New evidence and impact of electron transport non-linearities based on new perturbative inter-modulation analysis

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Introduction

In the Large Helical Device (LHD) a new type of frequency inter-modulation experiment is applied to the electron transport channel, thereby quantifying the amount of non-linear contributions within a single experiment [1]. The spatial dependency of the non-linearity is estimated using Volterra series, an extension of Taylor series. This calculated non-linear component shows some coherence with the measured turbulence level. Moreover, the effect of changing the equilibrium due to the non-zero mean perturbation is quantified showing a significant change between measured and reconstructed amplitude and phase profiles.

How to quantify non-linear contributions

Can we compare linearized physics models with experimental perturbative measurements:

- Yes: response is linear $\rightarrow$ same harmonic components temperature response as perturbation
- No: response is non-linear $\rightarrow$ new harmonic components in temperature response compared to perturbation

Hence, new harmonic components in the temperature response can be used to quantify plasma non-linearities. Assuming that the heat is locally absorbed near the deposition location ρ_0, the temperature response to a perturbation in heating power has the same harmonic components as that of the heating power modulation, i.e.,

$$T_d(\rho_0,t) = T_0 + \underbrace{A_1\cos(f_1 t)}_{P_1} + \underbrace{A_2\cos(f_2 t)}_{P_2} + h.o.c. \qquad (1)$$

Both the time-trace and harmonic components of the two modulated ECRHs and ECE measurements are shown in Fig. 1 and Fig. 2.

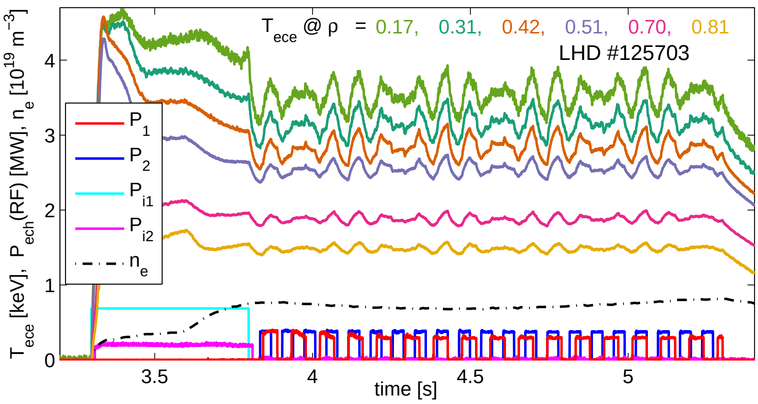

Figure 1: *Overview of LHD discharge #125703 showing the time-traces of the calibrated launched EC wave power generated by four gyrotrons; the electron temperature perturbations at different* ρ *measured with ECE; and the line-averaged density* n_e.

Interpretation harmonic components through Taylor series

Assume that the temperature is a general function of the plasma parameters and perturbative heating source quantified at the deposition location $T(\rho,t) = h\left(T_{eq}(\rho), T_d(t)\right)$. The Taylor expansion of this model around an equilibrium where $T_d = 0$ is given by [2]

$$T(\rho,t) = h\left(T_{eq},0\right) + \overbrace{\left.\frac{\partial h\left(T_{eq},T_d\right)}{\partial T_d}\right|_{T_d=0} T_d(t)}^{\text{linear contribution}}$$
$$\underbrace{+\frac{1}{2!}\left.\frac{\partial^2 h\left(T_{eq},T_d\right)}{\partial T_d^2}\right|_{T_d=0} T_d^2(t) + \frac{1}{3!}\left.\frac{\partial h^3\left(T_{eq},T_d\right)}{\partial T_d^3}\right|_{T_d=0} T_d^3(t) + \ldots,}_{\text{non-linear contributions}} \qquad (2)$$

or in short-hand notation $T(\rho,t) = h\left(T_{eq}\right) + K_1 \star T_d(t) + K_2 \star T_d^2(t) + K_3 \star T_d^3(t) + \ldots$, where ρ dependencies of the $K's$ have been omitted. Substituting (1) and expanding (2) gives

