Nonlinear collisionless Electron Cyclotron absorption in the pre-ionization stage

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A theoretical description is presented of the EC wave–particle interaction in the pre-ionization phase, much before collisions and other mechanisms can play a role. In the very first phase of a plasma discharge with EC-assisted breakdown, wave interaction with an electron crossing the microwave beam in single transit is in general non linear [1]. The electron motion can be described as that of a single particle in free space under the action of a localized e.m. beam in a static magnetic field. The EC beam is assumed to be injected into the plasma with a Gaussian-like profile with wave electric field amplitude $E = E_M \exp(-r^2/w^2)$, being r the beam radius, w the waist and E_M its maximum value. For a magnetized electron at room temperature and for EC beam parameters typical of ITER ($P = 1$ MW, width $w = 0.05 - 0.1$ m, $f = 170$ GHz) as well as of present experiments, the interaction regime is characterized by $\tau_{cycl} \ll \tau_{trap} \ll \tau_{flight} \ll \tau_{coll}$, with $\tau_{cycl} \simeq 6 \times 10^{-12}$ s the cyclotron period, $\tau_{trap} \approx 10^{-8}$ s the trapping time in the wave field, $\tau_{flight} \approx 1 - 5 \times 10^{-6}$ s the time flight in the beam, and $\tau_{coll} \approx 5 \times 10^{-4} - 5 \times 10^{-3}$ s the collision time. Being τ_{trap} much shorter than the time scale of the variation of the e.m. wave amplitude, the above conditions define an adiabatic nonlinear regime (widely investigated in the past, see e.g. [1–5]), that can be described theoretically via a Hamiltonian formalism, making use of the motion adiabatic invariants. A rigorous identification of the relevant elementary physical processes can then be performed within this framework.

The motion of an electron interacting with an EC wave with frequency ω close to a cyclotron harmonic $n\Omega$ in a static uniform magnetic field $\mathbf{B}_0 = B_0 \mathbf{e}_z$ can be described via a time-independent Hamiltonian with two degrees of freedom $H(z, \theta, \bar{P}_z, I) = \gamma - \nu_n I$ [3,4], with γ the relativistic factor. In the regime of interest here, i.e., very low electron temperature and density, in the weakly relativistic approximation and at first order in the normalized wave amplitude $\varepsilon = eE/mc\omega \ll 1$, the Hamiltonian reads $H(z, \theta, \bar{P}_z, I) = H_0(\bar{P}_z, I) + H_1(z, \theta, \bar{P}_z, I)$ with

$$H_0 = 1 + \bar{P}_z^2/2 + [I_r(\bar{P}_z)I - I^2/2](1 - N_\parallel^2 \nu_n^2), \qquad H_1 = \varepsilon(z)\, T_n(\bar{P}_z)\, (2I)^{n/2} \cos n\theta, \qquad (1)$$

where $I_r(\bar{P}_z) = (1 - \nu_n + N_\parallel \nu_n \bar{P}_z - \bar{P}_z^2/2)/(1 - N_\parallel^2 \nu_n^2)$ is the resonant action, $\nu_n = \nu/n = \omega/n\Omega$, I and θ are action-angle variables, $\bar{P}_z = P_z + N_\parallel \nu_n I$ is the parallel canonical momentum conjugate to z, and the polarization term reads $T_n = N_\perp^{n-1}(e_x - e_y + e_z N_\perp \bar{P}_z)/2$ for $n = 1, 2$. For the unperturbed system ($\varepsilon = 0$), θ is the phase of the gyromotion, $I = p_\perp^2/2$, and $P_z = p_z$, being $\mathbf{p}$ the normalized kinetic momentum.

Two examples of solution of the Hamiltonian equations of motion are shown in Figure 1 on the left for ITER parameters, 2^{nd} harmonic extraordinary mode (XM). The initial conditions correspond to thermal electrons with $T_e = 0.03$ eV, just outside the beam region, i.e., at $t = 0$, $z_0 = -4w$ ($w = 0.1$ m), $P_{z0} = \beta_{th}$ and $I_0 = \beta_{th}^2/2$ being $\beta_{th} = v_{th}/c = (T_e/mc^2)^{1/2}$ the thermal velocity normalized to the speed of light, and two different initial phase values θ_0. The electron is non linearly trapped and then untrapped during the EC beam crossing, and either recovers its initial energy or it gains a large amount of energy when exits the beam region. A small variation of the parallel canonical momentum is observed in the same conditions. The evolution of the action is also shown for various input power. The same quite large net action variation is found independently of the e.m. input power, provided that the power exceeds the trapping threshold value (≈ 400 kW for this case). Quite similar results are found at first harmonic, with a much larger energy variation.

