Observation of the dynamo effect in a Reversed Field Pinch

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Introduction

The tokamak and Reversed Field Pinch (RFP) fusion reactors are characterized by their similar magnetic configuration: a toroidal magnetic field generated by the surrounding poloidal coils combined with a strong toroidal current induced by the central solenoid. RFPs differ from tokamaks by the magnitude of the poloidal magnetic field, which is of the same order as the one of the toroidal magnetic field. Such a magnetic configuration is known to be susceptible to MHD instabilities when the toroidal current I_z is increased above a certain threshold for a given toroidal magnetic field B_z. These instabilities give in general rise to a complex chaotic interplay of helical structures of different spatial frequency, reorganizing the plasma into a stable state where the toroidal component reverses close to the boundary. The generation of this toroidal magnetic field, which has the originally imposed sign in the center and reversed close to the boundary, is referred to as the dynamo effect, a term '*borrowed*' from astrophysics [1]. This phenomenon, observed first in early RFP experiments [1–3], is extensively studied using different approaches. In [4] the parallel Ohm's law was used to point out the diamagnetic effect, while in [5] mean field theory was used and an α-model was invoked to explain the RFP reversal. It was also pointed out in [6] that cross-helicity plays a major role in the RFP dynamo. Nevertheless, after the observation of quasi-single-helicity states in [7] in 2000, the dominant electrostatic nature of the RFP dynamo was illustrated in [8,9].

The astrophysical description of the dynamo effect, which plays a major role in the generation of celestial and planetary magnetic fields, is the phenomenon of amplification of a weak magnetic field by the movement of a conducting fluid or plasma. However, the magnetic field in RFP fusion plasma is never weak. In this communication we investigate whether the velocity field of a RFP is capable of amplifying a weak magnetic field. We hereby reconcile the astrophysics and fusion community with respect to the presence, or not, of a dynamo in RFPs. We consider the interaction of three vector-fields (as in [10]): the velocity field $\boldsymbol{u}$, generated by an MHD instability resulting from its interaction with the magnetic field $\boldsymbol{B}$, and an initially weak magnetic field $\boldsymbol{D}$, passively advected by the velocity field $\boldsymbol{u}$. The dynamics are investigated using incompressible visco-resistive MHD simulations in cylindrical geometry [11] using a pseudo-spectral solver [12]. First results show that the RFP velocity field acts as a dynamo,

for sufficiently large magnetic Reynolds numbers.

Visco-resistive MHD equations

In the present work, we consider a plasma characterized by constant permeability μ, permittivity ε and conductivity σ. In the magnetohydrodynamic (MHD) description that we consider, the governing equations are the incompressible Navier-Stokes equations including the Lorentz force, and the induction equation. Normalizing these quantities by the Alfvén velocity $C_A = B_0/\sqrt{\rho\mu}$, a reference magnetic field B_0 and a conveniently chosen lengthscale $\mathscr{L}$ leads to the following expressions,

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla P + \boldsymbol{j} \times \boldsymbol{B} + \nu \nabla^2 \boldsymbol{u}, \tag{1}$$

and

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{B}) + \lambda \nabla^2 \boldsymbol{B}. \tag{2}$$

where ν is the kinematic viscosity, λ the magnetic diffusivity, and $\rho = 1$ the density. The current density is given by

$$\boldsymbol{j} = \nabla \times \boldsymbol{B}. \tag{3}$$

The passive magnetic field's evolution is described by the following induction equation,

$$\frac{\partial \boldsymbol{D}}{\partial t} = \nabla \times (\boldsymbol{u} \times \boldsymbol{D}) + \lambda' \nabla^2 \boldsymbol{D}. \tag{4}$$

where λ' is the magnetic diffusivity corresponding to $\boldsymbol{D}$. The velocity field $\boldsymbol{u}$, the magnetic field $\boldsymbol{B}$ and the passive magnetic field $\boldsymbol{D}$ are all divergence free,

$$\nabla \cdot \boldsymbol{u} = 0, \tag{5}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{6}$$

$$\nabla \cdot \boldsymbol{D} = 0. \tag{7}$$

Figure 1: Sketch of the cylindrical geometry.

