Enhanced radial velocity and damping rate of Geodesic Acoustic Modes in the presence of a temperature gradient

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Introduction

Geodesic acoustic modes (GAMs) represent the oscillating part of the Zonal Flow (ZF) and are generated by toroidal curvature effects through the coupling of the electric field (m,n)=0,0 mode and the (1,0) pressure perturbations. GAMs are strongly involved in the energy transport and in the turbulence suppression. In fact GAM oscillations contribute to the transfer of energy from the ZF to the pressure perturbation [1]. Moreover, GAMs can radially propagate the ZF with important consequences on the energy transport. However, to the present the radial propagation of the GAM is not well understood and several aspects need to be elucidated. Although most of experiments show a radial propagation outwards the tokamak device, some observations show an inward radial propagation of GAMs [2]. Moreover, it is unclear whether this velocity is constant or exhibits variations in the experiments and in particular the role of temperature gradient requires to be investigated. The linear theory of GAM velocity due to finite ion Larmor radius has been well developed in recent years predicting a group velocity $v_g \propto \omega_G k_r \rho_i^2$ for a GAM with a wavenumber k_r and a frequency ω_G. However, only a qualitative agreement with the experimental results has been obtained. Experimental results show a velocity much larger than that predicted by linear theory. Consequently in order to explain the gap between theory and experiments the research focuses on nonlinear aspects [3]. Although GAMs are a natural part of the turbulent system, it is nevertheless very useful to further investigate their linear behavior, in order to be able to judge how turbulence and GAMs influence each other. Recently, by studying the linear behavior of GAM in the presence of a temperature gradient a new damping mechanism has been identified [4] -the so called *Phase-mixing Landau damping (PL)-mechanism-* and discussed in Ref.s [4, 5, 6]. The local dependence of GAM frequency on the plasma parameters, such as temperature, generates a continuum spectrum in tokamak plasmas. As a consequence, in the presence of a temperature gradient, GAM is affected by phase mixing energy-conserving cascade. Thus, the combined effect of phase mixing and Landau damping has been proposed as a novel decay mechanism of GAMs at the tokamak edge, where the temperature gradients are very large. This proposed mechanism is consistent with the observed existence or nonexistence of GAMs in the different confinement regimes. Here, we focus rather on the real part of the GAM dispersion relation and we investigate the influence of the phase mixing effects on the GAM radial propagation [7].

Model and Simulations

First we briefly describe the equations used in the numerical code ORB5 [8] which now includes all extensions made in the NEMORB project [9]. This code uses a Lagrangian formulation based on the gyrokinetic Vlasov-Maxwell equations using a discrete version of gyrokinetic field theory. The code solves the full-f gyrokinetic Vlasov equation using a particle-in-cell δf method. The δf quantity represents the fluctuating part of the distribution function.

In this work we have used the electrostatic version of the model with a single ion species and adiabatic electrons. The corresponding polarization equation reduces to the standard linear quasi-neutrality condition, written in the long wave-length limit. In the code, the time t is normalized to the inverse of the ion cyclotron frequency $\Omega_i = eB_0/m_i$ (assuming $c = 1$), the radial direction is normalized to $\rho_s = \sqrt{k_B T_{e,0} m_i}/(eB_0)$ with $T_{e,0}$ the electron temperature, and the potential is given in $\phi_0 = k_B T_{e,0}/e$ units. The quantity B_0 is calculated at the magnetic axis, while $T_{e,0}$ is calculated in the middle of the radial domain. The ion Larmor radius is defined as $\rho_i = \sqrt{2}\sqrt{T_{i,0}/T_{e,0}}\rho_s$ with $T_{i,0}$ the ion temperature.

