Gradient-drift instability in Hall thrusters based on experimental data

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The gradient-drift instability of partially magnetized plasmas is studied for typical parameters and plasma profiles of stationary Hall thruster [1].We use the following local dispersion relation for electrostatic oscillations of plasma in external crossed electric, $\mathbf{E}_0 = E_0\mathbf{e}_x$, and magnetic, $\mathbf{B}_0 = B_{0x}\mathbf{e}_x + B_{0z}\mathbf{e}_z,\ B_{0z} \gg B_{0x}$, fields:

$$\begin{aligned} 1 &+ \frac{\omega_{pe}^2}{\omega_{Be}^2(1+k_\perp^2\rho_e^2)} - \frac{\omega_{pi}^2}{(\omega - k_x v_{0i})^2} + \\ &+ \frac{1}{k_\perp^2 d_e^2(1+k_\perp^2\rho_e^2)} \frac{\omega_{\star e}(1+k_\perp^2\rho_e^2) - \omega_D(1+2k_\perp^2\rho_e^2)}{(\omega-\omega_E)(1+k_\perp^2\rho_e^2) - \omega_D(1+2k_\perp^2\rho_e^2)} = 0. \end{aligned} \tag{1}$$

Here $\omega_{Be} = eB_0/m_e c$ is the electron cyclotron frequency, $\omega_{p\alpha} = \sqrt{4\pi n_0 e^2/m_\alpha}$ is the plasma frequency, $\alpha = (i,e)$, ω and $\mathbf{k}$ are the oscillations frequency and wavevector, $\rho_e = (T_e/m_e\omega_{Be}^2)^{1/2}$ is the electron Larmor radius, $d_e = (T_e/4\pi e^2 n_0)^{1/2}$ is the electron Debye length, v_{0i} is the equilibrium ion velocity, T_e, m_e are the electron temperature and mass, e is the elementary charge, c is the speed of light,

$$(\omega_{\star e}, \omega_D) = -\frac{2k_y c T_e}{eB_0}(\kappa_n, \kappa_B) = k_y(V_{\star e}, V_D), \quad \omega_E = -\frac{k_y c E_0}{B_0} = k_y V_E$$

are the electron drift, electron magnetic drift and $\mathbf{E}_0 \times \mathbf{B}_0$-drift frequency, correspondingly; $(\kappa_n, \kappa_B) = d\ln(n_0, B_0)/dx$ are the characteristic variation lengths of the equilibrium plasma density and magnetic field magnitude. The oscillations are considered to be electrostatic and two dimensional with the wavevector $\mathbf{k} = (k_x, k_y, 0)$. This dispersion relation was obtained in our previous paper [2] in the framework of two-fluid model under the assumption of cold unmagnetized ions and hot magnetized electrons. Such an approach is justified if the frequency ω and the wavenumber $\mathbf{k}$ of considered oscillations satisfy the following conditions:

$$\omega_{Bi} \ll \omega \ll \omega_{Be}, \quad \omega \gg k v_{Ti},$$

where $\omega_{B\alpha},\ \alpha = (i,e)$ are the ion and electron cyclotron frequencies, $v_{Ti} = (2T_i/m_i)^{1/2}$ is the ion thermal velocity, T_i, m_i are the ion temperature and mass. Dispersion relation (1) takes into account the electron inertia effects and the electron finite Larmor radius effects in the Padé approximation – terms proportional to $k_\perp^2\rho_e^2$.

We solve the dispersion relation (1) for equilibrium plasma profiles obtained in simulations [1] for SPT-100 thruster. The considered profiles of the magnetic field strength B_0, electrostatic potential ϕ ($\mathbf{E}_0 = -\nabla\phi$), plasma density n_0 and electron temperature T_e on x-coordinate along the thruster channel are shown in Fig. 1. Point $x = 0$ corresponds to the anode, point $x = 2.5$ cm – to the exit plane.

