Magnetic topology and the homoclinic tangle of the primary separatrix in divertor tokamaks

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ABSTRACT: Divertors are a regular feature of the modern day large tokamaks. Divertors are required for handling the plasma particle and heat exhausts on the walls in fusion plasmas. The single-null divertor can have two distinct magnetic topologies: open unbounded topology and closed compact topology. The simple map [1] generically represents open unbounded topology; and the symmetric quartic map [2] generically represents the closed compact topology. The new approach for calculation of homoclinic tangles of separatrices in Hamiltonian systems [3] is used. The homoclinic tangles of the primary separatrix of the single-null divertor tokamaks with the two distinct topologies are calculated and compared and contrasted.

In divertor tokamaks, open and unbounded topology is generically represented by the simple map [1] and the closed and compact topology is generically represented by the symmetric quartic map [2,10]. This study then will yield the generic features of the effect of topology on the homoclinic tangle of the ideal separatrix and the footprint from magnetic asymmetries. The trajectories of magnetic field lines are a 1½ degree of freedom Hamiltonian system where the toroidal angle φ plays the role of canonical time. There are three sets of canonical coordinates that can be deployed to calculate the trajectories of field lines. These are the magnetic or Boozer coordinates, the natural canonical coordinates (NCC), and the physical canonical coordinates [4]. In this study, the NCC is used because they are most suitable to represent doubly periodic magnetic asymmetries in tokamaks and they can be readily transformed to physical space [4]. Poloidal magnetic flux is the Hamiltonian for the trajectories of magnetic field lines.

The total Hamiltonian can be as $\psi_p(\psi_t,\theta,\varphi)=\overline{\psi}_p(\psi_t,\theta)+\widetilde{\psi}_p(\psi_t,\theta,\varphi)$ where the equilibrium poloidal flux, $\bar{\psi}_p$ is the unperturbed Hamiltonian and $\tilde{\psi}_p$ is the magnetic perturbation. The area-preserving or symplectic map equations for the trajectory of the i^{th} field line is given by

$$\psi_{t,i}^{(j+1)}=\psi_{t,i}^{(j)}-k\partial\psi_p\left(\psi_{t,i}^{(j+1)},\theta_i^{(j)},\varphi_i^{(j)}\right)/\partial\theta_i^{(j)}\ ,\ \theta_{t,i}^{(j+1)}=\theta_{t,i}^{(j)}-k\partial\psi_p\left(\psi_{t,i}^{(j+1)},\theta_i^{(j)},\varphi_i^{(j)}\right)/\partial\psi_i^{(j+1)}$$

and $\varphi_{t,i}^{(j+1)} = \varphi_{t,i}^{(j)} + k$. The equilibrium generating function for the simple map is given $\overline{\psi}_p(\psi_t, \theta) = \psi_t - \frac{2\sqrt{2}}{3}\sin^3(\theta)\psi_t^{3/2}$. The map equations are then given by $\psi_{n+1} = \psi_n + k\partial\overline{\psi}_p(\psi_t, \theta)/\partial\theta_n$ $\theta_{n+1} = \theta_n + k\partial\overline{\psi}_p(\psi_t, \theta)/\partial\psi_{n+1}$, and $\varphi_{n+1} = \varphi_n + k$. The map parameter *k* can be used to represent effects of perturbations as in the standard map [5,6]. Magnetic asymmetries can also be explicitly represented by $\widetilde{\psi}_p(\psi_t, \theta, \varphi) = \sum_{(m,n)} \delta_{mn}(\psi_t)\cos(m\theta - n\varphi + \phi_{mn})$ where (*m*,*n*) are the poloidal and toroidal mode numbers of the perturbation and the ϕ_{mn} are the phases. The objective of this work is to study the generic effects of two distinct types of magnetic topologies for single-null divertor tokamak. An important issue is to determine whether to represent the magnetic asymmetries by the map parameter *k* or explicitly as the above equation. It is important here to dwell on this issue. Representation of the asymmetries through *k* has definite advantages. It is completely generic and is independent of the structure and type of the asymmetries. At the heart of this issue is estimating the size of magnetic asymmetries corresponding to a given value of *k*. One path to resolving this is to calculate the width *w* of the stochastic layer in the neighborhood of the *X*-point as a function of the map parameter *k*. For small enough perturbation, the width of the stochastic layer scales as the $\delta^{1/2}$ in tokamaks [7]. This scaling can possibly be used to derive a scaling of δ with *k*. However, this could be an arduous, time-consuming, and computationally intensive task, and must be done for each map separately.