Figure 1: *GAM frequency ω as a function of the safety factor q for several values of the wave number k_r. The results of the simulations (points) are in good agreement with Eq. 1 with α_1 given in Eq. 2 (solid lines).*

In this paper we are interested in studying the radial GAM propagation and to this purpose we consider the real part of the GAM dispersion relation at the second order $k_r\rho_i$ of accuracy:

$$\omega^2 = \omega_G^2[1 + \alpha_1(k_r\rho_i)^2] \qquad \omega_G = \frac{\sqrt{7+4\tau_e}}{2}q\left(\frac{v_{T_i}}{R_0 q}\right)\left[1 + \frac{2(23+16\tau_e+4\tau_e^2)}{q^2(7+4\tau_e)^2}\right]^{1/2} \quad (1)$$

(for α_1 see below). In order to resolve these corrections in the simulations, we consider a temporal step of $25\Omega_i^{-1}$. We choose a plasma diameter $L_r = 2a/\rho_s = 320$, an inverse aspect ratio $\varepsilon = a/R = 0.1$ and a spatial grid of $(r \times \chi \times \phi) = (256 \times 64 \times 4)$ and a time step of $100\Omega_i^{-1}$. Note that convergence studies showed that 4 points in the toroidal direction ϕ are enough. Dirichlet boundary condition is imposed at the outer boundary $r = 1$, and Neumann boundary condition at the inner boundary $r = 0$. Simulations have been performed with 10^8 deuterium markers. Moreover we consider circular magnetic flux surfaces. In this limit, the flux surface coordinate $r = \sqrt{\psi/\psi_{edge}}$ is a good approximation of the usual cylindrical radial coordinate. In the first part of the work, temperature, density and q profiles have been considered flat. In Fig. 1, the GAM frequency ω is shown as a function of q for several k_r wavenumbers. By increasing the $k_r\rho_i$ value, the ω frequency increases in the simulations (points). These results are compared to the theoretical curves obtained by considering Eq. 1 with the following α_1 expression:

$$\alpha_1 = \frac{1}{2}\left[\frac{3}{4} - \left(\frac{7}{4}+\tau_e\right)^{-1}\left(\frac{13}{4}+3\tau_e+\tau_e^2\right) + \left(\frac{7}{4}+\tau_e\right)^{-2}\left(\frac{747}{32}+\frac{481}{32}\tau_e+\frac{35}{8}\tau_e^2+\frac{1}{2}\tau_e^3\right)\right] \quad (2)$$

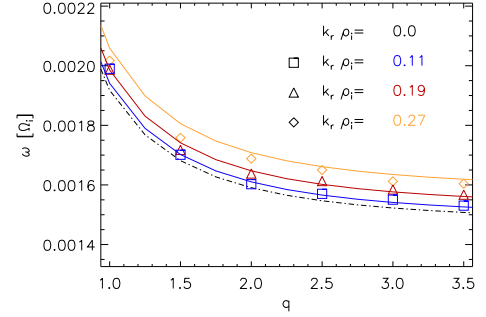obtained in Ref. [10] in which the authors give sufficient details for the calculation of ω to the second order of accuracy in order to resolve the contradiction in the literature about the trend of α_1. In the following we shall demonstrate that α_1 from Eq. 2 represents a good approximation among the several estimates in the literature (see Ref. [10] and reference therein). As a general remark, we find good agreement between theory and simulations.

In order to investigate the behavior of GAMs in the presence of a temperature gradient κ_T we recall that this latter is at the basis of the continuum spectrum of GAMs described by $\omega_G \propto \sqrt{T(r)}$. Thus, each radial point of the electric field perturbation oscillates with a different frequency generating higher k_r modes via phase mixing. In particular k_r changes in time according to $k_r = k_{r0} + \beta t$ with $\beta \propto \kappa_T$. Concerning the imaginary part of the dispersion relation we have demonstrated that the combined action of phase mixing and Landau damping generates a strong damping mecha-
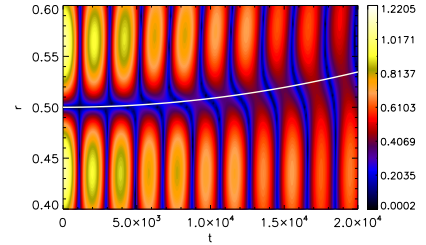

