

Fluid theory of gradient-drift instability of partially-magnetized plasmas in $\mathbf{E}\times\mathbf{B}$ fields with finite electron Larmor radius effects

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The gradient-drift instability in inhomogeneous partially-magnetized plasmas with transverse current is investigated in the framework of advanced two-fluid model, which includes finite electron temperature and finite electron Larmor radius (FLR) effects (in sense of Padé approximants). Such an instability is typical for plasmas immersed in crossed external electric and magnetic fields (in particular, for Hall ion sources, Penning discharges, closed-drift Hall plasma thrusters) [1]–[3] and can be a source of turbulence and anomalous electron mobility in such systems [4]–[6]. It is shown that, in general, the electron inertia and FLR effects stabilize the short-wavelength perturbations and, in some cases, can completely suppress the high-frequency short-wavelength modes leading to the development of long-wavelength low-frequency (in comparison with the lower-hybrid frequency, ω_{lh}) gradient-drift instability.

The analysis is performed in the framework of two-fluid theory. The ions are considered to be cold and unmagnetized, and their behavior is described by the equation of motion

$$\frac{\partial \mathbf{v}_i}{\partial t} + (\mathbf{v}_i \cdot \nabla) \mathbf{v}_i = \frac{e}{m_i} \mathbf{E}, \quad (1)$$

and by the continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = 0. \quad (2)$$

Here \mathbf{v}_i and n_i are the ion velocity and density, \mathbf{E} is the electric field, e is the proton charge and m_i is the ion mass.

As for the electrons, to include the effects due to their thermal motion we use the electron momentum equation in the following form

$$\frac{\partial \mathbf{v}_e}{\partial t} + (\mathbf{v}_e \cdot \nabla) \mathbf{v}_e = -\frac{e}{m_e} \left(\mathbf{E} + \frac{1}{c} \mathbf{v}_e \times \mathbf{B} \right) - \frac{\nabla p_e}{m_e n_e} - \frac{\nabla \cdot \pi_e}{m_e n_e}, \quad (3)$$

where \mathbf{v}_e , p_e , n_e are the electron velocity, pressure and density, c is the speed of light, \mathbf{B} is the magnetic field and π_e is the electron gyroviscosity tensor, which explicit expression in a curvilinear inhomogeneous magnetic field is given in [7]. The electron density is defined by the electron continuity equation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{v}_e) = 0. \quad (4)$$

The electron temperature $T_e = p_e/n_e$ is assumed to be homogeneous and its perturbation will be neglected too. To close the set of equations (1)–(4) the Poisson equation is used:

$$\nabla \cdot \mathbf{E} = 4\pi e(n_i - n_e). \quad (5)$$

We consider the problem in the simplified slab model in Cartesian coordinates $\{x, y, z\}$ with the x coordinate in the direction of the applied to plasma external electric field $\mathbf{E}_0 = E_0 \mathbf{e}_x$ (or the direction of plasma inhomogeneity), z coordinate along the predominant direction of magnetic field $\mathbf{B}_0 = B_0 \mathbf{e}_z$, y coordinate in the periodic azimuthal direction. Hereafter the subscript “0” implies the equilibrium (unperturbed) quantities.

For purely electrostatic perturbations $\mathbf{E}' = -\nabla\phi'$ with the frequency ω in the range between the ion and electron cyclotron frequencies and propagating strictly perpendicular to the magnetic field, $\mathbf{k} = \{k_x, k_y, 0\}$, the above model gives the following local dispersion relation

$$1 + \frac{\omega_{pe}^2}{\omega_{Be}^2} \cdot \frac{1}{1 + k_\perp^2 \rho_e^2} - \frac{\omega_{pi}^2}{(\omega - k_x v_{0i})^2} + \frac{1}{k_\perp^2 d_e^2} \cdot \frac{1}{1 + k_\perp^2 \rho_e^2} \cdot \frac{\omega_{*e} - \bar{\omega}_D}{\omega - \omega_E - \bar{\omega}_D} = 0. \quad (6)$$

