Predicting maximum ion energies in Target Normal Sheath Acceleration using a sheath theory for arbitrary electron distributions

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Target Normal Sheath Acceleration (TNSA) is a method for accelerating ions using high intensity laser pulses hitting solid density targets. Relativistic electrons travel through the target forming a space charge sheath at the rear surface. The electric field in this sheath accelerates ions to high energies. For pulse durations shorter than the electron traversal time the fast electrons forming the sheath will have a non-equilibrium distribution with a beam like component. For longer times the electrons can reach equilibrium in the form of the Maxwell-Jüttner distribution.

Most previous theories invoke a Boltzmann factor to model the electron density. This implies that the fast electrons have reached an equilibrium distribution. In this contribution a kinetic theory of a planar rear sheath for arbitrary electron distribution functions is presented. It is found that the far field is determined by the high energy tail of the distribution.

When accounting for electrons escaping the sheath region a finite potential drop over the sheath is found. This finite potential drop implies a maximum energy for ions being accelerated in the sheath field. The results are generalised to spherical sheaths. For a realistic electron distribution for short laser pulses, maximum ion energies of around 66MeV are predicted.

We assume the case of a planar target, consisting of electrons and heavy, immobile ions. The ion density n_i is assumed to be constant inside the target and zero in the region outside the target. The analysis makes use of the normalised quantities

$$\varphi = -\frac{e\Phi}{kT_h}, \qquad p = \frac{\tilde{p}}{\sqrt{m_e kT_h}}, \qquad x = \frac{\tilde{x}}{\lambda_D}, \qquad n_k = \frac{\tilde{n}_k}{n_0} \qquad (1)$$

and the dimensionless parameter $\alpha = \sqrt{\frac{m_e c^2}{kT_h}}$. Here we use a tilde to denote the unnormalised quantities. Φ is the electrostatic potential, $\tilde{p}$ and $\tilde{x}$ are the un-normalised momentum and the position, $\tilde{n}_k$ is the number density of species k, T_h is a characteristic temperature of the fast electron distribution function, λ_D is the electron Debye length, and n_0 is the equilibrium electron density inside the target.

The electron density n_e is divided into a cold background population and a hot electron beam $n_e = n_c + n_h$. The cold electrons are assumed to be in thermodynamic equilibrium and their density is given by the Boltzmann factor

$$n_c = n_{c0} \exp(-\varphi T_h / T_c) \qquad (2)$$

where T_c is the cold electron temperature. The hot electrons, on the other hand, are modelled kinetically using the time independent, relativistic, electrostatic Vlasov equation

$$\frac{p}{\gamma}\partial_x f_h + \varphi' \partial_p f_h = 0. \tag{3}$$

Here $f_h = f_h(x, p)$ is the hot electron distribution function, and $\gamma = (1 + (p/\alpha)^2)^{1/2}$ is the relativistic gamma factor. Vlasov's equation assumes that the hot electrons are collisionless. In addition we have Poisson's equation

$$\varphi'' = n(x) \tag{4}$$

Using the method of characteristics together with energy conservation Vlasov's equation can be formally solved, resulting in the hot electron density as a function of the potential

$$n_h(\varphi) = 2 \int_0^\infty f_{\text{inj}}\left(\sqrt{p^2 + 2\frac{\varphi}{\alpha}\sqrt{p^2 + \alpha^2} + \frac{\varphi^2}{\alpha^2}}\right) dp. \tag{5}$$

In this equation f_{inj} is the distribution function of electrons injected into the system by the laser. The total charge density across the target and the sheath is then given by

$$n = H(x) - n_{c0} \exp(-\varphi T_h/T_c) - n_h(\varphi) \tag{6}$$

where $H(x)$ is the Heavyside step function describing the normalised ion density which is unity inside the target and zero outside. Given this charge density, Poisson's equation can be integrated to provide the potential in the sheath as well as inside the target.

We assume that the injected hot electrons can be described by a shifted Maxwell-Jüttner distribution

$$f_{\text{inj}}(p) = \frac{n_{h0}}{\alpha K_2(\alpha^2)} \exp\left(-\alpha\sqrt{\alpha^2 + (p - p_0)^2}\right) \tag{7}$$

where p_0 is the directed momentum component. The normalised average kinetic E_k energy of the Maxwell-Jüttner distribution is made up of the thermal energy $\theta = 1/\alpha^2$ and the directed beam energy $\sqrt{1 + \theta p_0^2} - 1$,

