Modelling electrostatic solitary waves in positron-laden plasmas: shedding new light on an old problem

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Abstract

The propagation of linear and nonlinear electrostatic waves in an electron-positron-ion (e-p-i) plasma is studied, from first principles. The parametric effect of the relative concentration and temperature of the positron component is investigated. Earlier results are thus recovered, and in fact extended, by our analysis.

1. Introduction. Plasmas containing electrons, ions and positrons are widely found in astrophysical plasmas [1] and also in the laboratory [2]. Inspired by recent experiments on positron beam production [2], we have revisited by means of a systematic investigation, from first principles, the dynamics of electrostatic (ES) excitations in e-p-i plasmas. We have focused in particular on the role of the positron component (concentration, temperature) on the propagation characteristics of electrostatic (ion-acoustic) solitary waves. Our work has embraced, and indeed extends, earlier first-principle studies [3] on large-amplitude excitations; a rigorous link is thus established to small-amplitude theory, as will be shown in detail elsewhere [4].

2. Theoretical Model. We consider a collisionless plasma consisting of inertial ions (mass m_i, charge $+Z_i e$), inertialess electrons (mass m_e, charge $-e$) and positrons (mass m_p , charge $+e$). The scaled (dimensionless) fluid model for the e-p-i plasma reads:

$$\begin{aligned}
\frac{\partial n_i}{\partial t}+\frac{\partial (n_i u_i)}{\partial x} &= 0,\\
\frac{\partial u_i}{\partial t}+u_i\frac{\partial u_i}{\partial x} &= -\frac{\partial \phi}{\partial x},\\
\frac{\partial^2 \phi}{\partial x^2} &= -(1-\delta)n_i + e^{\phi} - \delta e^{-\phi/\theta}\,. \qquad (1)
\end{aligned}$$

where the time, space and velocity variables were scaled by the plasma period $\omega_{p,i}^{-1} = \left(\frac{n_{e0}e^2}{\varepsilon_0 m_i}\right)^{-\frac{1}{2}}$, the Debye length $\lambda_{D,e} = \left(\frac{\varepsilon_0 k_B T_e}{n_{e0}e^2}\right)^{\frac{1}{2}}$ and the characteristic speed $\omega_{p,i}\lambda_{D,e} = \left(\frac{k_B T_e}{m_i}\right)^{\frac{1}{2}}$. The parameters δ and θ represent the positron-to-electron density and temperature ratio(s), $\delta = \frac{n_p}{n_e}$ and $\theta = \frac{T_p}{T_e}$, respectively. Electrons and positrons are assumed to be in thermal equilibrium, hence $n_e = n_{e0}e^{\phi}$ and $n_p = n_{p0}e^{\phi}$, where n_{j0} denote the unperturbed densities ($j = e, p$).

3. Linear Analysis. Assuming small-amplitude harmonic variations, the (dimensionless) dispersion relation is found to be $\omega^2 = k^2(1-\delta)/\left[k^2+(1+\delta/\theta)\right]$, implying the "true" (i.e. positron-dependent, dimensionless) sound speed in e-p-i plasma to be $c_s = \sqrt{\frac{1-\delta}{1+\frac{\delta}{\theta}}}$. Both frequency and phase speed decrease with higher positron concentration, as witnessed in Fig. 1.

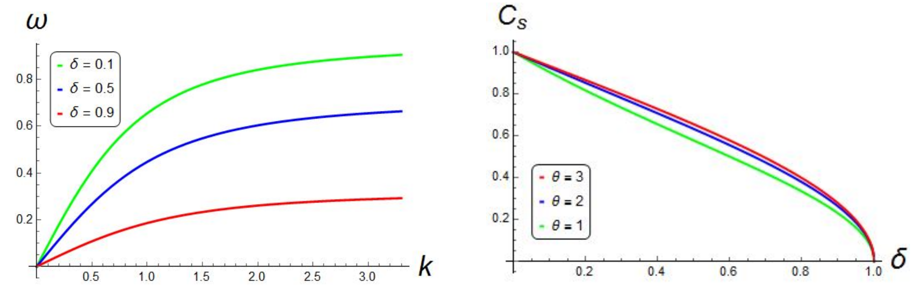

Figure 1: The vibration frequency is depicted for different values of δ, taking $\theta = 1$ (left panel). The true (i.e. positron-dependent) "sound speed" c_s is shown against δ, for different values of θ (right panel).