Here $\omega_{Be} = eB_0/m_e c$ is the electron cyclotron frequency, $\omega_{p\alpha} = \sqrt{4\pi n_0 e^2/m_\alpha}$ – the plasma frequencies, $\alpha = (i, e)$, v_{0i} – the equilibrium ion velocity, $d_e = (T_e/4\pi e^2 n_0)^{1/2}$ – the electron Debye radius, $\rho_e = (T_e/m_e \omega_{Be}^2)^{1/2}$ – the electron Larmor radius, $k_\perp^2 = k_x^2 + k_y^2$; $\omega_E = k_y V_{0E}$, $\omega_{*e} = k_y V_{*e}$, $\bar{\omega}_D = k_y V_D (1 + 2k_\perp^2 \rho_e^2)/(1 + k_\perp^2 \rho_e^2)$, and the electron drift velocities, V_{0E} , V_{*e} , V_D , are described by the expressions:

$$V_{0E} = -cE_0/B_0, \quad V_{*e} = -(cT_e/eB_0)d \ln n_0/dx, \quad V_D = -(2cT_e/eB_0)d \ln B_0/dx.$$

Dispersion relation (6) includes the equilibrium electron and ion flows perpendicular to the magnetic field, the electron inertia and FLR (in sense of Padé approximants) effects, and the Debye effects. This dispersion relation generalizes the earlier known dispersion relations of Refs. [3]–[6], [8]. In addition to the dispersion relations of Refs. [3]–[6] it takes into account the electron temperature effects (electron magnetic drift and FLR effects) and in addition to the dispersion relation of Ref. [8] – the electron inertia and FLR effects.

A lengthy but rather straightforward analysis shows that a *sufficient* condition of stability of considered plasma perturbations is

$$\left(\frac{eE_0}{T_e} + \frac{1 + 2k_\perp^2 \rho_e^2}{1 + k_\perp^2 \rho_e^2} \cdot \frac{d}{dx} \ln B_0^2 + \frac{k_x}{k_y} \frac{\omega_{Bi}}{c_s} \frac{v_{0i}}{c_s} \right) \left\{ \frac{d}{dx} \ln n_0 - \frac{1 + 2k_\perp^2 \rho_e^2}{1 + k_\perp^2 \rho_e^2} \cdot \frac{d}{dx} \ln B_0^2 \right\} \leq 0. \quad (7)$$

This condition guarantees that both long-wavelength, lower-frequency and short-wavelength, higher-frequency (of order ω_{lh} and higher) perturbations are stable. For azimuthal perturbations

with $k_x \ll k_y$ and negligible inhomogeneity of the magnetic field strength this condition takes the form which is complementary to the Simon-Hoh (S-H) instability condition $\mathbf{E}_0 \cdot \nabla n_0 < 0$ [9, 10]. Neglecting the FLR effects and equilibrium ions velocity in Eq. (7) one can obtain the extended S-H stability criterion for the inhomogeneous magnetic field: $(V_{0E} + V_D)(\kappa_n - 2\kappa_B) > 0$.

The general *necessary and sufficient* condition of gradient drift instability is formulated as

$$\begin{cases} \bar{\sigma}^2 > \mu_1, & \text{for } \lambda \leq 0; \\ \mu_1 < \bar{\sigma}^2 < \mu_2, & \text{for } 0 < \lambda < 1, \end{cases} \quad (8)$$

where $\lambda = 1 - \alpha k_y/k_{\perp}^2 \bar{\sigma}$, $\mu_{1,2} = [27 - 18\lambda - \lambda^2 \mp \sqrt{(27 - 18\lambda - \lambda^2)^2 - 64\lambda^3}] / 8\lambda^3$,

$$\bar{\sigma} = \frac{k_y}{\omega_0} \left[\frac{c}{B_0} \left(E_0 + \frac{2T_e}{e} \bar{\kappa}_B \right) + \frac{k_x}{k_y} v_{0i} \right], \quad \alpha = \frac{(\kappa_n - 2\bar{\kappa}_B)\omega_0}{\omega_{Bi}(1 + k_{\perp}^2 \rho_e^2)},$$

$$\kappa_{n,B} \equiv \frac{d \ln(n_0, B_0)}{dx}, \quad \bar{\kappa}_B = \frac{1 + 2k_{\perp}^2 \rho_e^2}{1 + k_{\perp}^2 \rho_e^2} \cdot \kappa_B, \quad \omega_0 \equiv \omega_{pi} \sqrt{\frac{1 + k_{\perp}^2 \rho_e^2}{1 + k_{\perp}^2 \rho_e^2 + \omega_{pe}^2/\omega_{Be}^2}}.$$

The full picture of instability is summarized in Fig. 1. The boundaries of instability region are described by the curves $\bar{\sigma}^2 = \mu_{1,2}$ – see Eq. (8). At $\lambda \leq 0$ the instability region is located above the curve $\bar{\sigma}^2 = \mu_1$ and at $0 < \lambda < 1$ – between the curves $\bar{\sigma}^2 = \mu_1$ and $\bar{\sigma}^2 = \mu_2$, which intersect at the point $\lambda = 1$. The region with $\lambda > 1$ is stable.