$$E_k = \theta + \sqrt{1 + \theta p_0^2} - 1. \tag{8}$$

In addition, let δ be the ratio of beam energy to the total kinetic energy

$$\delta = \frac{E_k - \theta}{E_k}. \tag{9}$$

Fig. 1 shows the potential profiles for $E_k = 1$, for different values of δ, and the corresponding electric fields. One can observe that the sheath potential is reduced for larger values of δ, i.e. when the hot electrons carry more of their energy in form of directed energy. At the target
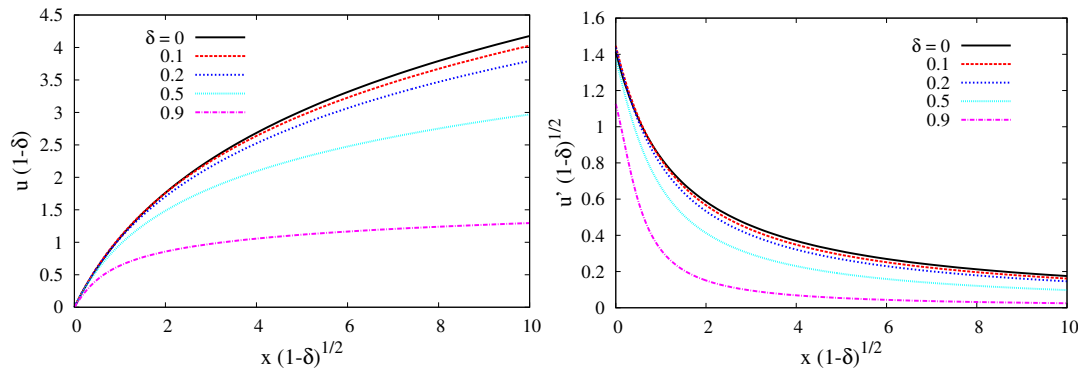
Figure 1: Normalised sheath potential profiles (left) and fields profiles (right) for $E_k = 1$ and different beam ratios δ.

surface, all profiles start off with a similar electric field, as can be seen in the right panel of Fig. 1 although the field is reduced slightly for the largest beam ratio. At some distance from the surface the field decreases with δ. This can be understood in terms of high energy tails of the electron distribution function. As the beam energy increases, the temperature decreases to keep the total energy constant. This means that there are less electrons in the high energy tails of the distribution. For lower temperatures the number of electrons that are located further from the target is reduced. Following Gauss' law this implies that the electric field is substantially reduced with the electron temperature.

In general we find that the sheath field is directly determined by the shape of the distribution function. A rapid decrease of the distribution function at high energies leads to a small electric field at large x and a slow increase of the potential with x. The field distribution becomes more kinked with increasing beam ratio, showing two distinct domains, an almost linear fall-off near the surface and a power law decay further away. The largest change in slope is located where the bulk of the electrons are reflected back towards the target's rear surface.

To determine a maximum sheath potential, and therefore the maximum ion energies, it is assumed that the high energy electrons can escape from the sheath by some mechanism and are therefore lost from the electron population. In order to allow the electrons to escape in a one dimensional model, we have to allow for a residual charge density n_r in the sheath region. This charge density allows a return current to be set up as the fastest electrons escape, preventing a further charge build-up of the target. The residual charge density can be very small and might be due to the pre-pulse or simply due to the non-ideal vacuum in the chamber. It is found that the dependence of the maximum ion enery on the residual charge density is logarithmic. As such, the exact numerical value of n_r has little influence on the results. We assume that n_r is due to the non-ideal vacuum in the experimental setup with a residual pressure of 10^{-4}Pa. The laser
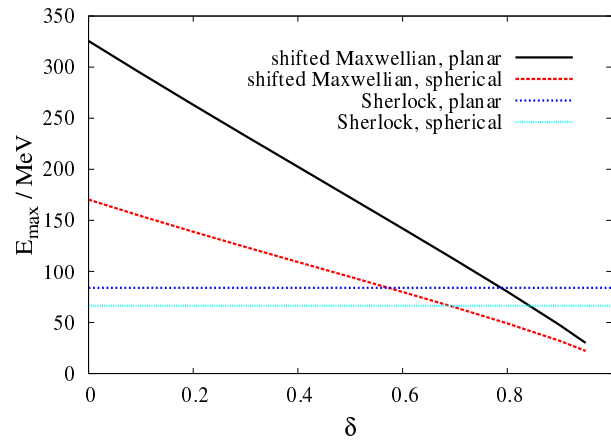
Figure 2: Maximum achievable ion energies for various models. Curves labelled "shifted Maxwellian" are calculated using the distribution function Eq. 7 and depend on the beam ratio δ. Curves labelled "Sherlock" are calculated using [2]. Energies are plotted for both the planar model and the spherical model.

has a wavelength of 1μm and an intensity of 10^{21}W cm^{-2}. Using the ponderomotive scaling the maximum achievable ion energies can be determined.

Fig. 2 shows the maximum achievable ion energies for fast electron distribution functions given by Eq. 7 compared with fast electron distrubtion given by Sherlock [2]. For a planar sheath Sherlock's distribution results in a normalised sheath potential of 6.31 and maximum energies of singly charged ions of about 84MeV. For a purely thermal distribution, Eq. 7 with $\delta = 0$, the potential drop is as high as 24.5 which corresponds to ion energies of 325MeV.

The model has been extended to account for geometrical effects in a spherical sheath. Using a spot radius $R = 5\mu$m and Sherlock's distribution function, we now obtain a maximum ion energy of 66MeV. In contrast for a pure Maxwellian, $\delta = 0$, the maximum achievable energy is reduced to 170MeV, and for an almost monoenergetic electron beam $\delta = 0.95$ this is reduced to as low as 22.2MeV.

These numbers should be seen as upper limits to the maximum ion energies. Assuming that Sherlock's distribution function, holds for ultra-short pulses, we can conclude that ion energies cannot exceed 66MeV. Of course other factors might reduce this limit further. Note that this result, for all other parameters held constant, scales directly with the fast electron temperature. An increase in laser intensity will therefore increase the ion energies by a factor given by the ponderomotive scaling.

References

[1] H. Schmitz, Phys. Plasmas **19**, 083115 (2012)

[2] M. Sherlock, Phys. Plasmas **16**, 103101 (2009)