Controlling Self-Injection in LWFA through Plasma Density Modulation

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Laser-wakefield accelerators (LWFA) present the exciting prospect of producing high quality electron beams from compact devices for radiation production and other applications in academia, industry, and healthcare. There is particular interest in the production of ultra-short (sub-femtosecond and attosecond) electron bunches and radiation pulses for use as probes of ultra-fast physical, chemical and biological processes.

Contemporary LWFA experiments typically function in the so-called "bubble" or "blowout" regime [1]. This is characterised by a short, relativistically intense laser driver, which causes complete evacuation of electrons from an approximately spherical region with a diameter comparable to the electron plasma wavelength. This bubble follows behind the driver at its group velocity. The plasma ions remain approximately stationary on this short timescale, and the resulting charge separation produces a linear, radial electric field within the ion cavity, with peak strength in the region of hundreds of GV/m. The displaced electrons flow as a dense sheath around the ion cavity, crossing the laser axis approximately one plasma wavelength behind the driver. Under certain conditions electrons from this sheath may be "self-injected" into the accelerating field from the back of the bubble. This obviates the need for an external electron source and the attendant complexities of synchronising such a source with the accelerator.

It has been demonstrated experimentally that self-injection may be induced by various methods, including the use of density gradients in the plasma, or simply increasing the laser intensity until some critical threshold is reached. Self-injection is routinely used for the production of LWFA electron bunches [2]; however, the fundamental mechanism by which self-injection occurs is not fully understood. In particular, control of the properties of the injected bunch remains an outstanding challenge (we direct readers to [3] for further detail).

We have developed a model [3] which describes the controlled self-injection of electron bunches in the LWFA. Arbitrary plasma density gradients cause changes in the propagation velocity of the accelerating field structure, which can lead to the injection of electron bunches. A threshold condition is found which can be applied to control the occurrence of self-injection and tune the bunch length through plasma density perturbations. We demonstrate this control through PIC simulation and show that close to threshold electron bunches with sub-femtosecond lengths may be injected in excellent agreement with our theoretical model.

Consider the behaviour of individual electrons as they propagate along the sheath, and their arrival at the rear of the bubble. Typical behaviour of the sheath electron population is shown in Fig. 1, where the comoving coördinate $\zeta = z - \beta_{\text{dr}} ct$, with $\beta_{\text{dr}} c$ the laser driver group velocity, such that the longitudinal electron position is shown with respect to the bubble structure. Fig. 1 demonstrates the effect of the cavity field, which accelerates the electrons forward as they pass around the rear half of the bubble. The fastest of these electrons attain peak velocities comparable to the velocity of the bubble. If their velocity exceeds the phase velocity of the back of the bubble $\beta_z > \beta_b$, they can penetrate forward into the accelerating phase and become injected. Using this simple condition for injection, a method of controlling either the bubble or electron velocities gives us control over the self-injection process. The peak velocity of the electron population β_{thr} may be determined from a PIC simulation with a plasma at constant background density or by analytical estimates [4].

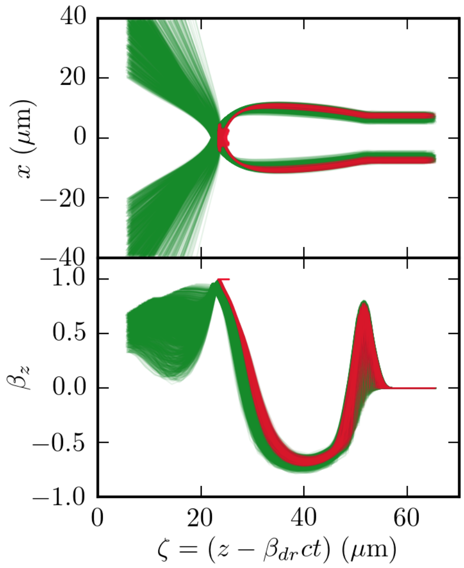

Figure 1: Example of electron motion around the LWFA bubble. The general population of electrons are indicated in green, injected electrons in red.

The bubble velocity is associated with the length of the bubble, since electrons driven from the axis by the laser at time t_ℓ form the back of the bubble at time $t_b = t_\ell + \tau(z)$, where τ is the time taken for electrons to return to the axis. Differentiating with respect to z gives

$$\frac{1}{\beta_b} = \frac{1}{\beta_{\text{dr}}} + c\frac{d\tau}{dz}, \tag{1}$$

with the laser group and bubble phase velocities defined as $c\beta_{\text{dr}} = (dt_\ell/dz)^{-1} = c\sqrt{1-\eta^2}$ and $c\beta_b = (dt_b/dz)^{-1}$, respectively, and η^2 the plasma density normalised to the critical density. The difference between the bubble phase velocity and the group velocity is determined by the rate of change of the flight time τ and thus the bubble length as it propagates. Such relationships between a driver and trailing structure are sometimes termed the “accordion effect” [5]. The bubble geometry follows from the trajectory traced out by the sheath electrons which oscillate transversely at the betatron frequency $\omega_\beta = \omega_p/\sqrt{\Gamma_e} = 2\pi\eta c/(\lambda\sqrt{\Gamma_e})$, where $\omega_p = \eta\omega$ is the plasma frequency and λ is the laser wavelength. The ratio of the frequencies depends on $\Gamma_e = \gamma_e$ in 2D, or $\Gamma_e = 2\gamma_e$ in 3D, where γ_e is the electron Lorentz factor. In the ponderomotive approximation, $\gamma_e \simeq \sqrt{1+a^2/2}$, where $a = eE/mc\omega$ is the peak normalised field amplitude. An electron initially located close to the laser axis returns to it after half an oscillation period giving $\tau = \pi/\omega_\beta$. Naturally, these are collective plasma effects and so (1) depends on the trajectories of all those electrons crossing the axis at z. It is therefore necessary to average (denoted $\langle\cdots\rangle$) over all possible initial electron positions $z - \lambda_p < \tilde{z} \leq z$, where $\lambda_p = \lambda/\eta$ is the plasma wavelength, yielding an expression for the bubble phase velocity

$$\beta_b = \beta_{\rm dr}\left[1 - \frac{\beta_{\rm dr}\lambda}{2}\left\langle\frac{\eta'\sqrt{\Gamma_e}}{\eta^2} - \frac{\Gamma_e'}{2\eta\sqrt{\Gamma_e}}\right\rangle\right]^{-1}, \tag{2}$$

where prime denotes differentiation with respect to z.