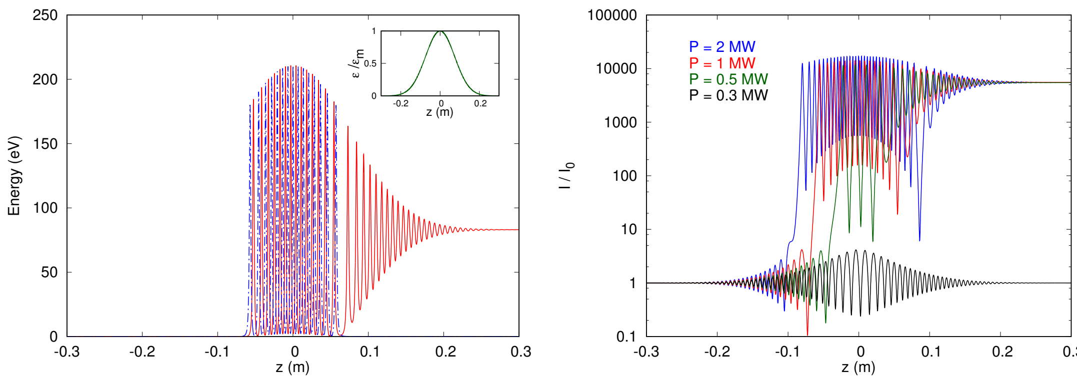

Figure 1: *Electron energy $mc^2(\gamma - 1)$ versus coordinate z for two different initial phase values θ (left). The electric field spatial variation is shown in the inset plot on the top right. Other parameters are $P = 1$ MW, beam width $w = 10$ cm, second harmonic, X-mode (XM) polarization, $N_\parallel = 0.3$, $\nu_n = 1$. Evolution of canonical action I normalized to initial value as a function of coordinate z for various input EC power, $P = 0.3, 0.5, 1, 2$ MW (right).*

The electron motion is characterized by a couple of fast action-angle variables (θ, I) and of slow canonically conjugate variables (z, P_z), and the action integral $J(z, P_z) \equiv n/2\pi \oint I\, d\theta$ is thus an adiabatic invariant of the motion for sufficiently slow e.m. field variation. Breaking of the invariant at separatrix crossing via trapping/untrapping allows to connect different regions in phase space and provides the mechanism for net energy variation [1–5]. The initial and final states are related by $\Delta\gamma = \nu_n \Delta I$, and $\Delta P_z^2 = \Delta\gamma[\Delta\gamma - 2(1/\nu_n - \gamma_0)]$ due to the constancy of the Hamiltonian. A net energy variation can occur if H_0 is double valued in action I, i.e. if $\partial H_0/\partial I = 0$ is satisfied. A particle with initial I_0 and $\bar{P}_{z0}$ initially on a given branch of the

Hamiltonian will enter the beam region, cross the separatrix for $\varepsilon = \varepsilon_s$ for increasing ε (if $\varepsilon_s \leq \varepsilon_M$), being trapped and then untrapped for decreasing ε at $\varepsilon = \varepsilon_s$, and exit the beam region with final values I_f and $\bar{P}_{zf}$, with $\bar{P}_{zf} \simeq \bar{P}_{z0}$. Note that $\Delta\gamma$ will be equal to zero if the particle at the last separatrix crossing will belong to the initial Hamiltonian branch.

The evolution of the slow variables is determined by the constancy of the adiabatic invariant J and of the Hamiltonian, and the ε and P_z values at the separatrix are related to the initial (or final) momentum values by the system of equations $I_{0,f} = |J_{0,f}(\varepsilon_s, \bar{P}_{zs})|$, and $H_0(I_{0,f}, \bar{P}_{z0,f}) = H_s(\varepsilon_s, \bar{P}_{zs})$, where $J_{0,f}(\varepsilon, \bar{P}_z)$ is the action integral in the initial and final stage of the motion and H_s is the Hamiltonian computed at the separatrix. The total variation of the action variable I is then equal to the jump of the adiabatic integral at the separatrix $\Delta I \equiv I_f - I_0 = J_f(\varepsilon_s, \bar{P}_{zs}) - J_0(\varepsilon_s, \bar{P}_{zs})$. The separatrix crossings of a very low energy particle with initial action $I_0 < I_r(\bar{P}_{z0})$, and momentum $\bar{P}_{z0}$, are given by $J_l = I_0$ for the first crossing, and either $J_l = I_f$ or $J_u = I_f$ for the last one, depending on the final branch of the Hamiltonian, lower or upper. In the first case $I_f = I_0$, so the electron recovers its initial energy ($\Delta\gamma = 0$), while in the last case it experiences a net energy variation

$$\Delta\gamma = \nu_n[J_u(\varepsilon_s, \bar{P}_{zs}) - J_l(\varepsilon_s, \bar{P}_{zs})] = 2\nu_n[I_r(\bar{P}_{zs}) - I_0]. \quad (2)$$

Note that the nonlinear energy variation (2) depends on the initial conditions only, and that for a given set of launching parameters the maximum energy gain occurs for particle with the lowest possible energy. The maximum field amplitude E_M value determines under the conditions for trapping to occur.

