Dissipation of Fine Velocity Structures in Weakly Collisional Plasmas

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The solar wind is usually considered a collisionless plasma, since collisions are considered far too weak to produce any significant effect on the plasma dynamics [1]. However, the estimation of the plasma collisionality is usually based on the restrictive consideration that particle velocity distribution function (VDF) is close to the thermodynamical equilibrium [2]. On the other hand, *in situ* spacecraft measurements [3] and kinetic numerical simulations [4] show the presence of significant non-Maxwellian signatures in the three-dimensional VDFs (temperature anisotropies and non-gyrotropies, particle beams, ring-like modulations, deformed "potato"-like shapes etc.). However, since collisional effects are correlated with the velocity gradients in the VDF [5, 6, 7, 8], the collisionless hypothesis may locally fail.

The presence of these out-of-equilibrium fine structures in velocity space is intrinsically due to the free-streaming term of the Vlasov equation, which naturally produces smaller and smaller scales in the presence of an initial perturbation. Furthermore, nonlinear wave-particle interactions and kinetic turbulence can also produce stronger structures in velocity space. Within the collisionless assumption, the free energy contained in the out-of-equilibrium structures can be converted into other form of ordered energy (for example electromagnetic energy through kinetic micro-instabilities). On the other hand, collisional effects - which in general tend to oppose to the formation of out-of-equilibrium velocity structures and restore the thermal equilibrium - can dissipate these structures, thus irreversibly degrading the information contained into such structures and, ultimately, heating the plasma. Hence, describing the effect of collisions on small scale structures in velocity structures is crucial for comprehending the competition of collisions and collisionless wave-particle interactions and the possible enhancement of collisional effects due to fine velocity space structures (i.e. strong velocity gradients).

In this perspective, in Refs. [7, 8], it has been shown that collisions are effectively enhanced when small scales structures are recovered in the particle VDFs. In particular, collisions - introduced through a collisional operator at the right side of the Vlasov equation - have been modeled by means of the Landau operator. The choice of the *correct* collisional operator represents a longstanding problem. Several derivations from the Liouville equation show that the most general collisional operators for plasmas are the Lenard-Balescu operator [9, 10] or the Landau operator [5]. Both operators are nonlinear Fokker-Planck-like operators, which involve velocity space derivatives and integrals in the three-dimensional velocity space. Furthermore both operators have good conservation properties and satisfy a H-theorem for the Gibbs-Boltzmann entropy. The Landau operator introduces an upper cut-off of the integrals at the Debye length to avoid the divergence for large impact parameters, while the Balescu-Lenard operator solves this divergence in a more consistent way through the dispersion function. Despite the Balescu-Lenard operator is more general compared to the Landau operator from this point of view, both operators are usually derived by assuming that the plasma is not extremely far from the equilibrium and clearly this condition could be not satisfied in strongly turbulent plasmas. Moreover, it is worth to note that the numerical approach of the Lenard-Balescu operator is more difficult with respect to the Landau operator, since the former operator also involves the evaluation of dispersion function.

The Landau operator has the following form:

$$\frac{\partial f(\mathbf{v})}{\partial t} = \pi \left(\frac{3}{2}\right)^{\frac{3}{2}} \frac{\partial}{\partial v_i} \int d^3v' \, U_{ij}(\mathbf{u}) \left[f(\mathbf{v}') \frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v}) \frac{\partial f(\mathbf{v}')}{\partial v'_j} \right] , \qquad (1)$$

being f normalized such that $\int d^3v f(\mathbf{v}) = n = 1$ and $U_{ij}(\mathbf{u})$

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij} u^2 - u_i u_j}{u^3} , \qquad (2)$$

where $\mathbf{u} = \mathbf{v} - \mathbf{v}'$, $u = |\mathbf{u}|$ and the Einstein notation is introduced. In Eq. (1), and from now on, time is scaled to the inverse Spitzer-Harm frequency ν_{SH}^{-1} [2] and velocity to the particle thermal speed v_{th}.

However, even by focusing only on the Landau operator, the computational cost to evaluate it numerically is huge. For example, if one considers a $3D$–$3V$ numerical phase space ($3D$ in physical space and $3D$ in velocity space) discretized with N gridpoints along each direction, the computation would require about N^9 operations at each time step (a three-dimensional integral must be evaluated on each gridpoint of the six dimensional grid). This represents a significant challenge for computational plasma physics and, nowadays, it is extremely difficult performing self-consistent simulations considering collisions. Therefore, in order to remedy to the computational complexity here described and to reduce the computational cost of the simulation, in Refs. [7, 8] we focused on a spatially homogeneous force-free plasma.

