An Automated Approach to Plasma Breakdown Design with Application to WEST

H. Heumann[1], E. Nardon[2], and the WEST team*

[1] *CASTOR, Inria SAM, Sophia Antipolis, & LJAD, University Cote d'Azur, Nice, France*

[2] *IRFM, CEA, Saint-Paul-lez-Durance, France*

Introduction Plasma breakdown in a tokamak requires a large toroidal electric field E_ϕ and a low poloidal magnetic field B_p, i.e. a so-called *field null region*. The latter should remain as extended as possible for a sufficient duration (typically a few tens of *ms*), all the more if one operates at low E_ϕ (e.g. in ITER where $E_\phi = 0.3V/m$). Finding appropriate settings (i.e. pre-magnetization coils currents and voltage waveforms) to produce and maintain a good field null region is not a trivial task, in particular in the presence of highly conducting passive structures which make the problem dynamic. WEST [3, 2] is a good example of this situation, due to two toroidally continuous copper plates which have been added for vertical stabilization: indeed, the current in the plates ramps up fast when E_ϕ is applied, which tends to degrade the field null region. Our automated approach to determining appropriate breakdown settings relies on a precise electromagnetic model of the machine (including the iron core) and solves a constrained optimization problem, where the objective function to be minimized quantifies the design goal: the averaged magnitude of B_p. The approach follows the lines of optimal control methods for plasma equilibria in [1] and [4].

Automated Approach to Plasma Breakdown Design The breakdown is governed by the eddy current model, which, in axisymmetry and in terms of the poloidal flux ψ, writes as

$$-\nabla\cdot\left(\frac{1}{\mu[\psi]r}\nabla\psi(t)\right) = \begin{cases} \sum_j \mathbf{S}_{ij}V_j(t) + \sum_k \mathbf{R}_{ik}\int_{\mathscr{C}_k}\partial_t\psi(t) & \text{in } i\text{-th coil } \mathscr{C}_i\,, \\ -\frac{\sigma}{r}\partial_t\psi(t) & \text{in passive structures } \mathscr{S}\,, \\ 0 & \text{elsewhere}\,, \end{cases} \tag{1}$$

with ψ vanishing at the magnetic axis and at infinity:

$$\psi(t)_{|r=0} = 0; \qquad \lim_{\|(r,z)\|\to+\infty}\psi(r,z,t) = 0\,. \tag{2}$$

The coefficients $\mathbf{S}_{ij}$ and $\mathbf{R}_{ik}$ follow from the electric circuits connecting the suppliers and the coils of the poloidal field system. The magnetic permeablility is a non-linear functional of ψ in

*http://west.cea.fr/WESTteam

the iron core $\mathscr{F}$. The initial poloidal flux at $t = t_0$ verifies

$$-\nabla \cdot \left(\frac{1}{\mu[\psi] r} \nabla \psi(t_0) \right) = \begin{cases} j_{\mathscr{C}_i}(t_0) & \text{in } i\text{th coil } \mathscr{C}_i\,, \\ j_{\mathscr{S}}(V_{\text{loop}}) & \text{in passive structures } \mathscr{S}\,, \\ 0 & \text{elsewhere}\,. \end{cases} \tag{3}$$

Hence, the free design or control parameters for the breakdown are the current densities $j_{\mathscr{C}_i}$ in coils at t_0 and the evolution of the voltages $V_i(t)$ in the suppliers of the poloidal field system. A discretization of (1) and (3), e.g. a finite element discretization in space combined with explicit Euler in time, allows to study breakdown scenarios in simulating the evolution of the poloidal flux for various choices of the design parameters. These equations are the so-called *direct mode*, and they can be used to replay a shot or to test settings in simulations.

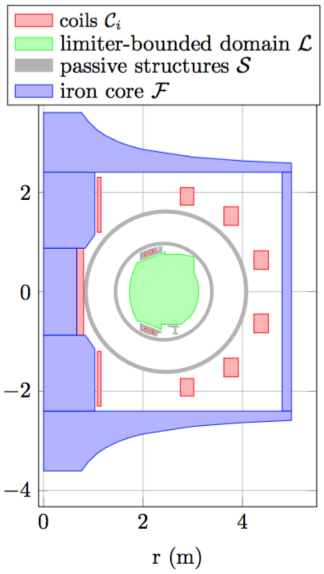

Figure 1: Illustration of the different subdomains on (1) and (3).

