Dimensionless Scaling of Intrinsic Toroidal Rotation

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Abstract. An empirical scaling for intrinsic rotation has been published [2]. We now look for a gyrokinetic basis for this scaling. We have shown some agreement with published gyrokinetic results and theory [2]. Now, TGLF is being brought to bear on this endeavor, so far with some qualitative agreement.

The plasma rotation profile is uncertain in detail for projections to ITER, relative to the temperature and density profiles, largely because of the uncertainty in the size scaling of intrinsic rotation and sensitivity of the core rotation profile, which can display large variations and change sign with a small change in plasma parameters [1]. Recently an empirical dimensionless scaling for the intrinsic toroidal rotation magnitude under axisymmetric tokamak conditions has been proposed [2], namely that $M_A \sim \rho^* \beta_N$, where M_A is the toroidal velocity, V_ϕ, scaled to a global Alfven velocity, ρ^* is the local ion gyroradius scaled to the minor radius, and β_N is the usual definition of normalized beta. This toroidal velocity is in the direction of the plasma current, co-Ip. This scaling projects to a modest rotation frequency for the ITER inductive scenario, ~ 4 krad/s at ρ ~ 0.7. Interestingly, this projected intrinsic rotation value is well within a factor of 2 of a projection arising from other completely different DIII-D experiments that utilize NBI modulation to back out an intrinsic torque and extrapolate to ITER [3]. This dimensionless scaling is shown to be directly related to the well-known machine parameter scalings of Rice and Parra [see Ref. 2], which scale the intrinsic toroidal velocity with stored energy, or ion temperature, respectively, divided by plasma current magnitude.

A previous investigation into the gyrokinetic basis of the empirical scaling of intrinsic toroidal rotation has shown that published theoretical results from simulations and analytic theory can show agreement with empirical scaling and even in intrinsic rotation profile comparison [4,5]. First, we restate the dimensionless scaling in terms of the local kinetic parameters, finding that $V_\phi \sim (nT)^{1/2} T/B_\theta \sim p_i^{3/2}/n_i B_\theta$ where n_i is ion density, T is ion temperature (using T=Ti=Te), $p_i = n_i T$, and B_θ is the poloidal magnetic field. The simplified momentum equation is

$$-\chi_\phi \, {\partial l}/{\partial r} + u^p l + \Pi_M = 0 \qquad (1)$$

where $\ell = Mn < R^2 > \omega_\Phi$ is the toroidal angular momentum (TAM) density, χ_ϕ the momentum diffusivity and u^P the convective velocity, a pinch for $u^P < 0$, and $\Pi_M = Mn < R\tilde{v}_\phi \tilde{v}_r >$ is the turbulent gyrokinetic momentum stress. The transport terms are also driven by turbulence. Taking a qualitative statement from gyrokinetic simulation scaling that $\Pi_M/Q_i \sim -\sqrt{2}/\varepsilon_a\,\omega_{c\theta}$ [6], where Q_i is the (outward) ion thermal energy flux, $\varepsilon_a = a/R_0$, and $\omega_{c\theta}$ is the gyrofrequency in the poloidal field, and using Shaing's "neoclassical quasilinear theory" [7] scaling of the pinch term $u^P/\chi_\phi = \partial[In(p_i^\alpha) + In(T_i^\gamma)]/\partial r$, we show that the solution of eqn (1) can be related to the empirical scaling with the Shaing exponents being $\alpha = 3/2$ and $\gamma = 0$ [5]. As an approximate experimental validation of the above gyrokinetic correlation from [6], in DIII-D this predicts $\Pi_M/Q_i \sim 3x10^{-7}s$ in magnitude, or $\Pi_M/Q_i \sim 0.3\,Nm/MW$, about 1/3 of the torque per power of the unidirectional neutral beams. Observationally in DIII-D, an intrinsic rotation of ~1/3 of that from the unidirectional beams is reasonable, when the NB power level generates the same $\beta_N \sim 1$ typical of intrinsic rotation conditions.

To further investigate how this empirical scaling might be described in gyrokinetic theory, we utilize the reduced gyrokinetic code, TGLF [8]. As we have observed [5], it appears that a dominant mechanism creating the co-Ip peak in intrinsic rotation between mid minor radius and the pedestal region is a balance between turbulence driven momentum diffusion, χ_φ, and Π_M due to ExB shear symmetry breaking. In order to look for this in TGLF we evaluate the transport coefficients and stress terms in eqn (1) individually, then hypothesize that intrinsic rotation is in a low drive limit so that these values can then be used together in eqn (1). Defining the toroidal rotation through $m = R_0\omega_\phi/c_s$, with $c_s = \bar{v}_i = \sqrt{T/M}$, TGLF is used to compute u^P from the ion momentum flux, Π_m, with nonzero m, and the diffusivity χ_φ from the flux Π_γ, with nonzero $\partial m/\partial r$, and the r.h.s. flux with nonzero $\partial E_r/\partial r$, Π_M. If all of these driving terms are zero TGLF gives a zero turbulent ion momentum flux. TGLF variables are gyro-Bohm normalized {mass,length,velocity,density,energy,field} -> {M_D,a,c_s,n,T_e,B_0}. All the turbulent transport quantities in eqn (1) ~ $(\rho^*)^2$. Since turbulence in TGLF is driven by the normalized density and temperature gradients, $-aLn = \partial\ell n(n)/\partial x$, and $-aLT = \partial\ell n(T)/\partial x$ (~ TGLF definitions), with x= r/a, we have done 2-d scans in these variables using TGLF to investigate the intrinsic rotation scaling. The ExB shear is defined to be [9] -

