

# Large Amplitude Kinetic Alfvén Excitations in non-Maxwellian Plasmas

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## Abstract

Kinetic Alfvén waves (KAWs) have been studied in a low- $\beta$  magnetized plasma, consisting of positively charged ions and two non-Maxwellian (kappa-distributed) electron populations. Linear and nonlinear analysis reveals the impact of suprathermal electrons on linear dispersion characteristics and on localized KAW modes (pulses). Super-Alfvénic speed negative potential structures are predicted via a Sagdeev-type method, whose characteristics are briefly discussed.

**Introduction.** In a strongly magnetized plasma i.e. when  $\beta \ll 1$ , kinetic Alfvén (KA) waves (KAWs) arise when the perpendicular wavelength is of the ion gyroradius  $\rho_i$  scale. In these conditions, light electrons follow the magnetic field lines. Electrons and ions respond differently to magnetic field perturbations, leading to charge separation and formation of KAWs.

Plasmas with a co-existence of two electron populations (at different temperature) are ubiquitous in Space. Studies of KAWs in such (two-electron-temperature,  $2eT$ ) plasmas start with Treumann *et al.*[1], who showed that the plasma  $\beta$  plays a crucial role in the occurrence of rarefactive and compressive density excitations, at either sub- or super-Alfvénic speeds. Berthomier *et al.* [2] investigated solitary KA waves in  $2eT$  plasma, and rigorously established the existence of either compressive or rarefactive KA solitary waves, in such a plasma configuration. Chakraborty and Das [3] reported the propagation of three wave modes, viz. kinetic Alfvén waves, ion-acoustic waves and electron-acoustic waves in  $2eT$  plasma, by adopting a Korteweg - de Vries/Zakharov-Kuznetsov (small-amplitude) approach. Very recently, Kaur and Saini [4] discussed the formation of small amplitude KAWs in a dusty  $2eT$  plasma, making use of the Korteweg - de Vries (KdV) equation. Positive potential solitary structure were predicted.

Space [5] and experimental [6] plasmas are often characterized by non-Maxwellian electrons, identified by a distinct long-tailed (suprathermal) feature in their velocity distribution. These are effectively modelled by the *kappa* ( $\kappa$ ) distribution function [7] known by now to affect not only linear wave characteristics [5, 8], but also the properties of solitary waves [8, 9].

In this paper, we investigate from first principles the characteristics of linear and nonlinear waves in a plasma with two kappa distributed (“hot” and “cold”) electron populations.

**Fluid Model equations.** A magnetized plasma, consisting of positively charged (cold) ions (mass  $m_i$  and number density  $n_i$ ), “cold” electrons (mass  $m_c$  and number density  $n_c$ ) and

relatively “hotter” electrons (mass  $m_h$  and number density  $n_h$ ) is considered. The plasma  $\beta$  parameter satisfies the condition  $m_{(c,h)}/m_i \ll \beta \ll 1$ , where  $\beta = \frac{8\pi n_{i0} K_B T_{eff}}{B_0^2}$ ; here,  $T_{eff}$  is the effective temperature,  $n_{i0}$  is the equilibrium ion number density,  $B_0$  is the background magnetic field directed along z-axis and  $K_B$  is Boltzmann’s constant. The basic governing equations are:

$$\begin{aligned} \frac{\partial n_i}{\partial t} + \frac{\partial(n_i v_{ix})}{\partial x} + \frac{\partial(n_i v_{iz})}{\partial z} &= 0, & \frac{\partial v_{ix}}{\partial t} + v_{ix} \frac{\partial v_{ix}}{\partial x} + v_{iz} \frac{\partial v_{ix}}{\partial z} &= -\frac{e}{m_i} \frac{\partial \phi_{\perp}}{\partial x} + \Omega_i v_{iy}, \\ \frac{\partial v_{iy}}{\partial t} + v_{ix} \frac{\partial v_{iy}}{\partial x} + v_{iz} \frac{\partial v_{iy}}{\partial z} &= -\Omega_i v_{ix}, & \frac{\partial v_{iz}}{\partial t} + v_{ix} \frac{\partial v_{iz}}{\partial x} + v_{iz} \frac{\partial v_{iz}}{\partial z} &= -\frac{e}{m_i} \frac{\partial \psi_{\parallel}}{\partial z} \\ \frac{\partial^4(\phi_{\perp} - \psi_{\parallel})}{\partial x^2 \partial z^2} &= \frac{4\pi e}{c^2} \left[ \frac{\partial^2 n_i}{\partial t^2} + \frac{\partial^2(n_i v_{iz})}{\partial t \partial z} \right], \end{aligned} \quad (1)$$

