Linear wave propagation in ablation waves driven by nonlinear heat conduction

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Ablative waves are a key mechanism for obtaining the high compressions necessary to the achievement of thermonuclear burn in inertial confinement fusion (ICF). Gas dynamics equations with nonlinear heat conduction is a fluid model commonly used for describing and understanding the hydrodynamics of ablation flows encountered in ICF. Linear wave propagation properties of this set of equations are at the root of theoretical models of stationary ablation [1, 2] and are determining for the hydrodynamic stability of ablation flows. Here we proceed to a local analysis of these equations in terms of linear hyperbolic waves, for unsteady, fully compressible and non-uniform ablative flows.

Governing equations

Consider the one dimensional motion of an inviscid heat-conducting fluid with a polytropic equation of state, $p = R\rho T$, $\mathscr{E} = RT/(\gamma-1)$, and a nonlinear heat conductivity $\kappa(\rho,T)$. The equations of motion along the x-axis of a Cartesian coordinate system may be written — in dimensionless form and in terms of the Lagrangian coordinate m such that $\mathrm{d}m = \rho\,\mathrm{d}x$ — as

$$\frac{\partial\rho}{\partial t} + \rho^2\frac{\partial v_x}{\partial m} = 0, \qquad \frac{\partial v_x}{\partial t} + \frac{1}{\gamma M^2}\frac{\partial p}{\partial m} = 0, \qquad \frac{\partial T}{\partial t} + (\gamma-1)\,p\,\frac{\partial v_x}{\partial m} + \frac{\gamma}{Pe}\frac{\partial \varphi_x}{\partial m} = 0, \tag{1}$$

with $p = \rho T$, $\varphi_x = -\rho\,\kappa(\rho,T)\,\partial T/\partial m$, and where the Mach and Péclet numbers, M and Pe, are defined after some characteristic values of the flow density, velocity, length and temperature. The linearization of these equations about a particular smooth solution (ρ^0, v_x^0, T^0), in terms of Eulerian perturbations and the coordinate m^0 such that $\mathrm{d}m^0 = \rho^0\,\mathrm{d}x$, takes the form of the system

$$\frac{\partial U^1}{\partial t} + \boldsymbol{A}^0\frac{\partial^2 U^1}{\partial {m^0}^2} + \boldsymbol{B}^0\frac{\partial U^1}{\partial m^0} + \boldsymbol{C}^0 U^1 = 0, \tag{2}$$

for the vector of linear perturbation variables $U^1 = \begin{pmatrix}\rho^1 & v_x^1 & T^1\end{pmatrix}^\top$, where

$$\boldsymbol{A}^0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\gamma}{Pe}\rho^0\psi^0_{T'} \end{pmatrix}, \tag{3}$$

$$\boldsymbol{B}^0 = \begin{pmatrix} 0 & {\rho^0}^2 & 0 \\ \left(\frac{z_T^0}{\rho^0}\right)^2 & 0 & \frac{{z_T^0}^2}{\rho^0 T^0} \\ \frac{T^0}{\rho^0}q_\rho^0 & (\gamma-1)\rho^0T^0 & q_T^0 \end{pmatrix}, \quad \text{with} \quad \begin{cases} z_T^0 = \rho^0\sqrt{T^0/\gamma M^2}, \\ q_\rho^0 = \frac{\gamma}{Pe}\frac{\rho^0}{T^0}\,\psi_\rho^0, \\ q_T^0 = \frac{\gamma}{Pe}\left[\frac{\partial}{\partial m^0}(\rho^0\,\psi^0_{T'}) + \psi^0_T\right], \end{cases} \tag{4}$$