Finding satisfactory control parameters by plain trial-and-error performing repeatedly numerical simulations of the underlying eddy current model can be very time consuming. Moreover, the non-linearities due the magnetic permeability in iron transformer tokamaks like WEST impede to build up an intuitive idea of the relationship between the control actuaries and the objective, which would allow to improve steadily the guesses. To overcome the difficulties of plain trial-and-error we prefer to formulate breakdown design as a constrained optimization problem,

$$\min_{\vec{V}(t), j_{\mathscr{C}_i}(t_0), \psi(t)} \frac{1}{2} \int_0^T \int_G |\nabla \psi(t_i)|^2 dr dz dt + \frac{1}{2} \vec{I} \mathbf{W}_0 \cdot \vec{I} + \int_0^T \frac{1}{2} \vec{V}(t) \mathbf{W}(t) \cdot \vec{V}(t) dt \,, \tag{4}$$
$$\text{such that } \psi(t), V(t) \text{ and } j_{\mathscr{C}_i} \text{ verify (1), (2) and (3)}$$

where the objective function to be minimized quantifies the design goal: the average size of the poloidal component of the magnetic field in the focus area G. Penalization terms involving weights $\mathbf{W}_0$ and $\mathbf{W}(t)$ ensure convexity and hence solvability. After discretization of (4) we end up with *finite dimensional* convex constrained optimization problems, that can be solved efficiently with *sequential quadratic programming*. We refer to [4, Section 3] and [1] for more details on the finite element discretization of (1), (2) and (3) and how this is extended to an efficient procedure for solving (4). Clearly, various other objective functions are possible in (4). Our current implementation of (4) in the axisymmetric free-boundary equilibrium code

FEEQS.M is very flexible in this respect and we can easily include any other design goal that is encoded as a function of ψ. FEEQS.M[1], is a MATLAB implementation of the methods for axisymmetric free boundary plasma equilibria that are described in [4]. The code utilizes in large parts vectorization, and therefore, the running time is comparable to C/C++ implementations. FEEQS.M provides interfaces to the *virtual* tokamak SIMULINK workflow at IRFM that is used to develop control strategies for the WEST.

Analysis of WEST shot 50635 The prediction accuracy of control parameters via the proposed optimization framework hinges strongly on the quality of the numerical simulation of the breakdown itself. Therefore, we first show that our simulations reproduce fairly well magnetic measurements (see Fig. 2) and shapes that were observed on the fast camera (see Fig. 3) during breakdown experiments on WEST. The replay of WEST shot 50635 starts 20ms before the breakdown phase (i.e. before the time $T_{ignitron}$ at which a strong negative voltage is applied on the central solenoid). The replay is done by initiating currents in the coils and passive structures from experimental data and then applying the experimental voltages in the power supplies.

We do not see any plasma current on the magnetics nor do we see any sign of closed flux surfaces on the camera images. We suspect here a vertical stability issue: the ramp up of currents in the stabilizing plates makes the field configuration vertically unstable. The first row in Figure 4 shows colormaps of the radial field B_r. Vertical stability requires $dB_r/dz > 0$ in our convention. Based on FEEQS.M modelling, it has been decided to remove the lower stabilizing plate in WEST. Indeed, with this modification FEEQS.M suggests that the vertical stability problem may be solved, although it is hard to draw a hard conclusion.

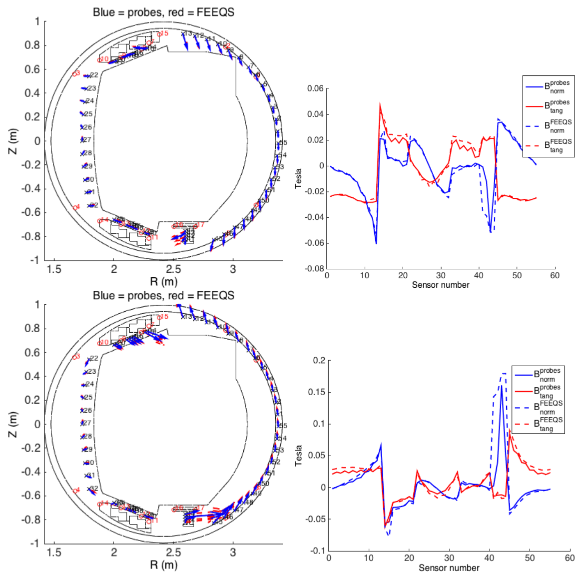

Figure 2: Experimental data and simulated data at $T_{ignitron} - 20$ ms and $T_{ignitron} + 80$ ms.