4. Nonlinear Analysis and Parametric Investigation. Adopting the Sagdeev pseudopotential method [3, 5], we anticipate (arbitrary amplitude) stationary-profile solutions as functions of $X = x - Vt$, where the velocity V is a real parameter (to be determined by physical considerations). The original fluid model is thus reduced to a pseudo-energy balance equation:

$$\frac{1}{2}\left(\frac{d\phi}{dX}\right)^2 + S(\phi, V, \delta, \theta) = 0, \tag{2}$$

where the Sagdeev-type pseudopotential function (note the i, e, p contributions) is given by:

$$S(\phi, V, \delta, \theta) = V^2(1-\delta)\left(1-\sqrt{1-\frac{2\phi}{V^2}}\right) + 1 - e^{\phi} + \delta\theta(1-e^{-\frac{\phi}{\theta}}). \tag{3}$$

The pseudo-energy balance equation (2) can be solved numerically for the potential ϕ, leading a bipolar electric field form $E = -\nabla\phi$, as shown in Fig. 2, for indicative parameter values.

Various comments are in row, some of which have been overlooked in earlier works.

The finite (non-zero) root of $S(\phi, V, \delta, \theta)$, say ϕ_{max}, which represents the pulse amplitude [5], is a growing function of V. On the other hand, as the positron concentration is increased, slightly larger and narrower pulses are created. These are inferred from e.g. Fig. 3.

The maximum amplitude ϕ_{max} also grows, for higher positron concentration; see Figure 4.

Velocity range. The analysis shows that pulses may exist for values of V between a threshold V_{min} and V_{max} (for given values of δ and θ, that is). The former (V_{min}) corresponds to the sound speed (see above) – hence, pulses are *always super-acoustic* – while the latter (V_{max}) determines

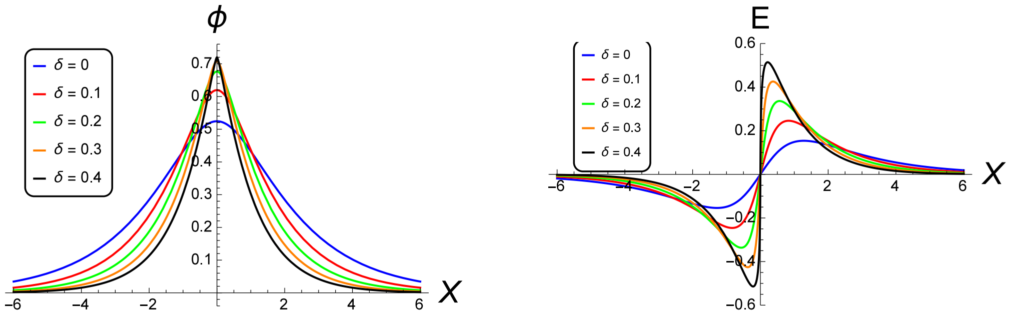

Figure 2: The electric potential pulse ϕ (a) and the bipolar electric field $E = -\nabla\phi$ are depicted, for various values of δ, taking $V = 1.2$ and $\theta = 1$.

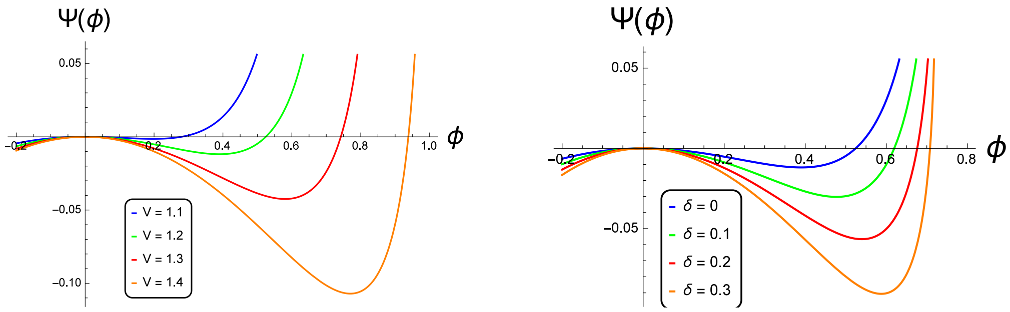

Figure 3: The Sagdeev pseudopotential is depicted versus ϕ: (left panel) for $V = 1.1, 1.2, 1.3, 1.4$ with $\delta = 0$ and $\theta = 1$; (right panel) for $\delta = 0, 0.1, 0.2, 0.3$, with $V = 1.2$ and $\theta = 1$.