where the electric field components derive from potential functions as  $E_x = E_{\perp} = -\partial \phi_{\parallel}/\partial x$  and  $E_z = E_{\parallel} = -\partial \psi_{\parallel}/\partial z$  respectively,  $\Omega_i = eB/m_i$  denotes the ion cyclotron (gyro-)frequency and the plasma fluid quantities bear their usual meaning. The neutrality condition  $n_i \simeq n_c + n_h$  is assumed. The electron number density is given by  $n_j = n_{j0} \left( 1 - \frac{e\psi_{\parallel}}{K_B T_j (\kappa_j - \frac{3}{2})} \right)^{-\kappa_j + 1/2}$ , for  $j = c, h$  (“hot”, “cold”), in terms of the respective temperature  $T_j$  and nonthermal index  $\kappa_j$ .

**Linear Dispersion Relation.** Eqs. (1) are linearized, by assuming all the perturbed quantities to be proportional to  $\exp[i(k_{\perp}x + k_{\parallel}z - \omega t)]$ , where  $k_{\perp}$ ,  $k_{\parallel}$  and  $\omega$  are the perpendicular and parallel wave vectors and frequency respectively, leading to the dispersion relation

$$\omega^2 = \omega_{\pm}^2 = \frac{1}{2} k_{\parallel}^2 v_A^2 (1 + k_{\perp}^2 \rho_{eff}^2 + \beta_{eff}) \left\{ 1 \pm \left[ 1 - 4\beta_{eff} (1 + k_{\perp}^2 \rho_{eff}^2 + \beta_{eff})^{-2} \right]^{1/2} \right\}, \quad (2)$$

where  $\beta_{eff} = \frac{n_{i0}}{\left[ \frac{n_{c0}}{C_{s1}^2} \frac{2\kappa_c - 1}{2\kappa_c - 3} + \frac{n_{h0}}{C_{s2}^2} \frac{2\kappa_h - 1}{2\kappa_h - 3} \right] v_A^2} = \frac{C_{eff}^2}{v_A^2}$  and  $\rho_{eff}^2 = \frac{n_{i0}}{\left[ \frac{n_{c0}}{\rho_{i1}^2} \frac{2\kappa_c - 1}{2\kappa_c - 3} + \frac{n_{h0}}{\rho_{i2}^2} \frac{2\kappa_h - 1}{2\kappa_h - 3} \right]}$ . are “effective” (averaged) plasma- $\beta$  and gyroradius analogues. We have also defined  $C_{s1}^2 = \frac{K_B T_c}{m_i}$ ,  $C_{s2}^2 = \frac{K_B T_h}{m_i}$ ,  $v_A^2 = \frac{B_0^2}{4\pi n_{i0} m_i}$ ,  $\rho_{i1}^2 = \frac{C_{s1}^2}{\Omega_i^2}$  and  $\rho_{i2}^2 = \frac{C_{s2}^2}{\Omega_i^2}$ . Suprathermal electrons evidently affect the dispersion characteristics (via  $\kappa_{c,h}$ ). In the limit  $\beta_{eff} \ll 1$ , relation (2) recovers  $\omega^2 = k_{\parallel}^2 v_A^2 (1 + k_{\perp}^2 \rho_{eff}^2)$ .

**Parametric Analysis.** In Fig. 1(a), we have shown the combined effect of cold electron number density ( $n_{c0}$ ) and superthermality on the wave’s phase speed. The phase speed is clearly reduced as the cold (superthermal) electron component density increases, with respect to the Maxwellian no-cold-electron case ( $\kappa_c \rightarrow \infty$ ,  $n_{c0} = 0$ ) here given by the solid (blue) curve. This may be due to the fact that the inclusion of cold species makes the wave heavier and suppresses the parallel current, thus slowing down the wave. A decrease in  $\kappa_c$  (i.e., farther off the Maxwellian) also reduces the phase velocity of the wave as shown in Fig. 1(b): KAWs appear to be slowed down in a superthermal plasma. Finally, an increase in cold electron temperature  $T_c$  appears to accelerate KA waves, as evident in Fig. 1(c).