In the case of a constant density plasma and non-evolving driver ($\eta' = \Gamma_e' = 0$), the expression reduces to $\beta_b = \beta_{\rm dr}$, as expected. For a positive density gradient $\eta' > 0$ the bubble length decreases, increasing β_b despite the reduction in the group velocity $\beta_{\rm dr}$. Indeed, β_b can exceed unity for a sufficiently large positive density gradient, completely suppressing electron self-injection as electrons can never enter the bubble [6]. Conversely, with a negative gradient the bubble lengthens, reducing β_b. We assume a matched, minimally evolving laser pulse, such that the Γ_e' term can be neglected. This is the desirable case for controlling self-injection, otherwise careful modelling of the laser evolution is required, making prediction and control of injection far more complex.

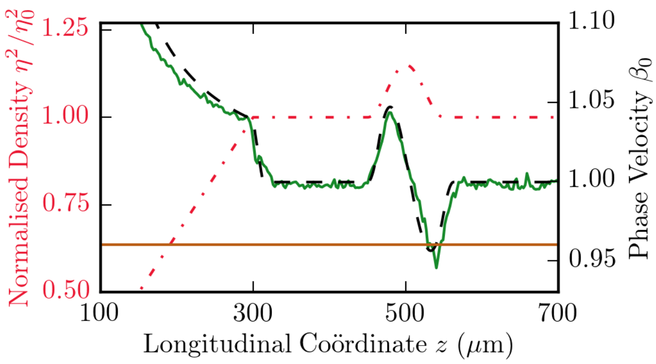

Figure 2: Bubble phase velocity β_b from our model (black) and PIC simulation (green) for a small sinusoidal density perturbation after an initial up-ramp. The plasma density (red) and threshold velocity $\beta_{\rm thr}$ (orange) are shown for context.

Applying the injection condition $\beta_b < \beta_{\rm thr}$ to (2) and rearranging yields an injection condition in terms of a density gradient:

$$\left\langle\frac{\lambda_p\eta'}{\eta}\right\rangle < \frac{2}{\sqrt{\Gamma_0}}\left(\frac{1}{\beta_{\rm dr}} - \frac{1}{\beta_{\rm thr}}\right). \tag{3}$$

The quantity $\lambda_p\eta'/\eta$ approximates the relative change in η over one plasma wavelength.

Fig. 2 shows bubble behaviour as found through an EPOCH [7] simulation of a $\sin^2$ density perturbation, in excellent agreement with our model prediction of the behaviour. The threshold velocity $\beta_{\rm thr}$ for the plasma conditions is shown in orange, and we find that in this case the bubble phase velocity drops below the injection threshold and so we expect an injection event to take place. The sudden deviation in the measured bubble phase velocity is due to the distortion of the field at the back of the bubble which occurs as the electron bunch is injected.

The length of the injected bunch may be predicted from the interval over which the bubble velocity is less than the threshold velocity. Assuming this occurs between two points z_0, z_1, the injection interval will be

$$\Delta t_{\text{inj}} = \frac{1}{c}\int_{z_0}^{z_1} \frac{1}{\beta_b(z')}\mathrm{d}z' = \frac{z_1 - z_0}{c\bar{\beta}_b}, \quad (4)$$

where $\bar{\beta}_b$ is the harmonic mean of β_b over the injection length. Approximating the velocity of the injected electrons as c yields the estimate $c\Delta t \simeq (z_1 - z_0)/(2\bar{\gamma}_b^2)$ for the bunch length, where $\bar{\gamma}_b$ is the Lorentz factor associated with the average velocity $\bar{\beta}_b$.

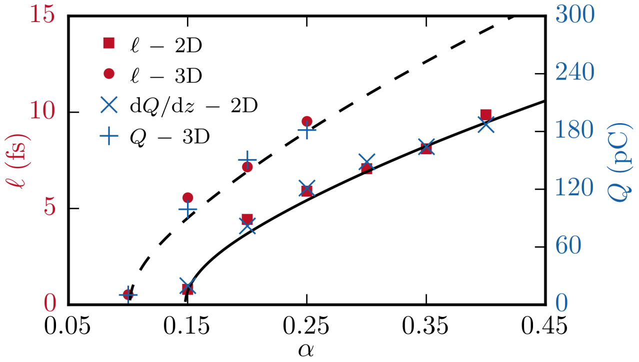

Figure 3: Parameter scan results showing injected bunch length and charge for both 2D (■,×), and 3D(●,+) PIC simulations, in comparison to our model prediction.

We performed a parameter scan by varying the density perturbation amplitude, and hence gradient, the results of which are shown in Fig. 3. We find that our model gives excellent agreement with the bunch lengths found from the PIC simulation. Additionally, the bunch charge scales proportionally to the bunch length, suggesting that in this close-to-threshold regime the injection process is not limited by beam loading. Of particular interest is the demonstration that close to threshold, it is possible to inject ultra-short bunches, that are only a few hundred attoseconds in duration. Such control of the bunch parameters is of huge potential importance for the development of electron and radiation sources based on LWFA technology.

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