# Finite orbit width effects on resonant transport regimes of neoclassical toroidal viscous torque within the Hamiltonian approach

C. G. Albert[1], M. F. Heyn[1], S. V. Kasilov[1,2], W. Kernbichler[1], A. F. Martitsch[1]

[1] *Fusion@ÖAW, Institut für Theoretische Physik - Computational Physics, Technische Universität Graz, Petersgasse 16, 8010 Graz, Austria*

[2] *Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Akademicheskaya Str. 1, 61108 Kharkov, Ukraine*

## Introduction

Recently analytical theory of neoclassical toroidal viscous torque (NTV) has been extended to take finite orbit width into account [1]. Such effects can be important for tokamaks with resonant magnetic perturbations (RMPs) such as e.g. in Ref. [2]. Within the Hamiltonian approach for resonant transport regimes [3] this case is naturally included if quantities are evaluated for full orbits. Firstly this concerns computation of canonical actions, angles and frequencies, secondly canonical (bounce) averages of the magnetic perturbation, and thirdly conservation laws involving integration over canonical angles used to compute particle transport and torque. For equilibrium fields of characteristic length larger than the orbit width the first modification should lead only to small changes while increasing complexity in pre-computation and interpolation of frequencies. The latter two modifications can however be done without substantial modifications of the original methods and should lead to more accurate results for perturbations with radial variations on the scale of the orbit width. Here quasilinear results for NTV torque are computed within this approximation for a typical case of a medium-sized tokamak with RMPs.

## Integral torque and orbit averages

To take the full orbit into account, flux surface averaged conservation laws should be replaced by volumetric conservation laws where we perform integrals along the full orbit including radial drift away from the flux surface of $r = r_\varphi$, where $r_\varphi$ is given by an implicit relation to the canonical toroidal momentum $p_\varphi = -\frac{e_\alpha}{c}\psi_{\mathrm{pol}}(r_\varphi)$ including species charge $e_\alpha$, speed of light $c$, and $\psi_{\mathrm{pol}}$ the poloidal flux function. It should be emphasized that while $r$ is a radial coordinate, the quantity $r_\varphi$ measures the toroidal momentum of orbits and coincides with the radius of the banana tip for trapped orbits, where their parallel velocity vanishes. Fixing $r = r_\varphi$ as in the small orbit width approximation would result in the torque generated by orbits of toroidal momentum $p_\varphi$ to be localized at $r_\varphi$. In contrast to that a full orbit contributes to sources on all flux surfaces that it passes, the integral quasilinear toroidal torque evaluated over the full plasma volume is

$$T_\varphi^{\mathrm{int}} = \int \mathrm{d}^3 r\, T_\varphi = -\sum_{\mathbf{m}} n \int \mathrm{d}^3\theta \int \mathrm{d}^3 J\, \frac{\pi}{2} \left|H_{\mathbf{m}}\right|^2 \delta(\Omega) m_k \frac{\partial f_0}{\partial J_k}, \tag{1}$$

where integration over the toroidal torque density $T_\varphi$ defined in Eq. (18) of Ref. [3] is performed over phase-space spanned by actions $J = (J_\perp, J_\vartheta, p_\varphi)$ and canonical angles $\theta$ with harmonics of a Hamiltonian perturbation $H_\mathbf{m}$ in canonical angles and using the resonance condition $\Omega = m_j\Omega^j = 0$ for canonical frequencies $\Omega^j = \Omega^j(J)$ and harmonic indices $m_j$ (see [3]). Here and in Eq. (1) summation is performed over repeated indexes and $n = m_3$ is the toroidal mode number. By the definition of the partial integral torque $T_\varphi^{\mathrm{int}}(r_a, r_b)$ over the volume between two flux surfaces of radius $r_a$ and $r_b$, Eq. (1) can be written as a sum over contributions in finite radial regions, $T_\varphi^{\mathrm{int}} = \sum_{k=1}^{N} \Delta T_\varphi^{\mathrm{int}}(r_{k-1}, r_k)$, with $r_0 = 0$ and $r_N$ at the separatrix, where

$$\Delta T_\varphi^{\mathrm{int}}(r_a, r_b) \equiv \frac{\pi^{3/2} n_\alpha c v_{T\alpha}}{e_\alpha \mathrm{sgn}(\psi'_{\mathrm{pol}})} \int \mathrm{d}r_\varphi \sum_{\mathbf{m}} \sum_{\mathrm{res}} n \int_0^\infty \mathrm{d}u\, u^3 e^{-u^2} \Delta\tau_{ab} |H_\mathbf{m}|^2 \left| \frac{\partial \Omega}{\partial \eta} \right|^{-1}_{\eta_{\mathrm{res}}} (A_1 + A_2 u^2). \quad (2)$$