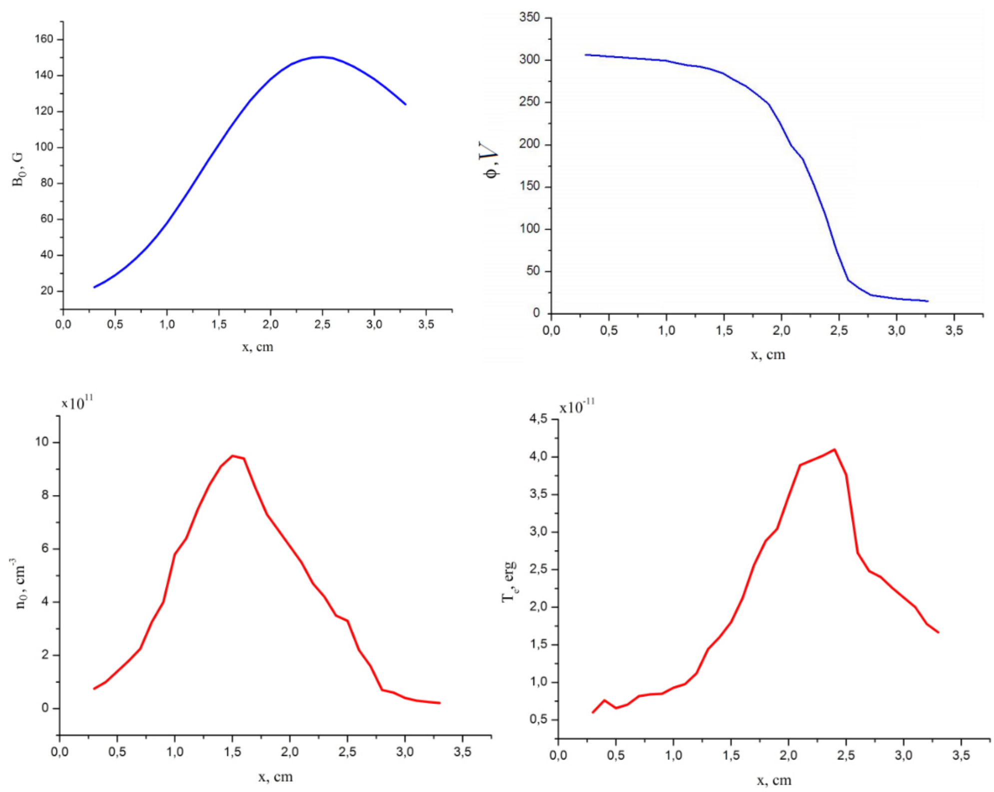
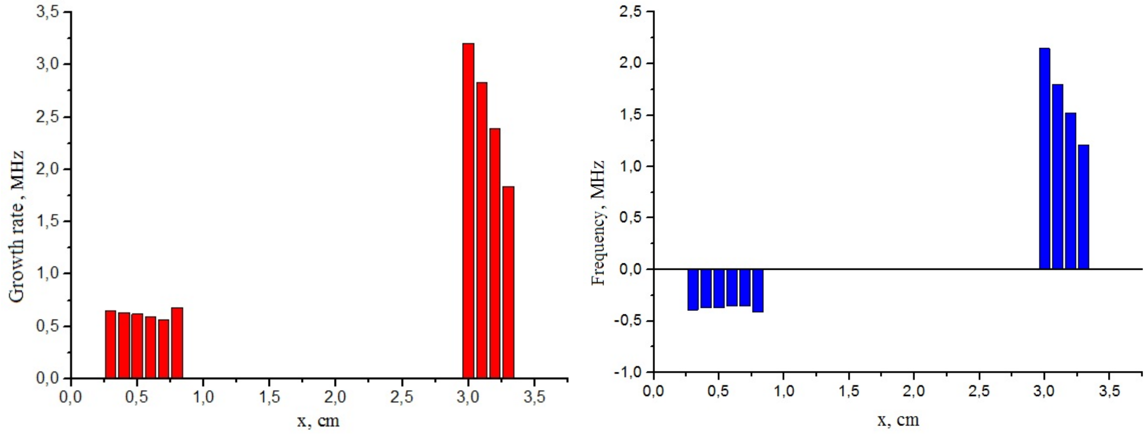


Figure 1: Magnetic field, electrostatic potential, plasma density and electron temperature profiles.

In this paper we neglect the equilibrium ion velocity, considering predominantly azimuthal perturbations $k_y \gg k_x$.

The growth rates and the frequencies of most unstable modes calculated along the thruster channel are shown in Fig. 2. Two types of spaced-apart instabilities are observed: the instability near the anode and the instability near the exit plane of the thruster; the main part of the acceleration channel appears to be stable with respect to the considered perturbations.