In the plasma a uniform current density j_0 in the z-direction and an axial magnetic field B_{z0} are imposed, resulting in a helically shaped magnetic field. The current density j_0 will induce a poloidal magnetic field B_{p0} parallel to the boundaries, where the velocity is imposed to be zero. The value of the poloidal parallel magnetic field at the boundary is fixed and its value is determined by j_0.

Equations (1),(2) and (4) are solved using a pseudo-spectral method in a periodic domain of size $\pi \times \pi \times 4\pi$ with $256 \times 256 \times 1024$ grid points. Boundary conditions are imposed using a volume penalization method in order to build the cylindrical domain. Detailed description and
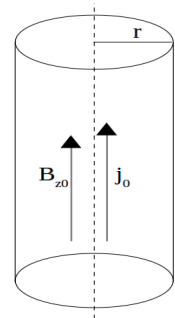
validation of the method can be found in [12], and an application of the method to investigate RFPs in toroidal domains is reported in previous work [11, 13]. The implementation of the Dirichlet boundary condition for the passive magnetic field is identical to that of the velocity.

Results

We consider first the case where the velocity field is "frozen". This consists in using in equation (4) a time-invariant velocity field obtained by resolving first, equations (1) and (2) until reaching a statistically stationary state. This is shown in Figure 2(a), where kinetic and magnetic energies evolve first to reach a stationary phase, then the frozen velocity field is taken at $t = 1500\tau_A$ and simulations of equation (4) for different values of λ' are carried out. The exponential evolution of the passive magnetic energy $\langle D^2 \rangle$ shows wether the dynamo effect exists ($\langle D^2 \rangle(t)$ is increasing) or not ($\langle D^2 \rangle(t)$ is decreasing). Figure 2(b) shows the different simulations that allowed us to explore the critical Reynolds number $R'_m = u\mathscr{L}/\lambda'$, above which the dynamo effect is observed. The frozen velocity method is suitable for laminar flows with small Lundquist number $S = C_A \mathscr{L}/\lambda$, where few kinetic modes dominate, and dynamo action is more probable due to lack of kinetic fluctuations. Simulations of higher Lundquist number are carried out using a dynamic velocity field, where equations (1), (2) and (4) are resolved simultaneously. Figure 3(a) shows the evolution of different modes' energy for $S \approx 4000$ and $R'_m \approx 220$. For the passive magnetic energy, mode $n = 1$ is dominant for $t > 200\tau_A$, while different kinetic modes
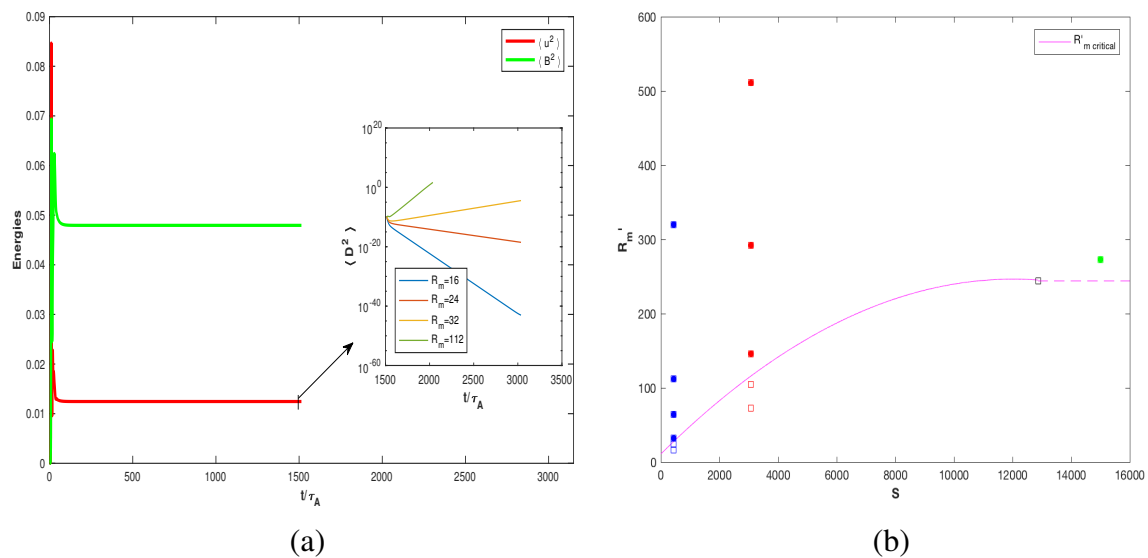