Here the integral over $\theta$ has been evaluated, variables $J_\perp, J_\vartheta, p_\varphi$ changed to $u, \eta, r_\varphi$ (see [3]) with $\eta = \eta_{\mathrm{res}}$ fixed at the resonance, $n_\alpha$ is the particle number density, $v_{T\alpha}$ the thermal velocity, $u = v/v_{T\alpha}$ the normalized velocity, and $\Delta\tau_{ab}$ the time that the orbit spends inside the radial interval $(r_a, r_b)$. We define thermodynamic forces via the unperturbed distribution function $f_0$,

$$A_1 = \frac{1}{n_\alpha}\frac{\partial n_\alpha}{\partial r} + \frac{e_\alpha}{T_\alpha}\frac{\partial \Phi}{\partial r} - \frac{3}{2T_\alpha}\frac{\partial T_\alpha}{\partial r}, \qquad A_2 = \frac{1}{T_\alpha}\frac{\partial T_\alpha}{\partial r}, \quad (3)$$

with temperature $T_\alpha = m_\alpha v_{T\alpha}^2/2$, species mass $m_\alpha$ and potential $\Phi$, and are evaluated at $r_\varphi$. Despite the fact that all mentioned formulas are valid for all orbit sizes, in Ref. [3] an approximation of small orbit width has been used. Here, using a full orbit average to compute $H_\mathbf{m}$ can change $T_\varphi^{\mathrm{int}}$ in comparison to the small orbit width approximation. We write the Hamiltonian perturbation $\tilde{H}(r, \vartheta, \varphi) = \sum_n H_n(r, \vartheta) e^{in\varphi}$ by a Fourier series in the toroidal angle $\varphi$ of straight field line flux coordinates $x = (r, \vartheta, \varphi)$ defined for the unperturbed axisymmetric equilibrium. Velocity space dependencies of $\tilde{H}$ via constants of motion are omitted in the notation. For sufficiently small radial gradients of unperturbed quantities and toroidal harmonic number $n$, harmonics $H_\mathbf{m}$ in canonical angles are formally similar to the case of small orbit width,

$$H_\mathbf{m} = \left\langle H_n(r(\vartheta), \vartheta) e^{inq(\vartheta(\tau) - \vartheta_0) - i(m_2 + nq\delta_{\mathrm{tp}})\omega_b \tau} \right\rangle_b, \quad (4)$$

where $m_2$ is the canonical poloidal (bounce) harmonic, $\tau$ is the time in the orbit, $0 < \tau < \tau_b$, $\omega_b = 2\pi/\tau_b$ is the bounce frequency, $q$ the safety factor, and the poloidal angle $\vartheta$ is evaluated along the orbit starting from $\vartheta_0$. In contrast to the case of small orbit width, changes of the quantity $H_n$ on the radial scale similar to the orbit width are taken into account here by evaluation of the radial position $r(\vartheta)$ during integration along the orbit.

**Results for RMPs in a medium-sized tokamak**

Computations for finite deuterium ion orbit width have been performed in an extended version of the code NEO-RT for a non-axisymmetrically perturbed plasma in a medium-sized tokamak

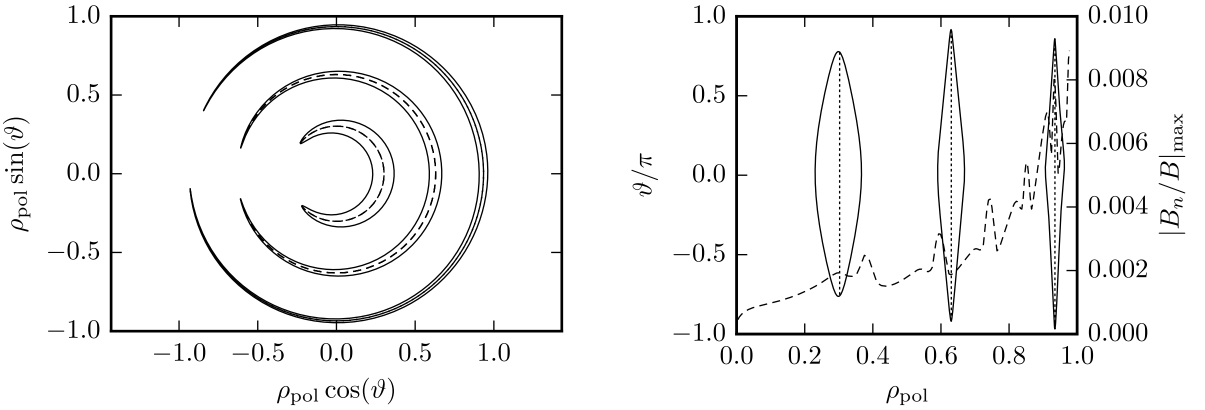


Figure 1: Left: Examples of different ion orbits of at thermal velocity. Wide orbits at small $\rho_{\mathrm{pol}}$ becoming eventually potato orbits limit the range of applicability near $\rho_{\mathrm{pol}} = 0$. Near the separatrix $\rho_{\mathrm{pol}} = 1$ orbits are narrow, but also the radial scale of the fields becomes smaller. Right: Comparison of the scale of orbits (solid) to the maximum perturbation amplitude (dashed line). Averaging over orbits can introduce a significant modification of the perturbation strength.

