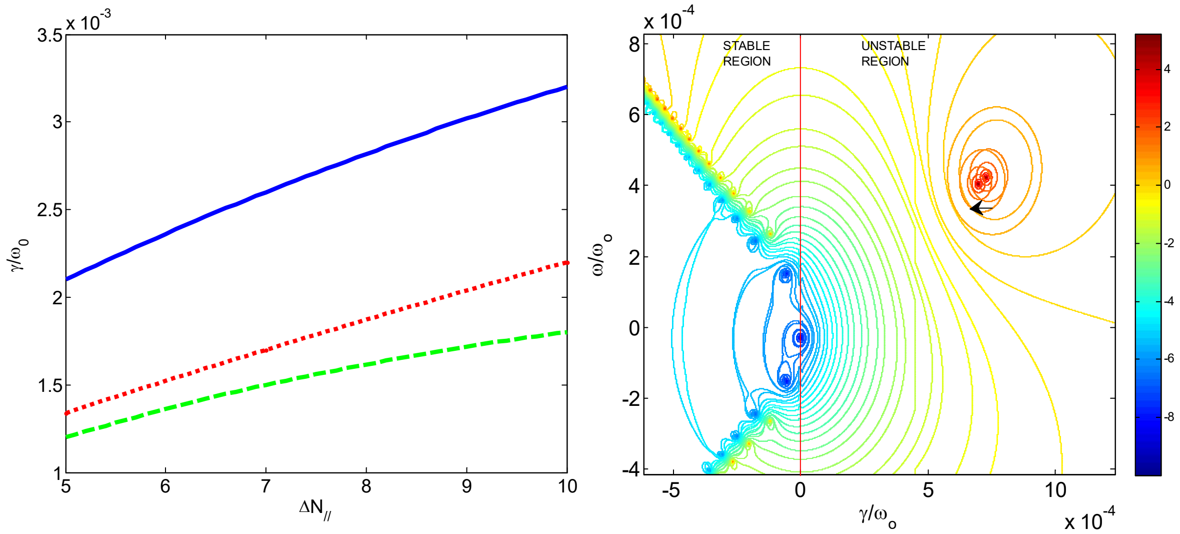

Figure 2 – Left: growth rates of the low frequency modes normalized to the angular frequency ω_0 as a function of $\Delta N_z = k_z c/\omega_0$ for a collisionless plasma with parameters as in Fig. 1. Numerical solutions of Eq. (5), continuos line, are compared with analytical solutions from [7], dashed line, and with numerical solutions of the full kinetic dispersion equation from [3], dotted line. Right: Contour plot of $\mathrm{Log}_{10}(1/|x|)$ where x is the lhs of Eq. (5). We compare a collisionless and collisional Deuterium plasma with parameters as in Fig. 1 with $\Delta N_z = 1$. The vertical line separates stable ($\gamma < 0$) from unstable ($\gamma > 0$) region. The arrow highlights the shift of the unstable zero toward the stable region.

References

[1] C. S. Liu, V. K. Tripathi *Phys. Rep.* **130** (1986) 143

[2] M. Porkolab, et al. *Phys. Rev. Lett.* **38** (1977) 230

[3] R. Cesario, et al., *Nucl. Fusion* **54** (2014) 043002

[4] R. Cesario, et al., *Nature Comms*, **1** (2010) 55

[5] F. Napoli, et al., *Plasma Phys. Contr. Fusion* **55** (2013) 095004

[6] C. Castaldo, et al., *Nucl. Fusion* **56** (2016) 016003

[7] A. Zhao, Z. Gao, *Nucl. Fusion* **53** (2013) 083015

[8] P. L. Bhatnagar, E. P. Gross, M. Krook, *Phys. Review* **94** (1954) 511

[9] C. Gormezzano, et al., *Fus. Sci. Technol.* **45** (2004) 3

[10] F. Napoli, et al., *AIP Conf. Proc.* **1580** (2014) 450

[11] M. Porkolab, *Phys. Fluids* **17** (1974) 1432