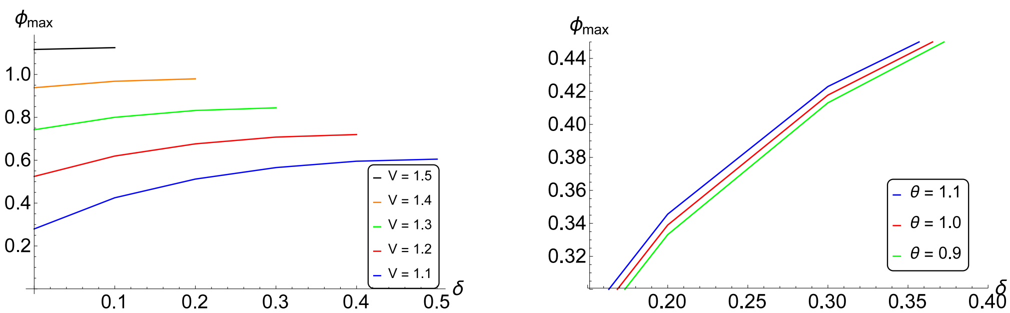

Figure 4: The root of S, ϕ_{max}, is depicted against the relative positron concentration δ, for different values of: (a) the pulse speed (for $\theta = 1$); (b) the temperature ratio θ (for $V = 1.2$).

the infinite compression limit [5], where the dependent variables are no longer real-valued, and the model collapses. This velocity range can be seen in Figure 5. As the positron concentration is increased, the existence region (V_{min}, V_{max}) shrinks and, in fact, vanishes as $\delta \to 1$ (*e-p* plasma limit). The above observations are in agreement with earlier findings [3] (although the upper limit V_{max} was actually not discussed in the latter reference).

Finally, it may be added that considering a higher positron-to-electron temperature ratio θ leads to faster solitons; however, this effect is weaker than that of n_p; cf. Figs. 4b and 5.

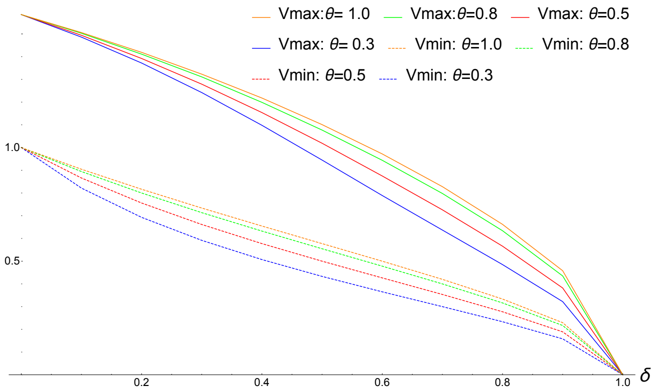

Figure 5: Plot showing the maximum and minimum values for the velocity of soliton pulses as δ varies from $0-1$. Recall that pulses occur between the lower and upper curves.

Conversely, for given value of V, there is an interval of positron density values $\delta \in (0, \delta_{max})$ wherein pulses may occur. It turns out that, the larger the pulse speed V, the lower δ_{max} will be: see left panel in Fig. 4. There are no pulses beyond δ_{max}.

5. Summary. We have investigated, from first principles, the impact of an additional positron component on ion-acoustic waves propagating in an *e-p-i* plasma. A parametric analysis of the positron concentration and temperature effect(s) provides evidence of a strong impact of the former and a weaker one of the latter on bipolar E-field characteristics. Further investigation, including a more detailed parametric analysis and also a weak-amplitude slightly supersonic nonlinear analysis (leading to the same qualitative conclusions though, strictly speaking, for small-amplitude pulses), was omitted here for brevity, and will be reported soon elsewhere [4].

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References

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