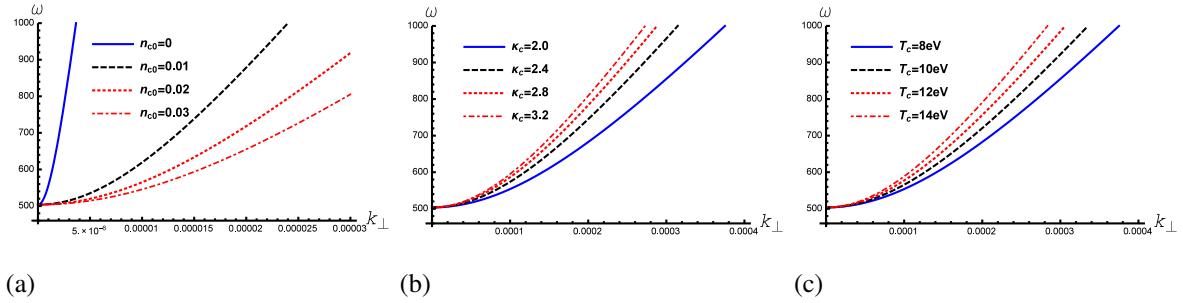


Figure 1: The wave frequency  $\omega$  is depicted against the perpendicular wavevector  $k_{\perp}$ ; for  $k_{\parallel} = 10^{-6} \text{ cm}^{-1}$  ( $\ll k_{\perp}$ ). In (a), the solid (blue) curve corresponds to  $n_{c0} = 0 \text{ cm}^{-3}$ ,  $\kappa_h \rightarrow \infty$ ,  $T_c = 8 \text{ eV}$ ,  $T_h = 1100 \text{ eV}$ ,  $n_{i0} = 75 \text{ cm}^{-3}$ ,  $n_{h0} = 0.07 \text{ cm}^{-3}$ ; for the rest of the curves,  $\kappa_h = 4$ . Density values in  $\text{cm}^{-3}$ . In (b) and (c), the solid (blue) curve corresponds to  $\kappa_c = 2$ ,  $n_{c0} = 2.5 \text{ cm}^{-3}$ .

### Nonlinear Analysis.

To proceed with the analysis, we have rescaled Eqs. (1), as follows (capital letters are used for dimensionless variables):  $N_i = \frac{n_{(i,c,h)}}{n_{(i,c,h)o}}$ ,  $V_{i(x,y,z)} = \frac{v_{i(x,y,z)}}{C_s}$ ,  $T = \omega_{pi}t$ ,  $(X, Y, Z) = \frac{(x,y,z)}{\lambda_D}$  and  $(\phi, \psi) = \frac{e(\phi_{\perp}, \psi_{\parallel})}{K_B T_{eff}}$ , where  $T_{eff} = T_c T_h n_{i0} / (n_{c0} T_h + n_{h0} T_c)$  and  $T_c(T_h)$  is the temperature of cold (hot) electrons,  $C_s = \left( \frac{K_B T_{eff}}{m_i} \right)^{1/2}$  is the ion-acoustic (plasma “sound”) speed,  $\omega_{pi} = \left( \frac{4\pi n_{i0} e^2}{m_i} \right)^{1/2}$  is the ion plasma frequency and  $\lambda_D = \left( \frac{K_B T_{eff}}{4\pi n_{i0} e^2} \right)^{1/2}$  is an effective Debye-screening length.

We have adopted the (Sagdeev) pseudopotential methodology, by assuming that all state variables depend on the single moving coordinate  $\xi = l_x X + l_z Z - MT$  (propagation in the  $x$ - $z$  plane is assumed). Here,  $l_x$  and  $l_z = \pm(1 - l_x^2)^{1/2}$  represent direction cosines and  $M$  is the ion-acoustic Mach number  $M = \frac{V}{C_s}$ , where  $V$  is the soliton speed and  $C_s$  was defined above. A lengthy but straightforward procedure leads to a pseudo-energy balance equation in the form of the ordinary-differential equation (ODE):  $\frac{1}{2} \left[ \frac{d\psi}{d\xi} \right]^2 + U(\psi) = 0$ , where  $U(\psi)$  is given by

$$U(\psi) = -\frac{\gamma(1 - M_A'^2)}{4M_A'^2 l_z^2 \left( \frac{(1+Q)\beta}{2} - \frac{M_A'^2(1+\delta)^2 P_3}{P_1^3} \right)^2} \left[ M_A'^4 \left\{ 1 + \frac{(1+\delta)^2}{P_1^2} - \frac{2(1+\delta)}{P_1} \right\} - 2\beta M_A'^2 \times \right. \\ \left. \times \left\{ \left( \frac{(1+Q)}{1+\delta} - \frac{1}{P_1} \right) \left[ P_2 - \left( \frac{1}{\alpha_c} + \frac{\delta}{\alpha_h} \right) \right] - Q\psi \right\} + \frac{(1+Q)\beta^2}{4(1+\delta)^2} \left\{ \left[ P_2 - \left( \frac{1}{\alpha_c} + \frac{\delta}{\alpha_h} \right) \right]^2 \right\} \right],$$