For calculation of the homoclinic tangles, a recently developed method by Punjabi and Boozer for the DIII-D tokamak [8,9,10] is used. This new approach is based on two topological invariants: the symplectic invariant and the preservation of the neighborhood. When the separatrix manifold is mapped forward and backward a single toroidal circuit, these manifolds meet in theφ=mod(φ,π)=0 plane, and form homoclinic tangle. Figure 1(a-b) depicts ideal separatrix of the SM. Fig. 2(a-f) depict the homoclinic tangles of the SM and SQM for various values of *k* after a single toroidal circuit in the φ-plane. These figures show that for SM, the principal lobe near the X-point is prominently elongated in radial direction as *k* increases ; while the principal lobe near the X-point for the SQM moves in the poloidal direction. It is reasonable to infer these difference is due to difference in topology. We plan to study this issue in depth in future. This work is supported by grants DE-FG02-01ER54624, DE-FG02-04ER54793. This research used resources of the NERSC, supported by the Office of Science, US DOE, under Contract No. DE-AC02-05CH11231.

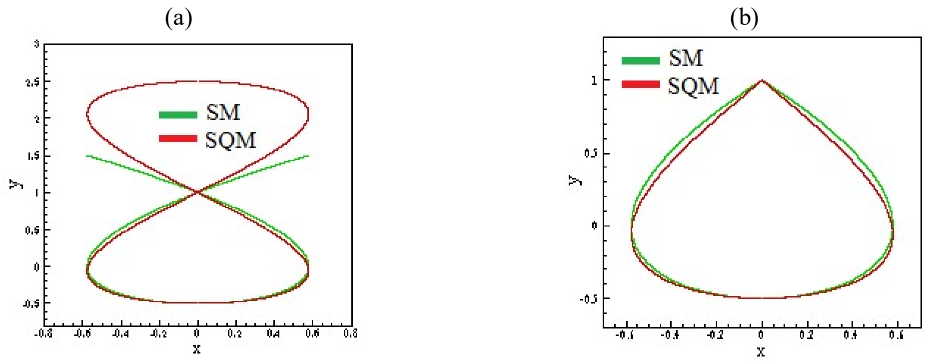

Fig.(a-b): Ideal separatrices for the SM and SQM.