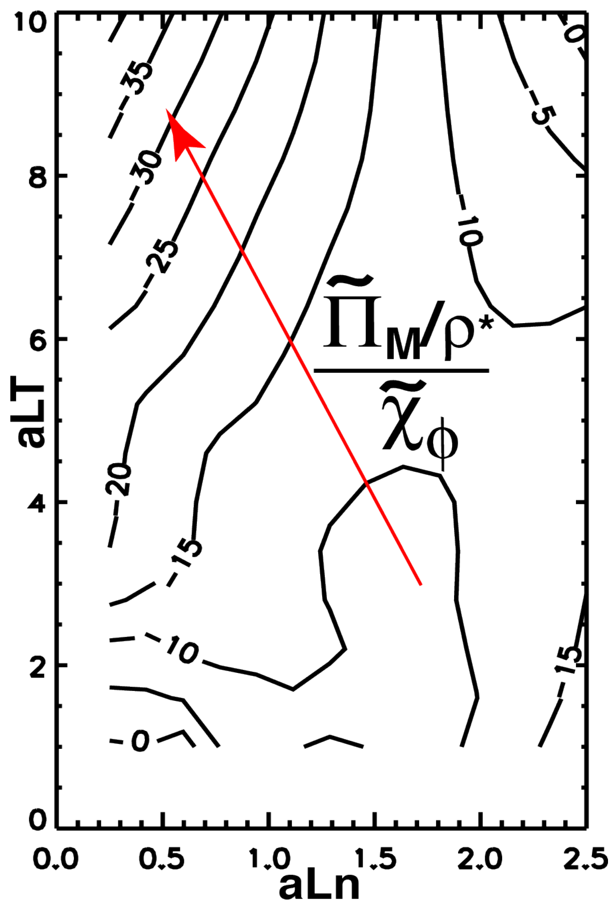

Figure 1 TGLF scan in [aLn,aLT] computing ExB shear-driven momentum flux divided by the TGLF-determined momentum diffusivity.

$$\gamma_E = -\frac{x}{q}\frac{\partial}{\partial x}\left(\frac{qE}{rB}\right) = -k_q\frac{E}{B} - \frac{\partial}{\partial r}\left(\frac{E}{B}\right) \quad (2)$$

where $k_q = (\hat{s} - 1)/r$ and $\hat{s}$ is the magnetic shear term. The ExB shear given by (2) is computed and used at every point in the 2 parameter TGLF gradient scan. The magnetic shear is held constant in these scans. The magnetic shear term can be dominant in the region 0.5 < x < 0.9 where $\hat{s}$ is large, and is likely the way the common $1/I_p$ effect in intrinsic rotation enters via TGLF. For the electric field we use $(E/B = V_\theta + (r/R_0 q)V_\phi + (\partial p_i/\partial r)/(en_iB)$, from radial force balance, with the sign values for normal DIII-D conditions. The poloidal velocity is taken from the neoclassical value in the plateau regime, ~ aLT. In computing $\gamma_E$we neglect the V_ϕ term because of the small multiplier and we neglect second derivatives of n and T. The normalized ExB shear thus obtained has the functional form $\tilde{\gamma}_E = \gamma_E[a/c_s] = \rho^* G\,(aLn, aLT, \hat{s})$. We use this value in TGLF when scanning the gradients and computing

Π_M due to ExB shear, where typically an inward turbulent flux is generated by positive $\tilde{\gamma}_E$ [9].

In figure 1 we plot contours of the momentum flux scaled by diffusivity from a TGLF gradients scan that covers typical values for intrinsic rotation conditions in the spatial range of interest [5]. Here, $\Pi_M/\chi_\phi = (\tilde{\Pi}_M/\tilde{\chi}_\phi)(anT/ac_s)$, and here $\hat{s} = 1.5$. The arrow shows the direction of increasing inward momentum flux (< 0). Along this arrow greater temperature gradient and smaller density gradient lead to more inward flux, and hence larger momentum gradient to balance. For orientation, the ITG-dominant GA standard case [9] has aLn=1, aLT= 3 and $\hat{s} = 1$. Essentially, the turbulence drive threshold is at aLT just below 2. For the range of relatively small $\tilde{\gamma}_E$ we find that $\tilde{\Pi}_M$ is linear in $\tilde{\gamma}_E$, that is, not strong enough to suppress the turbulence [9]. Thus, $\tilde{\Pi}_M$ is linear with ρ^* in this regime. With no convective term in eqn (1), the gradient in TAM must balance the quantity in figure 1, that is

$$\partial m/\partial x \approx \left[\tilde{\Pi}_M/\rho^*\right]_0 \rho/\chi_\phi \qquad (3),$$