and

$$\boldsymbol{C}^0 = \begin{pmatrix} \rho^0 \frac{\partial v_x^0}{\partial m^0} & \rho^0 \frac{\partial \rho^0}{\partial m^0} & 0 \\ -\frac{1}{\gamma M^2} \frac{T^0}{\rho^0} \frac{\partial \rho^0}{\partial m^0} & \rho^0 \frac{\partial v_x^0}{\partial m^0} & \frac{1}{\gamma M^2} \frac{\partial \rho^0}{\partial m^0} \\ C_{31}^0 & \rho^0 \frac{\partial T^0}{\partial m^0} & C_{33}^0 \end{pmatrix}, \quad \text{with} \quad \left\{ \begin{array}{l} C_{31}^0 = \frac{\gamma}{Pe} \left[\frac{\partial \psi_\rho^0}{\partial m^0} - \frac{1}{\rho^0} \frac{\partial \psi^0}{\partial m^0} \right], \\ C_{33}^0 = (\gamma - 1)\, \rho^0 \frac{\partial v_x^0}{\partial m^0} + \frac{\gamma}{Pe} \frac{\partial \psi_T^0}{\partial m^0}. \end{array} \right. \tag{5}$$

In (4), z_T^0 denotes the *isothermal acoustic impedance*, while q_ρ^0 and q_T^0 are homogeneous to *velocities in the coordinate m* with the notations

$$\psi_\rho^0 = -\rho^0 \frac{\partial \kappa}{\partial \rho}(\rho^0, T^0) \frac{\partial T^0}{\partial m^0}, \quad \psi_T^0 = -\rho^0 \frac{\partial \kappa}{\partial T}(\rho^0, T^0) \frac{\partial T^0}{\partial m^0}, \quad \psi_{T'}^0 = -\kappa(\rho^0, T^0), \tag{6}$$

and are identically zero for a uniform mean flow.

Hyperbolic waves

A local analysis of the reduced first order system

$$\frac{\partial U^1}{\partial t} + \boldsymbol{B}^0 \frac{\partial U^1}{\partial m^0} = 0, \tag{7}$$

is performed, leading to (local) linear-wave characteristic equations

$$\mathrm{d}w^1{}_k \equiv l^{k\top} \mathrm{d}U^1 = 0, \quad \text{along } \mathscr{C}_k\text{:} \quad \mathrm{d}m^0/\mathrm{d}t = \lambda_k, \quad \text{for } k = 1,2,3, \tag{8}$$

where l^k denotes the left eigenvector of $\boldsymbol{B}^0$ associated to the eigenvalue λ_k.

Low heat-propagation (LH) regime. When $|q_\rho^0|, |q_T^0| \ll z_T^0$, approximate characteristics, at leading order, consist in

$$\left. \begin{array}{lll} \mathrm{d}w^1{}_1^{\mathrm{LH}} \equiv \mathrm{d}p^1 + \gamma M^2 \sqrt{\gamma} z_T^0 \,\mathrm{d}v_x^1 = 0, & \text{along } \mathscr{C}_1^{\mathrm{LH}}\text{:} & \mathrm{d}m^0/\mathrm{d}t = \sqrt{\gamma} z_T^0, \\ \mathrm{d}w^1{}_2^{\mathrm{LH}} \equiv \mathrm{d}s^1 = 0, & \text{along } \mathscr{C}_2^{\mathrm{LH}}\text{:} & \mathrm{d}m^0/\mathrm{d}t = \left(q_T^0 - q_\rho^0\right)/\gamma, \\ \mathrm{d}w^1{}_3^{\mathrm{LH}} \equiv \mathrm{d}p^1 - \gamma M^2 \sqrt{\gamma} z_T^0 \,\mathrm{d}v_x^1 = 0, & \text{along } \mathscr{C}_3^{\mathrm{LH}}\text{:} & \mathrm{d}m^0/\mathrm{d}t = -\sqrt{\gamma} z_T^0. \end{array} \right\} \tag{9}$$

The characteristics $\mathscr{C}_{1,3}^{\mathrm{LH}}$ are the *isentropic-acoustic wave* characteristics as coming out from considering (7) with $q_\rho^0 = q_T^0 = 0$, while the characteristics $\mathscr{C}_2^{\mathrm{LH}}$ correspond to a modification of the gas-dynamics entropy-wave characteristics: $\mathrm{d}s^1 = 0$, along $\mathscr{C}_0^S$: $\mathrm{d}m^0/\mathrm{d}t = 0$.