(a)   (b)

Figure 2: (a) Evolution of the kinetic and magnetic energies for $S \sim 1000$, and of the passive magnetic energy for different $R'_m = u\mathscr{L}/\lambda'$, (b) different runs function of S and R'_m. Empty squares are the non-dynamo cases.

(a)   (b)
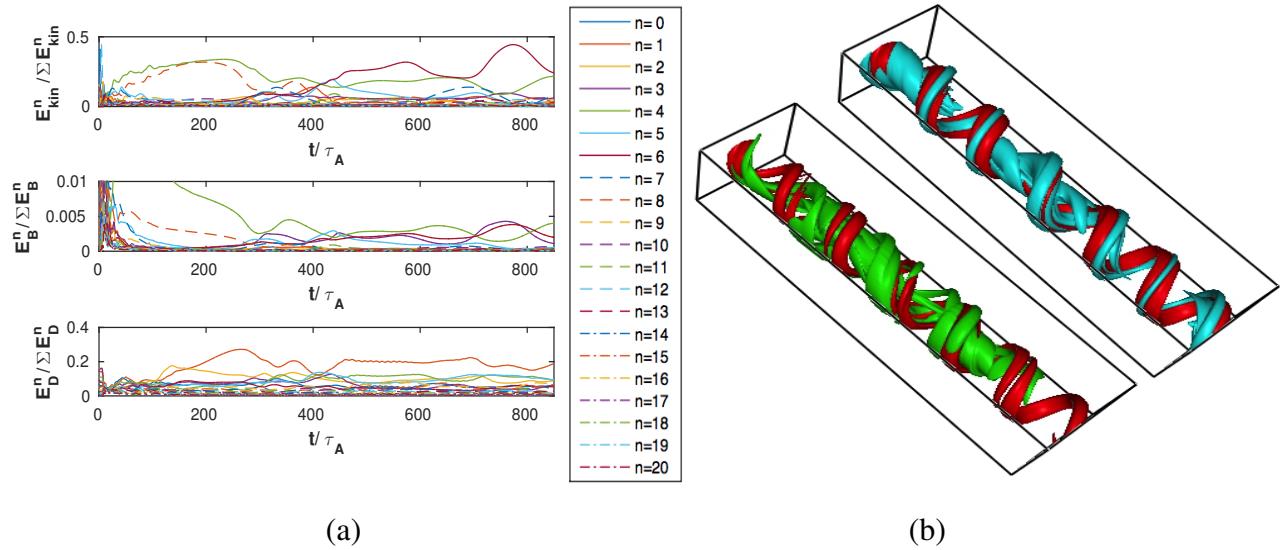
Figure 3: (a) Evolution of different normalized toroidal modes for $S \sim 4000$ and $R'_m = 220$, (b) isosurfaces of kinetic (red), magnetic (blue) and passive magnetic (green) energies.

dominate during this period. Hence no clear correlation observed between kinetic and passive magnetic modes. The 3D visualization in figure 3(b) shows that $\boldsymbol{D}$ has a helical structure similar to that of $\boldsymbol{u}$.

Conclusion

In the present work we investigated wether the RFP velocity field acts as a true dynamo or not, and hence reconcile the fusion and astrophysical communities. Results show that the RFP velocity field is able of amplifying a weak magnetic field, and thereby proves the existence of a dynamo effect, even in the astrophysical sense.

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