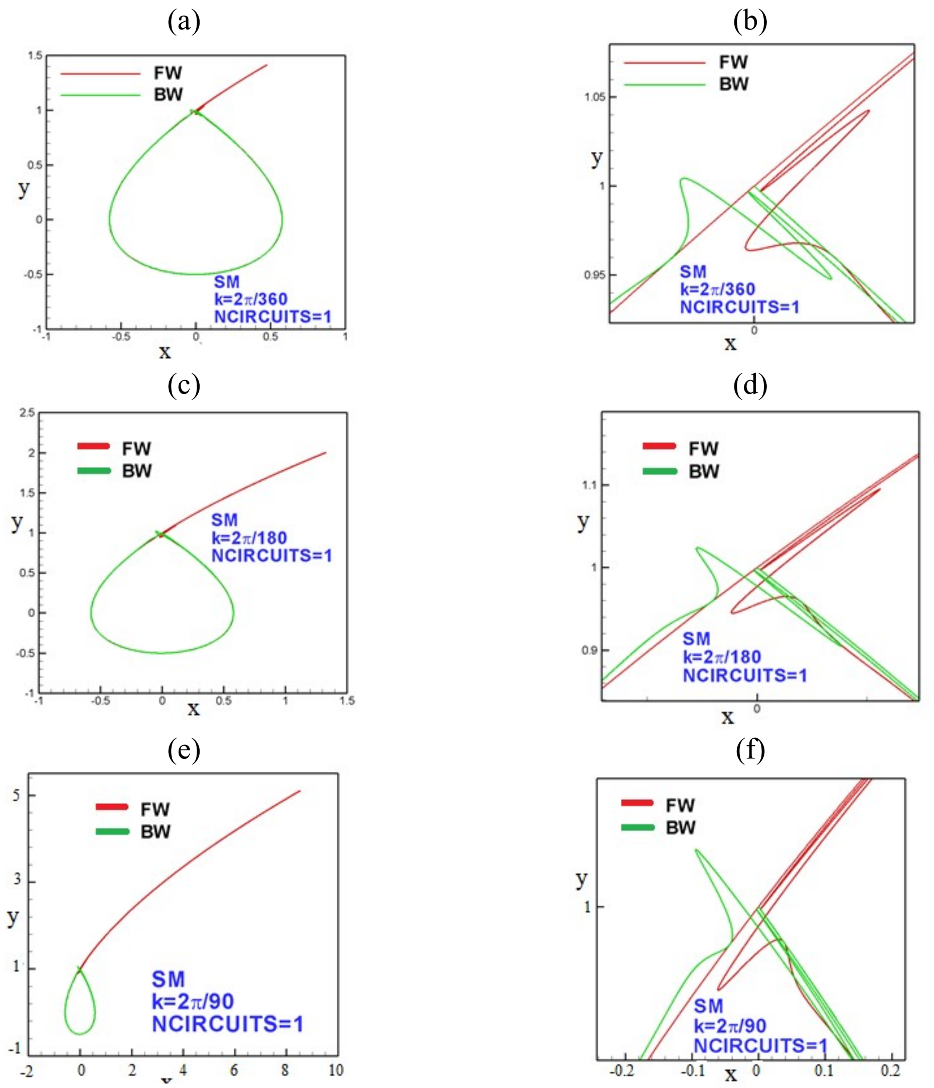

Fig.2(a-f): Homoclinic tangles of the SM separatrix for $k=2\pi/360$, $k=2\pi/180$ and $k=2\pi/90$ after a single toroidal circuit in the $\varphi=0$ plane.

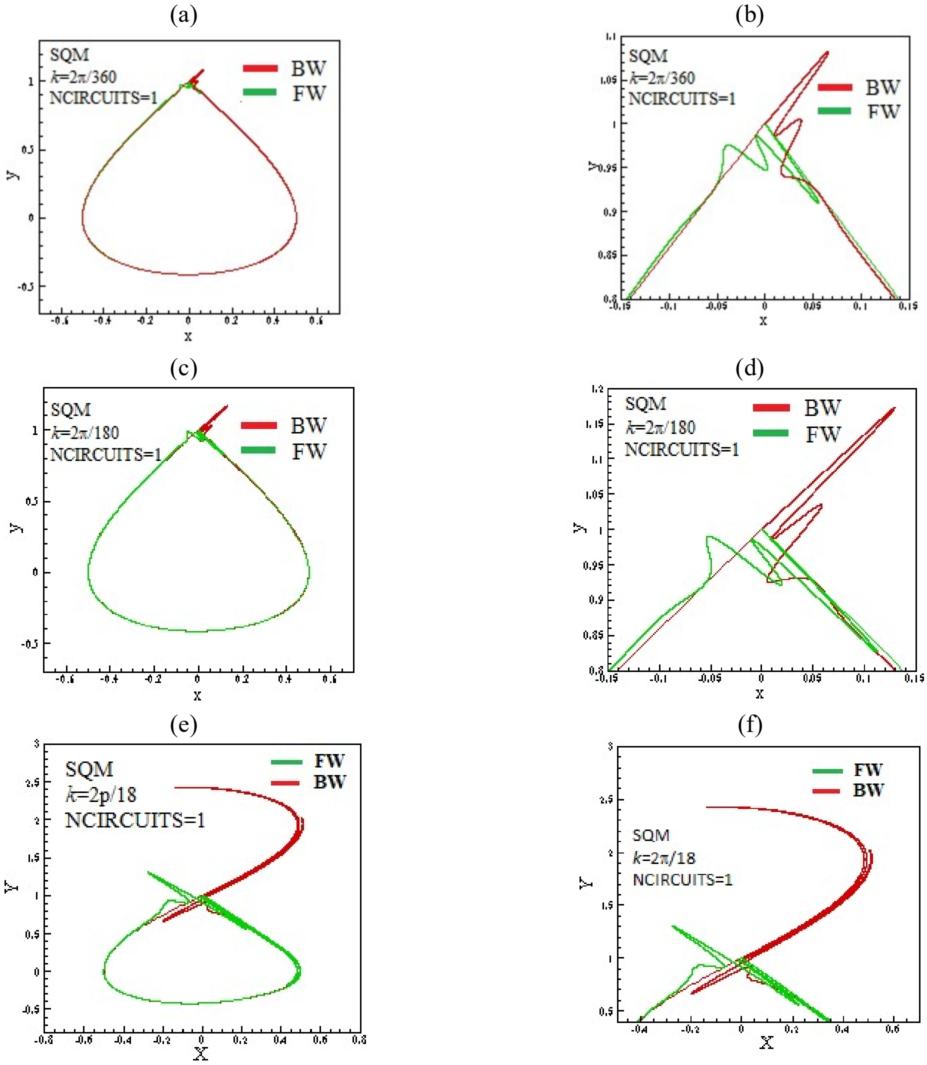

Fig.3(a-f): Homoclinic tangles of the SQM separatrix for $k=2\pi/360$, $k=2\pi/180$ and $k=2\pi/18$ after a single toroidal circuit in the $\varphi=0$ plane.

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