References

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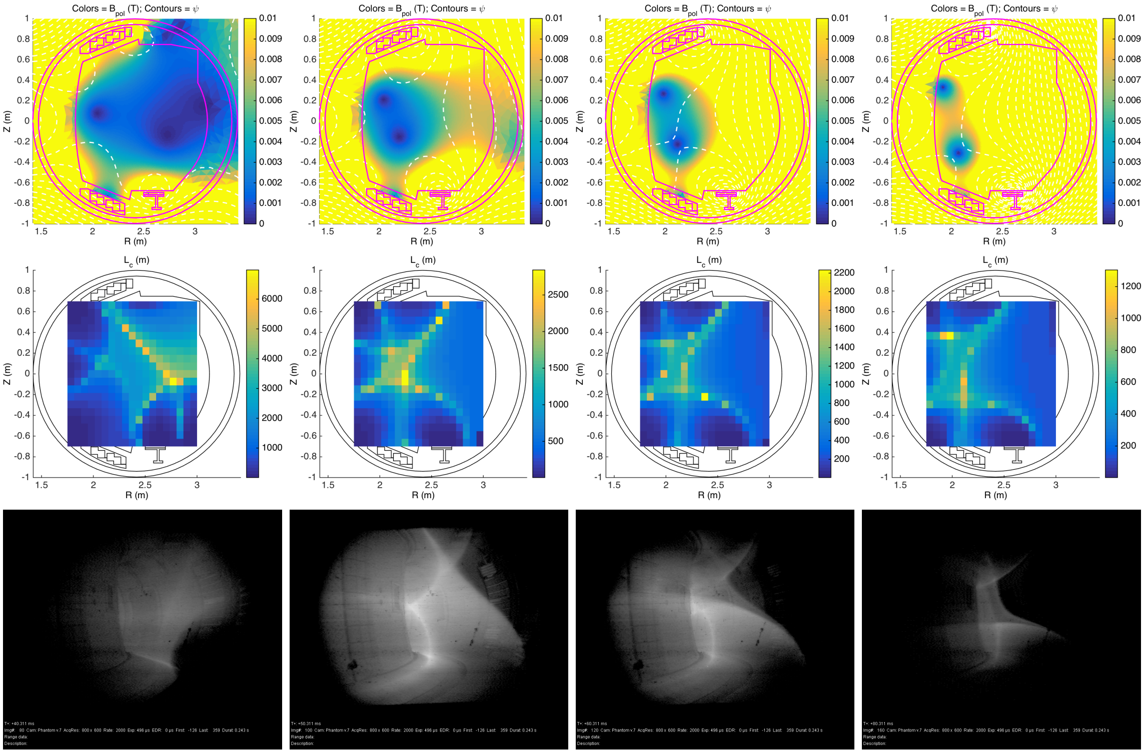

Figure 3: Model vs. camera images at $T_{ignitron}+40,50,60,80$ms: First row: poloidal field magnitude (color map) and iso-flux contours (white dashed lines); Second row: field lines connection length; Third row: snapshots from the fast camera.

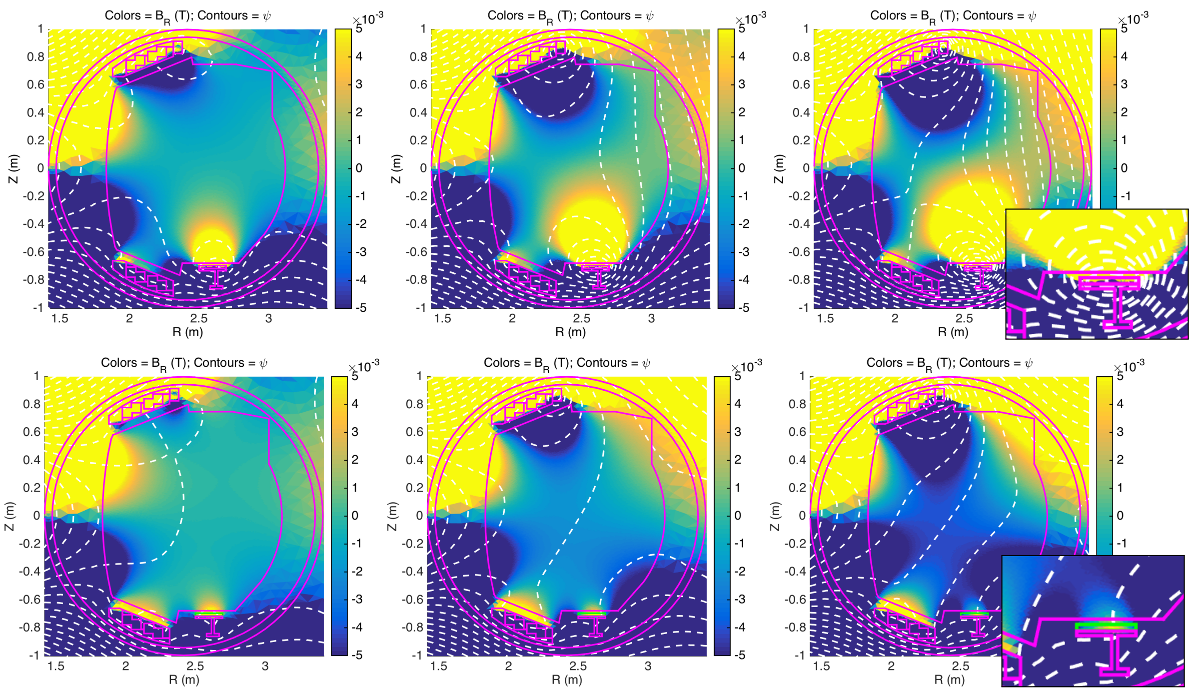

Figure 4: Colormaps of the radial field B_r for 1.) the *replay* of shot 50635 at $T_{ignitron}+40,50,60$ms (first row); 2.) for a scenario prediction of FEEQS.M without lower plate (light green) at $T_{ignitron}+30,40,50$ms (second row).