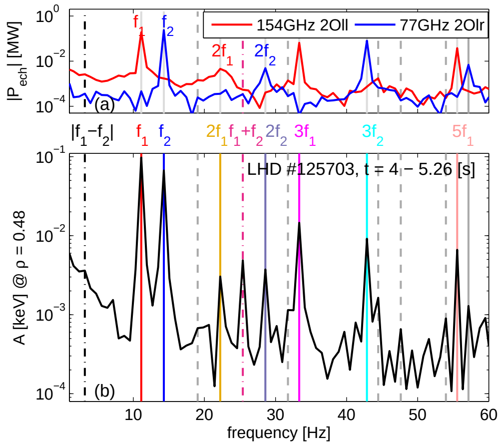

Figure 2: *Amplitude spectra of (a) the calibrated EC power and (b) the ECE-measurements at* $\rho = 0.48$*. The solid lines show the contributions at the perturbed harmonics. The dashed-dotted lines show the locations of the primary inter-modulations and the grey-dashed lines show the secondary inter-modulations.*

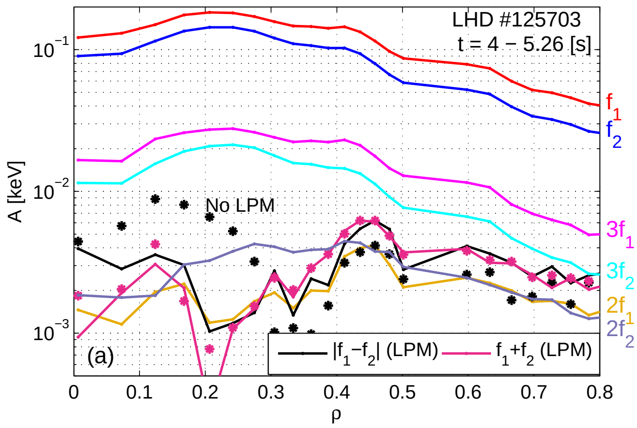

Figure 3: *Profiles of (a) amplitude. The solid lines of* $|f_1 \pm f_2|$ *show the estimates compensated for the transient with the Local Polynomial Method (LPM) and the stars* $(*)$ *without.*

$$
\begin{array}{rcl}
T(\rho,t) = & h\left(T_{eq}\right) + K_1 T_0 + K_2 T_0^2 & (a) \\
+ & (K_1 + 2K_2 T_0)\left(A_1 \cos(f_1 t) + A_2 \cos(f_2 t)\right) & (b) \\
+ & \frac{1}{2} K_2 \left(A_1^2 \cos(2 f_1 t) + A_2^2 \cos(2 f_2 t)\right) & (c) \\
+ & K_2 A_1 A_2 \left(\cos((f_1 - f_2)t) + \cos((f_1 + f_2)t)\right) & (d) \\
+ & h.o.t & (e)
\end{array}
\quad . \qquad (3)
$$

In standard experiments using a single block-wave modulation source, or sources with the same waveform, f_2 is always a multiple of f_1. As such contributions due to $(3b)$, $(3c)$, and $(3d)$ cannot be uniquely distinguished. Instead, we use two sources with different frequencies f_1 and f_2 such that the intermodulation frequencies $f_1 + f_2$ and $f_1 - f_2$ do not coincide with multiples of f_1 and f_2. Consequently, new harmonic perturbations can only occur at $f_1 + f_2$ and $f_1 - f_2$, due to plasma non-linearities.

In the experiment at LHD, the intermodulation component is clearly visible at $f_1 + f_2 = 25.4$ Hz and changes as function of ρ as shown in Fig. 3. Moreover, as the non-linear component is 2 orders lower than the main harmonic components f_1 and f_2 the non-linearity must be classified as being weak.