Our results indicate that, when the VDF shows small scales structures, the approach towards the equilibrium occurs on several characteristic times, related to the dissipation of different velocity space structures. It is important to highlight that the presence of several characteristic times is observed in the entropy growth, while the temperature is not affected by the presence of small scales structures, thus confirming that, in out-of-equilibrium systems, the entropy is the proper variable which describes the evolution towards the equilibrium. These characteristic times are generally faster than the times associated with the dissipation of global features, such as temperature anisotropies. Furthermore, they are inversely proportional to the steepness of velocity gradients in the VDF: finer are the VDF structures, stronger are the velocity gradients in the VDF and faster is the dissipation due to collisions. The presence of velocity gradients hence speeds up the growth of the entropy of the system. To be more quantitative, within the quasi-Maxwellian assumption described above, characteristic times of collisional processes are about the Spitzer-Harm time ν_{SH}^{-1}, being $\nu_{SH} \simeq 8 \times (0.714\pi n e^4 \ln\Lambda)/(m^{0.5}(3k_B T)^{3/2})$, where n, e, $\ln\Lambda$, m, k_B and T are respectively the particle number density, the unit electric charge, the Coulombian logarithm, the Boltzmann constant and the plasma temperature. Our analysis shows instead that collisional characteristic times associated with the presence of fine velocity structures are significantly smaller (from one to three orders of magnitude) than the Spitzer-Harm time. These evidences indicate that when the particle VDFs exhibit small velocity scale deformations, the quasi-Maxwellian approximation, on which the Spitzer-Harm collisional evolution is based, is no longer appropriate.

In order to connect our analysis to the general case of the solar-wind, we also compared the approach towards the equilibrium - in terms of the presence of fast characteristic times - of a strongly distorted VDF, obtained from the hybrid Vlasov-Maxwell [11] numerical simulations of SW-like decaying turbulence [4], with the a VDF obtained by smoothing the small scale structures of the former one through a fitting procedure. This procedure is usually adopted working on low-resolution VDFs which comes from *in-situ* measurements. It has been shown that, when small-scale structures in the VDF are artificially smoothed out through the fitting procedure, the physics related to fine velocity structures is definitively lost. Hence, high-resolution measurements of the particle VDFs are crucial for an accurate description of the solar wind in order to address important questions related to particle heating [12].

In Ref. [8] we focused on the importance of considering nonlinearities in the collisional operator. Indeed, by comparing the fully nonlinear Landau operator with a linearized version of the Landau operator, we showed that both operators are able to detect the presence of several characteristic times associated with the dissipation of fine velocity structures and the approach towards the equilibrium is qualitatively similar in both cases. The magnitude of the characteristic times is much different if nonlinearities are neglected: characteristic times are always bigger (i.e. collisions are slower) when nonlinearities are neglected. Therefore, considering nonlinearities is relatively important for correctly comparing collisional times with other dynamical times, such as - for example - instabilities growth rates.

We would point out that, since the Landau operator is computationally demanding, self-consistent high-resolution simulations cannot be currently afforded and we restricted to the case of a force-free homogeneous plasma, where both force and advection terms have been neglected. This approximation represents a caveat of the work here presented and future studies will be devoted to the generalization of the results here shown to the self-consistent case. Even though, we would remark that our analysis indicates that collisions work much faster on fine velocity structures and this properties does not depend on the other terms in the Vlasov equation but only on the physical mechanisms which rules the collisional dissipation and the thermalization of the system.

Finally, let us highlight that, in principle, the combination of the ubiquitous turbulence of plasmas such as the solar wind and the weak collisionality may constitute a new scenario to describe the plasma heating. Indeed, the turbulence transfers efficiently energy towards smaller scales and produces strong distortions in the VDF (i.e. strong gradients). Moreover, nothing forbids the production of finer and finer scales in the turbulent cascade. Then, the presence of strong gradients in the VDF naturally activates the collisions which, as we showed, dissipate fine structures very fast, thus producing a local source of dissipation and heating.

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