High heat-propagation (HH) regime. For $|q_T^0| \gg z_T^0$, approximate characteristics come out, at leading order, as

$$\left. \begin{array}{lll} \mathrm{d}w^1{}_1^{\mathrm{HH}} \equiv (q_\rho^0/q_T^0)\, T^0 \,\mathrm{d}\rho^1 + \rho^0 \,\mathrm{d}T^1 = 0, & \text{along } \mathscr{C}_1^{\mathrm{HH}}\text{:} & \mathrm{d}m^0/\mathrm{d}t = q_T^0, \\ \mathrm{d}w^1{}_2^{\mathrm{HH}} \equiv \left|1 - q_\rho^0/q_T^0\right| T^0 \,\mathrm{d}\rho^1 + \gamma M^2 z_{\mathrm{HH}}^0 \,\mathrm{d}v_x^1 = 0, & \text{along } \mathscr{C}_2^{\mathrm{HH}}\text{:} & \mathrm{d}m^0/\mathrm{d}t = z_{\mathrm{HH}}^0, \\ \mathrm{d}w^1{}_3^{\mathrm{HH}} \equiv \left|1 - q_\rho^0/q_T^0\right| T^0 \,\mathrm{d}\rho^1 - \gamma M^2 z_{\mathrm{HH}}^0 \,\mathrm{d}v_x^1 = 0, & \text{along } \mathscr{C}_3^{\mathrm{HH}}\text{:} & \mathrm{d}m^0/\mathrm{d}t = -z_{\mathrm{HH}}^0, \end{array} \right\} \tag{10}$$

with the notation $z^0_{\rm HH} = \sqrt{\left|1 - q^0_\rho / q^0_T\right|}\, z^0_T$. The characteristics $\mathscr{C}^{\rm HH}_{2,3}$ are modified forms of the *isothermal-acoustic wave* characteristics. The remaining characteristics $\mathscr{C}^{\rm HH}_1$ describe the *supersonic propagation*, at the velocity q^0_T, of *heat-flux perturbation inhomogeneities*: cf. the expression of the heat-flux perturbation gradient

$$\frac{\partial \varphi^1_x}{\partial m^0} = \frac{\partial \psi^0_\rho}{\partial m^0}\rho^1 + \frac{\partial \psi^0_T}{\partial m^0}T^1 + \underbrace{\psi^0_\rho \frac{\partial \rho^1}{\partial m^0} + \left[\frac{\partial}{\partial m^0}(\rho^0 \psi^0_{T'}) + \psi^0_T\right]\frac{\partial T^1}{\partial m^0}}_{\partial {w^1}^{\rm HH}_1/\partial m^0} + \rho^1 \psi^0_{T'} \frac{\partial^2 T^1}{\partial {m^0}^2}. \qquad (11)$$

For a fluid-density dependent heat conductivity ($q^0_\rho \neq 0$), the corresponding waves, or *'heat conductivity waves'*, carry *both* density and temperature perturbations. When $q^0_\rho = 0$, these waves are isochoric, carry only temperature perturbations, and are thus propagating *thermal waves*, whereas waves of the characteristics $\mathscr{C}^{\rm HH}_{2,3}$ are merely isothermal acoustic waves.

Application to self-similar waves of radiation conduction

The above analysis, applied to four representative self-similar ablative waves of radiation conduction [3] obtained with $\kappa(\rho,T) = \kappa_* \rho^{-2} T^{13/2}$, leads to the following findings.

Numerical computation of the eigenvalues λ_k and associated eigenvectors l^k of the matrices $\boldsymbol{B}^0$ of (2) show that the eigenvalues λ_k, $k = 1,2,3$, are all real and that the approximate characteristic equations (9) and (10) agree fairly well with the computed values of λ_k and l^k over, respectively, the compressed-fluid and conduction regions. Furthermore, in the conduction regions, the heat-conductivity wave characteristics $\mathscr{C}_1$ reveal a supersonic upstream propagation of heat flux perturbations up to the ablation front, even for fast expansion flows presenting a Chapman–Jouguet (CJ) point (figure 1*b*).