Effect of changing equilibrium on profiles

In $(3b)$ can be seen that the non-linearity in combination with a non-zero mean perturbation $T_0 \neq 0$ modifies the profile (change of equilibrium). This change can be studied by measuring

K_2 from (3d). This results in Fig. 4.

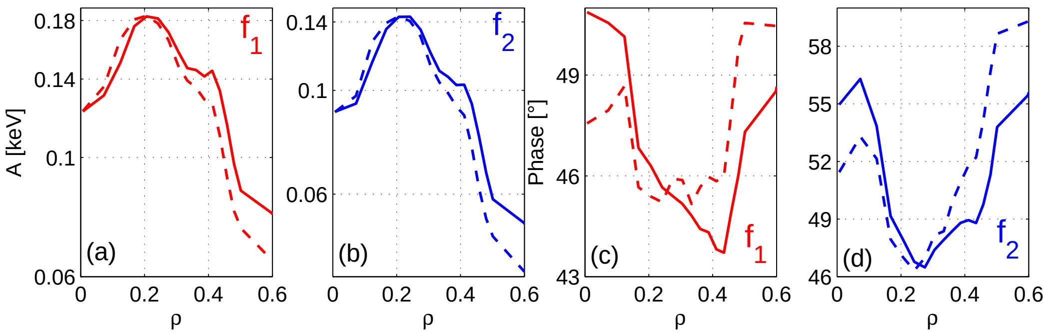

Figure 4: *(a-d) Amplitude and phase profiles of f_1 and f_2 of the original measured profiles (full), and the profiles compensated for the non-linearities using K_2 in Volterra form (dashed).*

Comparison local non-linear contribution and turbulence level

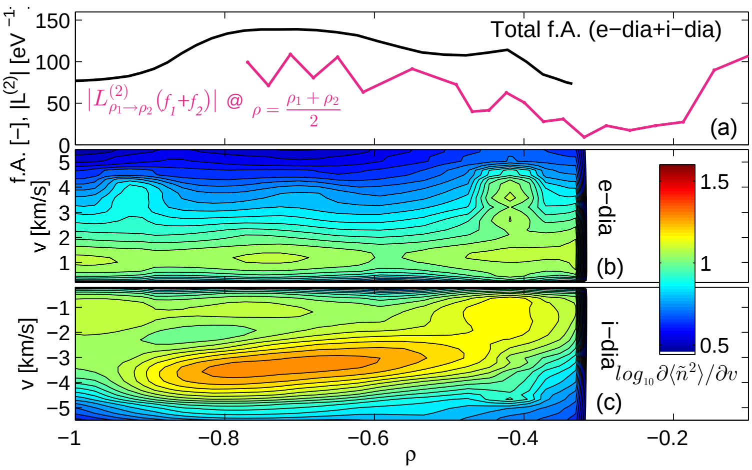

Figure 5: *(a) Amplitude of non-linear component* $L^{(2)}_{\rho_1\to\rho_2} = \frac{T(\rho_2,f_1+f_2)-L^{(1)}_{\rho_1\to\rho_2}(f_1+f_2)T(\rho_1,f_1+f_2)}{2T(\rho_1,f_2)T(\rho_1,f_1)}$, $L^{(1)}_{\rho_1\to\rho_2}(f_1+f_2) \approx \frac{K_1(\rho_2,f_1+f_2)}{K_1(\rho_1,f_1+f_2)}$ *versus total fluctuation amplitude. (b-c) Phase velocity profile* $\partial\left\langle\tilde{n}^2\right\rangle/\partial v$ *(in lab. frame direction), with in red the regions of large fluctuations (turbulence), see details [3]*

The non-linear component is compared to the turbulence level showing some coherence, but further research is necessary to confirm this relationship. Making a number of additional assumptions the non-linear (plasma) component independent of the size of the perturbation can be estimated. This results in the purple line in Fig. 5.

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