where  $P_1 = \left[ \left( 1 - \frac{\alpha_c \psi}{\kappa_c - \frac{3}{2}} \right)^{-\kappa_c + \frac{1}{2}} + \delta \left( 1 - \frac{\alpha_h \psi}{\kappa_h - \frac{3}{2}} \right)^{-\kappa_h + \frac{1}{2}} \right]$ ,  $P_2 = \int P_1 d\psi$ ,  $P_3 = \frac{dP_1}{d\psi}$  and  $\gamma = \frac{C_s^2}{c^2}$ .

We have defined  $Q = \frac{l_x^2}{l_z^2}$  and  $M_A' = M_A / l_z$ , where the “Alfvén Mach number” is defined as  $M_A = \left( \frac{\beta}{2} \right)^{1/2} M = \frac{V}{v_A}$  with respect to the Alfvén velocity  $v_A$ .

A meticulous numerical investigation of the properties of solitary waves, arising as solutions

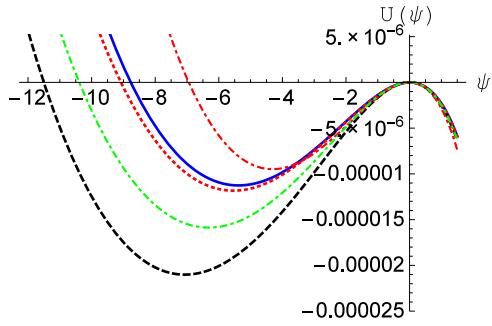


Figure 2: Behavior of the Sagdeev potential  $U(\psi)$  vs  $\psi$  for negative potential structures at  $\kappa_c = 3$ ,  $\kappa_h = 3$ ,  $\alpha_c = 0.1$ ,  $\alpha_h = 0.01$ ,  $Q = 25$ ,  $l_z^2 = 0.15$ ,  $\beta = 0.3$ ,  $\delta = 0.1$  and  $M_A'^2 = 1.2$ , represented by solid (blue) curve. Dashed (black) curve corresponds to  $\kappa_c = 3.5$ , dotted (red) to  $\kappa_h = 23$ , dotted-dashed (red) curve to  $M_A'^2 = 1.3$  and dotted-dashed (green) to  $\beta = 0.32$ .

of the above ODE, has been carried out, and the details (omitted here) will be reported in a lengthy report, in preparation. Negative polarity potential structures ( $\psi$  pulses) and bipolar electric field structures are obtained numerically, corresponding to the negative values (and roots) of the Sagdeev pseudopotential  $U$ ; see Fig. 2. Based on an analysis of  $U$ , we see that the amplitude and depth of the potential well significantly increases, as either of  $\kappa_c$ ,  $\kappa_h$  and plasma  $\beta$  increase(s); this effect is depicted in the dashed (black) curve, dotted (red) curve and dotted-dashed (green) curves respectively in Fig. 2. On the other hand,  $\kappa_h$  appears to have a very small effect on solitary KA wave characteristics. Finally, an increase in  $M_A'$  leads to a decrease in both amplitude and depth of  $U$ , as depicted by the dotted-dashed (red) curve in Fig. 2.

## References

- [1] R. A. Treumann, M. Güdel and A. O. Benz, *Astronomy and Astrophysics* **236**, 242 (1990).
- [2] M. Berthomier, R. Pottelette and R. A. Treumann, *Phys. Plasmas* **6**, 467 (1999).
- [3] D. Chakraborty and K. P. Das, *Phys. Plasmas* **10**, 2236 (2003).
- [4] Nimardeep Kaur and N. S. Saini, *Astrophys. Space Sci.* **361**, 331 (2016).
- [5] M. Hapgood *et al.*, *Planet. Space Sci.* **59**, 618 (2011).
- [6] G. Sarri *et al.*, *Physics of Plasmas*, **17**, 010701 (2010).
- [7] M. A. Hellberg *et al.*, *Phys. Plasmas*, **16**, 094701 (2009).
- [8] I. Kourakis, S. Sultana & M.A. Hellberg, *Plasma Phys. Cont. Fusion*, **54**, 124001 (2012).
- [9] N. S. Saini and I. Kourakis, *Plasma Phys. Cont. Fusion* **52**, 075009 (2010).