where $[\Pi_M/\rho^*]_0$ is the TGLF ~ linear dependence of the flux on ρ* as used in the plot. Eqn (3) demonstrates a strong correlation of V_ϕ with T_i. The small shear linear scaling of $-\tilde{\Pi}_M$ with $\tilde{\gamma}_E = \rho^* G$, means $\partial V_\phi/\partial x \sim -a\omega_{c\theta}(\rho^*)^2 G$. With significant magnetic shear, the dominant term in G is $[ak_q + (a/R_0) + 0.5](aLT) \sim -(\partial T/\partial x)/T$. Using this term in (3) results in $\partial V_\phi/\partial x \sim (1/eaB)\partial T/\partial x$. A clear correlation between the T_i profile and the intrinsic velocity profile is to be anticipated. If the pedestal region velocity is determined by ion orbit loss, then the boundary conditions for velocity and temperature are correlated and the correlation will be carried in magnitude clearly through the profile. The remaining intrinsic rotation dependence needed to match the empirical scaling is $(T/n)^{1/2}/B_\theta$. We hypothesize that the $(1/eaB)$ term above could be replaced by $(1/eaB_\theta)$ through the effect of the k_q factor in $\tilde{\gamma}_E$, but this is still to be investigated in further TGLF scans.

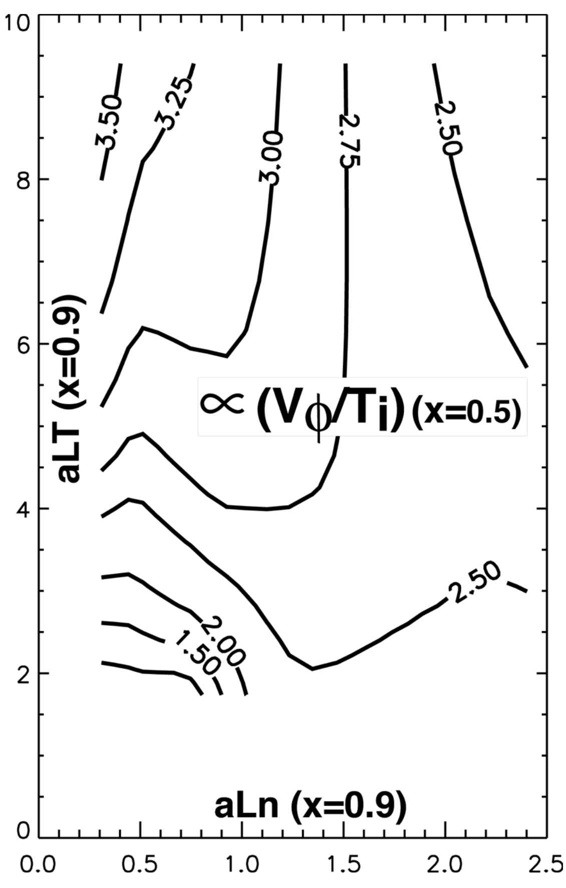

Figure 2. Computed scaled peak velocity for a set of linear temperature and density profiles.

TGLF is a local, gradient driven model that computes local turbulent fluxes. The empirical scaling is stated in terms of the local values of V_ϕ, T_i, and n, not the gradients. To make progress in the connection it would seem that profiles must be considered. Toward this approach we take families of linearly decreasing T_i and n profiles in the region $0.5 < x < 0.9$, having different slopes, with the value of T_i and n each normalized to 1 at $x = 0.5$. For each set of profiles, we compute $\partial V_\phi/\partial x$ vs. x, locally using the computed values indicated in figure 1, then integrate spatially to get the

V_ϕ profile. The results of this exercise are shown in figure 2, where the contours are proportional to V_ϕ/T_i evaluated at $x = 0.5$, and the axes specify the profiles by plotting the normalized gradients at $x = 0.9$, where these have their largest values. Beyond the linear T_i scaling there is a general trend of higher intrinsic rotation with larger aLT and lower aLn due to the greater turbulence drive, and these trends go in the direction of supplying the additional $(T/n)^{1/2}$. These initial investigations with TGLF indicate that empirical scalings should consider the local experimental profiles, and in theoretical treatments the full velocity profile should be computed and compared with experimental values. The missing factor of the poloidal field will be investigated with magnetic shear scans.

Conclusion The empirically determined non-dimensional scaling for intrinsic toroidal rotation is recast into a form that shows the dependence of velocity on temperature and density, so that we can investigate the relation of the scaling to gyrokinetic theory and modeling. We find that the scaling qualitatively agrees with the gyrokinetic forumlation when using two published results [6,7]. To make quantitative progress we utilize TGLF scans of the normalized density and temperature gradients. When the turbulent momentum stress is dominated by the magnetic shear term of γ_E , we show that the dominant part of the intrinsic velocity scaling, $V_\phi \sim T_i$, is immediately recovered. TGLF does indicate how the other density and temperature terms could enter, but to make progress with TGLF we must utilize density and temperature profiles, in order to relate n and T to their gradients beyond one specific minor radius. This will allow relating the empirical scaling to the local fluxes determined in TGLF.

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