Figure 2: *Zoom of electric field amplitude in the (t,r) plane for $a\kappa_T = 1$ and $\tau_e = 1$. The white trajectory has been obtained by Eq. 4.*
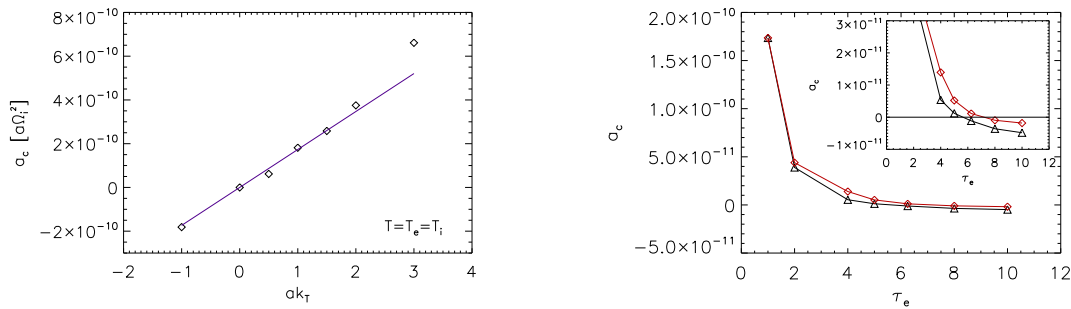
nism of GAMs resulting in the so called *PL-mechanism*. But the phase mixing can also influence the ω frequency increasing more and more the effect of the term $k_r \rho_i$ in Eq. 1 (see Ref. [7]) with a consequent impact on the radial group and phase velocity:

$$\omega^2 = \omega_G^2(r)\left[1 + \alpha_1(k_{r0} + \beta t)^2\rho_i^2\right] \rightarrow v_g = \alpha_1\omega_G(k_{r0} + \beta t)\rho_i^2, \quad v_p = \frac{\omega_G[1 + 0.5\alpha_1(k_{r0} + \beta t)^2\rho_i^2]}{(k_{r0} + \beta t)} \tag{3}$$

We note that group and phase velocities are not constant but depend on time. Consequently we can write the following accelerations for GAMs:

$$a_c = \frac{\partial v_g}{\partial t} = \frac{\partial}{\partial t}\frac{\partial \omega}{\partial k_r} = \alpha_1\omega_G\beta\rho_i^2 \qquad \frac{\partial v_p}{\partial t} = \frac{1}{2}\omega_G\alpha_1\beta\rho_i^2 - \frac{\omega_G\beta}{(k_{r0} + \beta t)^2} \tag{4}$$

In order to verify and put in evidence GAM acceleration we consider a GAM packet that evolves in the same equilibrium considered for the benchmark, but with the following temperature profile $T(r) = \exp(-\kappa_T l \tanh((r - r_0)/l))$ with $l = 0.225$, $r = 0.5$ and $a\kappa_T = -a\nabla T(r_0)/T(r_0) = 1$. Moreover we choose $q = 3$. For this case in Fig. 2 we show time evolution of the central node of the electric field that corresponds to follow the time evolution of the peak of the potential perturbation. The node initialized with a zero group velocity accelerates outwards in agreement with the white trajectory by a_c expression in Eq. 4 with $\beta = 0.5\omega_G\kappa_T$ as approximated at the center of the temperature profile. To further investigate the GAM radial propagation, in Fig. 3 (left panel) we show the acceleration as a function of the temperature gradient. The simulation points and the straight blue line obtained by the theory show an acceleration that linearly increases with κ_T. Finally, in Fig. 3 (right panel) we plot the acceleration values as a function of τ_e obtained from the simulations (red line) and from the theory by using the expression for α_1

Figure 3: *(left) Acceleration values as a function of $a\kappa_T$ obtained from the theory (blue line) and from the simulations (black points) for $\tau_e = 1$. (right) Acceleration values as a function of τ_e obtained from the simulations (red line) and from the theory with phase mixing effects and by using Eq. 2 for α_1.*

in Eq. 2 (black line). It is interesting to observe that there is a value of τ_e at which acceleration and consequently α_1 changes sign. We find a good agreement between simulations and theory by using α_1 obtained in Ref. [10]. We obtain a change in the acceleration direction for $\tau_{e0} \approx 6$.

Conclusions

The effects of phase mixing on the GAM dispersion relation in the presence of a temperature gradient have been investigated. In particular, we have shown for the first time that in the presence of a temperature gradient the frequency of the GAM is not constant but evolves in time because of the increase of the radial wavenumber. As a consequence, also the radial velocity of GAMs increases in time. Thus, this study reduces the discrepancy between the linear theory and the experiments, in which strong velocities of GAMs are generally observed. Moreover, the acceleration, amplifying the radial displacement of the GAM, gives us an operative method to select between several perturbative techniques found in literature.

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