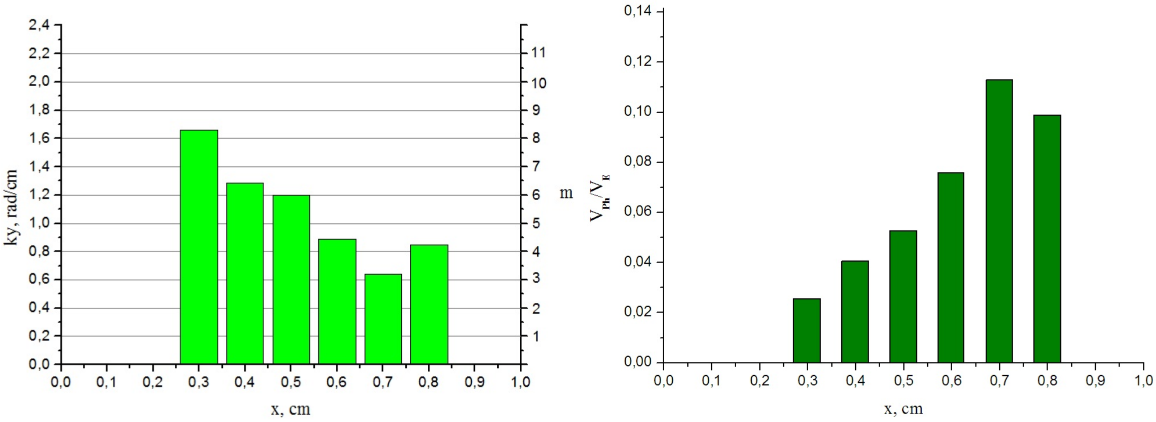
The unstable modes localized beyond the exit plan exhibit higher growth rates and higher frequencies in comparison with the modes in the near-anode region and correspond to the short-wavelength perturbations with the azimuthal mode numbers $m \sim 100$ ($m = k_y R$, R – outer radius of thruster channel).



(a) (b)

Figure 2: Growth rates (a) and frequencies (b) of the most unstable modes along the thruster channel.

Near the anode only the long-wavelength perturbations $m \sim 3-8$ are unstable – see Fig. 3 (a). The existence of the long-wavelength azimuthal instability in the near-anode region is an important result since it may be related to the formation of coherent rotating structures like spokes, observed in the experiments with Hall plasmas (see, e.g., [3, 4]). The suggestion that drift gradient modes are responsible for spoke phenomena was made in Ref. [5], where the long-wavelength limit of the dispersion relation (1) was considered. In support of this hypothesis



(a) (b)

Figure 3: Wave-numbers (a) and phase velocity (b) of the most unstable modes in the near-anode region.

is also the smallness of the phase velocity of the considered modes in comparison with the stationary $\mathbf{E}_0 \times \mathbf{B}_0$ electron drift velocity $V_{ph} = 0.03-0.11V_E$ ($V_E = cE_0/B_0$) – see Fig. 3 (b), that is consistent with the spoke property.

The obtained frequencies, however, significantly exceed the observed frequencies of spokes. A possible solution can be in the transition from continuous values of k_y to discrete values of azimuthal wavenumbers m, leading to a significant drop of frequency, which corresponds to the maximal growth rate – see Fig. 4. To verify this suggestion one needs to perform a global analysis.

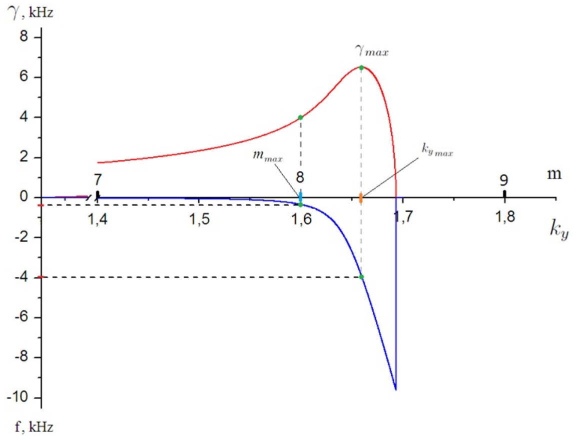


Figure 4: Dependence of growth rates (red line) and frequencies (blue line) of unstable modes on wavenumber k_y at fixed point near the anode. Dashed lines show the drop of frequency of the most unstable mode at the transition to discrete m

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