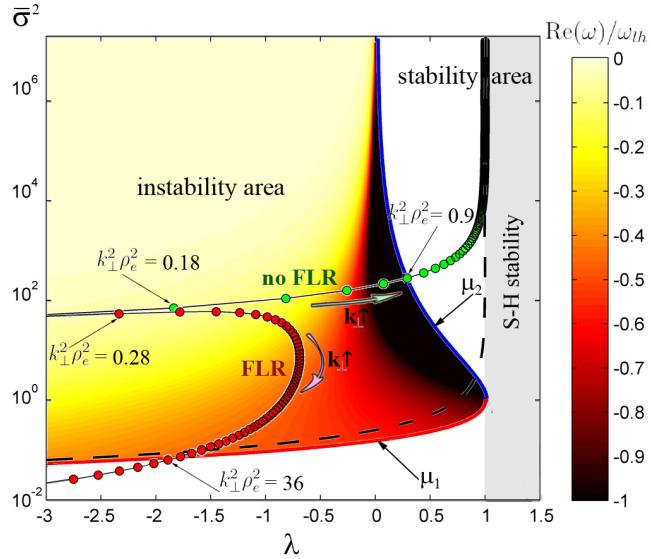


Figure 1: *Stability diagram and contour plots of frequencies of unstable modes in the plane $\bar{\sigma}^2 - \lambda$. White color indicates the stability region, the instability region is colored. Dashed curve shows the stability boundary with no account of electron inertia; the corresponding instability region locates above this curve. Oscillations with frequency higher than ω_{lh} are shown with black color.*

Now we fix the equilibrium parameters and follow the parametric curve $\bar{\sigma}^2(\lambda)$ given by parameter k_{\perp} on the stability diagram. For azimuthal modes the curve is described by the relation

$\bar{\sigma}^2 = \alpha\sigma/(1 - \lambda)$. With no FLR effects α and σ depend not on k_{\perp} and the dependence $\bar{\sigma}^2(\lambda)$ – see black line with green circles in Fig. 1 – appears to repeat the behavior of the stability boundary with no account of electron inertia, $\bar{\sigma}^2 = 1/4(1 - \lambda)$, [8]. Thus, with no account of electron inertia the whole spectrum of perturbations $k_{\perp} \in (0, \infty)$ is unstable at $\alpha\sigma > 1/4$; at $\alpha\sigma < 1/4$ all modes are stable. When we take into account the inertia effects the cut off of the short-wavelength modes occurs with the maximal possible $k_{\perp max}$ for unstable modes defined by the relation $\alpha\sigma/(1 - \lambda) = \mu_2$; consequently, the growth rate of instability is limited from above. Also the additional region of instability $\mu_1 < \bar{\sigma}^2 < 1/4(1 - \lambda)$ near the instability threshold (small $\bar{\sigma}$) appears.

The FLR effects make the picture much more complex, since with their account α and σ depend not only on equilibrium plasma state, but are also functions of k_{\perp} . As a consequence, the function $\bar{\sigma}^2(\lambda)$ is not necessarily an increasing monotonic function and its behavior is strongly determined by equilibrium parameters. In the example given in Fig. 1 the FLR effects totally change the behavior of $\bar{\sigma}^2(\lambda)$: it decreases with the growth of k_{\perp} and the stabilization of the short-wavelength modes takes place due to the intersection of $\bar{\sigma}^2(\lambda)$ with the lower stability boundary, μ_1 – see black line with red circles in Fig. 1. As one can see from Fig. 1, the frequency of unstable modes near μ_1 is low in comparison with the lower-hybrid frequency. The additional analysis shows that even small FLR effects can completely stabilize high-frequency short wavelength modes near the instability threshold.

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