A detailed investigation of the above outlined process for the first two harmonics can be performed in the phase space of the fast variables for fixed ε and $\bar{P}_z$ (see e.g. [3,4]). A separatrix is found only if $\varepsilon \leq \varepsilon_{cn} = \beta_{cn} I_r^{2-n/2}/T_n$, with $\beta_{c1} = (2/3)^{3/2}$ and $\beta_{c2} = 1/2$. For low ε values the phase space topology for the resonant case is pendulum-like, while it shows quite different features for ε close to ε_{cn}. The adiabatic integrals J_l and J_u on the lower and upper branch of the separatrix are given in [3], and satisfy the relation $J_l + J_u = 2I_r(\bar{P}_{zs})$.

Simple analytical estimates for the energy variation can be derived for electrons with very low initial energy. It is found that $\bar{P}_{zs} \simeq \bar{P}_{z0}$ except for the first harmonic oblique propagation for which the $\bar{P}_z$ variation is small but finite. Figure 2 shows a very good agreement between the analytical estimates and the results obtained by solving the Hamilton equations of motion for various initial phase values and same initial I_0 and P_{z0}. The energy increases linearly with the frequency mismatch from the resonance $1 - \nu_n$, up to a maximum value which depends

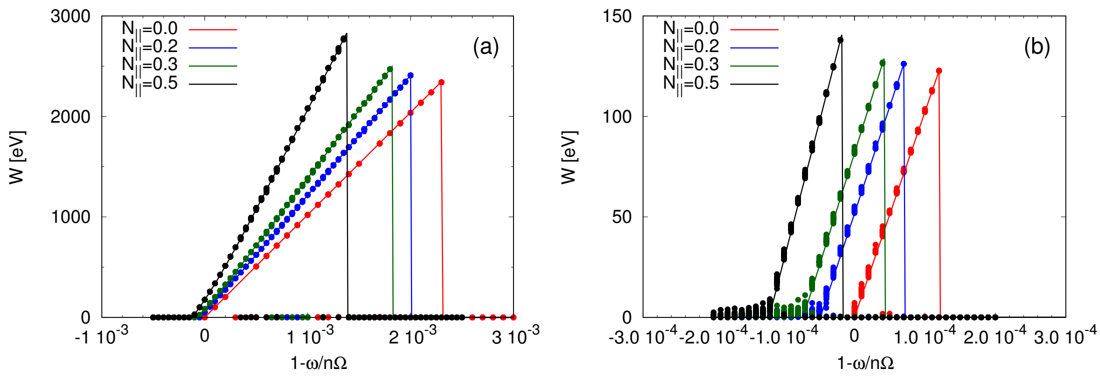

Figure 2: *Electron final energy W as a function of the EC frequency ratio for $P = 1$ MW, $w = 10$ cm, $f = 170$ Ghz and quasi XM1 (a), XM2 (b) polarization. Solid curves correspond to the analytical estimate of the maximum energy, points to the numerical solution of Hamiltonian equations with 50 uniformly distributed θ_0 initial values and $I_0 = \beta_{th}^2/2$, $P_{z0} = \beta_{th}$. Note the different scales for the two harmonics.*

on ε_M, and drops to zero afterwards. Since very low energy particles cross the separatrix for $\varepsilon \approx \varepsilon_{cn} < \varepsilon_M$, the maximum energy $W = mc^2(\gamma - 1)$ for increasing $1 - \nu_n$ can be estimated for $\varepsilon_M = \varepsilon_{cn}$ as $W_{max} \simeq 2mc^2[\varepsilon_M T_n/(\beta_{cn}(1 - N_\parallel^2))]^{2/(4-n)}$. For a quasi X-mode ($e_x = e_z = 0$) the maximum energy W_{max} scales with the injected EC beam parameters as

$$W_{max,XM1} \simeq 15.6 P^{1/3}/[fw(1 - N_\parallel^2)]^{2/3}, \qquad W_{max,XM2} \simeq 2.1 P^{1/2}/[fw(1 - N_\parallel^2)], \tag{3}$$

where W, P, f, w are measured in keV, MW, GHz and meters, respectively. The derived simple analytical estimates for the maximum energy gain as a function of power, frequency and beam spot size are in good agreement with numerical particle simulations.

In conclusion, the analysis confirms that cold electrons can easily gain energies well above the ionization energy in most conditions, as observed in many experiments [6, 7].

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