Heat diffusion effects estimated from a local quantitative analysis of the second-order operator

$$\mathscr{L}(m^0, t, \partial_{m^0}.) = -\boldsymbol{A}^0 \frac{\partial^2 .}{\partial {m^0}^2} - \boldsymbol{B}^0 \frac{\partial .}{\partial m^0}, \qquad (12)$$

indicate a transmission, across the conduction region, of heat conductivity waves with characteristic lengths exceeding several conduction region lengths.

Conclusion

The present analysis highlights the importance of flow stratification on wave propagation in ablation flows driven by nonlinear heat conduction. The combined effects of nonlinear heat conduction and flow non-uniformity in the conduction regions of such flows induce the existence, in addition to quasi-isothermal acoustic waves, of supersonic upstream-propagating *heat conductivity waves*. This feature is absent from the standard model of stationary radiative ablation [1, 2] that assumes a quasi-isothermal expansion region where waves may only propagate

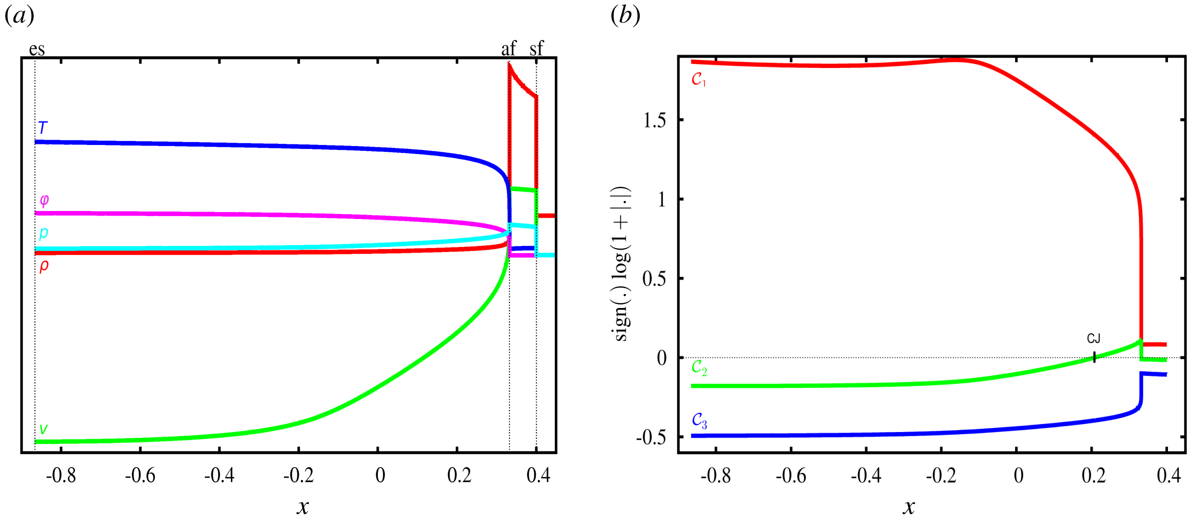

Figure 1: Fast expansion ablative wave. (*a*) Profiles in the Cartesian coordinate x of the flow density ρ, velocity v, temperature T, heat flux φ, and pressure p. From left (downstream) to right (upstream): the conduction region comprised between the flow external surface (es), where the external heat flux is applied, and the ablation front (af), followed by the compressed-fluid region extending up to the fore-running shock-wave front (sf). (*b*) Characteristic wave x-velocities relative to the ablation front for each of the characteristic families $\mathscr{C}_k$, $k = 1, 2, 3$, with identification of the ablative wave Chapman–Jouguet point (CJ).

under the form of isothermal acoustic waves and temperature fluctuations are only diffused. Estimated damping of these heat conductivity waves shows that the upstream propagation of temperature *and* density fluctuations up to the Chapman–Jouguet points of fast-expansion flows is effective for wavelengths exceeding several conduction-region lengths. This possible supersonic propagation of density perturbations could trigger conduction region flow instabilities [4]. This improved linear wave description of ablative waves should be helpful whether for analyzing or modeling ablation flows and their hydrodynamic instabilities, and this for any type of nonlinear heat conduction.

References

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