with RMPs. For this purpose a first order correction in Larmor radius is used to approximate full orbits. The radially local toroidal torque density is given by the derivative of Eq. (2) with respect to $r_b$ weighted by the derivative of $r$ with respect to the volume, $\mathrm{d}r/\mathrm{d}V$. The normalized radial variable $\rho_{\mathrm{pol}} = \sqrt{\psi_{\mathrm{pol}}/\psi_{\mathrm{pol}}^a} \in (0,1)$ is used in figures, where $\psi_{\mathrm{pol}}^a$ is the value of the poloidal flux function at the separatrix. Figure 1 shows examples of full trapped orbits of different $p_\varphi$ and their relative size with respect to the perturbation amplitude. The range of validity in this particular computation is limited by the first-order radial width reaching unphysical values of $r < 0$ or beyond the separatrix. For ions at thermal velocity $v_{T\alpha}$ a lower radial limit is reached for strongly counter-passing orbits with negative parallel velocity $v_\parallel \approx -v_{T\alpha}$. The upper limit is realized for orbits with $v_\parallel > 0$ reaching beyond the separatrix. Toroidal torque density $T_\varphi$ and partial integral torque $\Delta T_\varphi^{\mathrm{int}}(0,r)$ evaluated up to a certain radius are shown in Figure 2 for both, small orbit width approximation and finite orbits. Finite orbit width leads to radial shifts and smoothing of $T_\varphi$. The overall result for the integral torque $T_\varphi^{\mathrm{int}}$ is lower by a factor of two for the computation with finite orbit width. In particular resonances of counter-passing orbits which are shifted radially inwards from $r_\varphi$ lead to a reduction of toroidal torque, since the perturbation amplitude is strongly reduced at smaller radii.

**Conclusion**

The possibility of taking finite orbit width into account for the computation of neoclassical toroidal viscous torque in resonant transport regimes within the Hamiltonian approach has been demonstrated. The procedure has been applied to a representative medium-sized tokamak

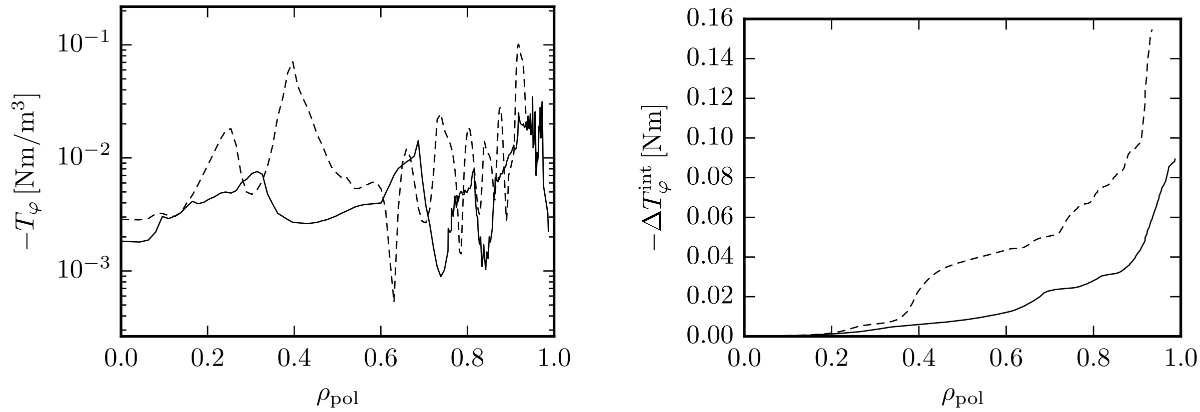


Figure 2: Radial dependency of torque density (left) and integral torque evaluated up to the radial position on the x-axis (right) for small (dashed) and finite orbit width (solid). Contributions from a resonance of counter-passing orbits at $\rho_{\mathrm{pol}} \approx 0.4$ vanish for finite orbit width due to a smaller pertubation amplitude at the actual orbit radius closer to the magnetic axis. Finite orbit width has a strong influence on results for both, torque density and integral torque.

plasma with resonant magnetic perturbation and compared to computations in the small orbit width approximation. In this case a strongly modified torque density profile is observed together with a reduction of the integral toroidal torque by a factor of two. The former can be explained by a radial redistribution of toroidal torque that does not influence global integral torque, while the latter is likely caused by a different effective perturbation strength experienced by radially displaced resonant orbits. The magnitude of the observed effect leads to the conclusion that finite orbit width effects could considerably influence ion NTV torque in cases similar to the one discussed here. Note that in case the effect of finite orbit width on NTV is significant, this width should also be taken into account in computations of the perturbation field.

**Acknowledgements**

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission. The authors gratefully acknowledge support from NAWI Graz and funding from the OeAD under the grant agreement Wissenschaftlich-Technische Zusammenarbeit mit der Ukraine No UA 04/2017

**References**

[1] K. C. Shaing and S. A. Sabbagh, Physics of Plasmas **23**, 072511 (2016).

[2] A. F. Martitsch *et al.*, Plasma Phys. Contr. Fusion **58**, 074007 (2016).

[3] C. G. Albert *et al.*, Phys. Plasmas **23**, 082515 (2016).