# Local and integral disruption forces in tokamaks

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**1. Introduction**. Extreme loads on the vacuum vessel wall during disruptions is already an issue for JET [1, 2]. Their mitigation is an urgent task to be solved to ensure the integrity of plasma-facing components in ITER [2, 3]. Recent simulations [4, 5] indicate that two yet undeveloped sensitive areas in the task deserve careful study. First is the induction of strong poloidal currents in the wall during current quench (CQ) [4]. Second is the generation of large-amplitude force on the wall during thermal quench (TQ) [5] at still unchanged net plasma current. It has been confirmed analytically that both effects can have a strong impact on the integral forces [6]. Here the analysis is extended on the local distributions.

The study is focused on analytical calculation of the distribution of the disruption forces over the poloidal angle for both TQ and CQ. The presented approach is based on the Maxwell equations and force balance required for plasma equilibrium in a tokamak. The rapid events are considered when the plasma-produced field does not penetrate through the vessel outwards because of the skin effect in the wall. Finally, the analytical expressions for the local forces on the wall are derived within the standard large-aspect-ratio tokamak model. Here, the plasma is treated as an axially symmetric toroid separated from the wall by a vacuum gap. There is no halo current in such system. The toroidal and poloidal currents in the wall have a purely inductive nature. This corresponds, at least, to fast events in JET that produce the electromechanical loads predominately due to eddy currents [7]. In other cases, this covers an early stage of the disruption before the plasma-wall contact.

**2. Formulation of the problem**. We consider the surface density of the force on the wall,

$$\mathbf{f}_w \equiv \int_{wall} (\mathbf{j}\times\mathbf{B})\, d\ell_\perp \,, \tag{1}$$

where the integration is performed over the length (thickness) across the wall. With substitution

$$\mu_0 \mathbf{j}\times\mathbf{B} = -\nabla \mathbf{B}^2/2 + (\mathbf{B}\cdot\nabla)\mathbf{B}\,, \tag{2}$$

which is the consequence of the Maxwell equations, and by using the equality

$$\int_{wall} \nabla f d\ell_\perp = \mathbf{n}_w f\Big|_{in}^{out} + \int_{wall} \nabla_\parallel f d\ell_\perp \,, \tag{3}$$

where $\mathbf{n}_w$ is the (outwardly directed) unit normal to the wall and

$$\nabla_{\|} \equiv \nabla - \mathbf{n}_w \partial / \partial \ell_{\perp} , \tag{4}$$

we obtain from (1)

$$\mu_0 \mathbf{f}_w = -0.5 \mathbf{n}_w \mathbf{B}^2 \Big|_{in}^{out} + (\mathbf{n}_w \cdot \mathbf{B}) \mathbf{B} \Big|_{in}^{out} . \tag{5}$$

Only the terms representing a possible jump of $\mathbf{B}$ across the wall are retained here. This corresponds to disregard of the last term in (3) proportional to the wall thickness $d_w$ that is always much smaller than the wall minor radius $b_w$ in tokamaks. Also, it is assumed that $\mathbf{B}\Big|_{in}^{out}$ is tangential to the wall. The approximations are well justified if a large force is expected.

Below we discuss the force normal to the wall,

$$\mathbf{f}_w = \mathbf{n}_w \delta p_m , \tag{6}$$

which is determined by the jump in the magnetic pressure $p_m \equiv \mathbf{B}^2 / (2\mu_0)$. This is applied to events rapid enough for treating the wall as an ideal conductor. Then

$$- f \Big|_{in}^{out} = \delta f \equiv f(t) - f(t_0) , \tag{7}$$

where $t_0$ is the time moment before the disruption, and both quantities in $\delta f$ are taken at the same point at the inner side of the wall because the field $\mathbf{B}$ remains unchanged behind the wall.

With axisymmetry, for a circular plasma we have (approximately) at the circular wall

$$\mathbf{B}^2 = \kappa^2 B_J^2 (1 + 2\varepsilon_w \Lambda_w \cos\theta) + B_0^2 (1 - 2\varepsilon_w \cos\theta) , \tag{8}$$

where $B_J \equiv \mu_0 J / (2\pi b)$ is the averaged poloidal magnetic field at the plasma boundary, $J$ is the plasma current, $b$ is the plasma minor radius, $\kappa \equiv b / b_w$, $\varepsilon_w \equiv b_w / R$, $R$ is the major radius, $\Lambda_w$ is similar to the 'Shafranov's $\Lambda$' [8], but describing the poloidal field distribution at the wall, $\theta$ is the poloidal angle with $\theta = 0$ at the outer midplane, and the second summand is the contribution from the toroidal field, which is $B_t = B_0 R / r$ in the plasma-wall vacuum gap with $r / R = 1 + \varepsilon_w \cos\theta$ at the wall. Accordingly,

$$p_m = p_{m0} + p_{m1} \cos\theta \tag{9}$$

with $p_{m0}$ and $p_{m1}$ independent of $\theta$:

$$2\mu_0 p_{m0} = \kappa^2 B_J^2 + B_0^2 , \tag{10}$$

$$2\mu_0 p_{m1} = 2\varepsilon_w (\kappa^2 B_J^2 \Lambda_w - B_0^2) . \tag{11}$$

Variations of these quantities can be expressed through the plasma parameters by using the results of the plasma equilibrium theory [8] plus the boundary conditions at the wall (here, resulting in the flux conservation). Our goal is to find the disruption-produced $\delta p_m$ in (6).

**3. Calculations**. The disruptions and the disruption mitigation events are described by the rapid drops of plasma pressure $p$ (TQ) and current $J$ (CQ). These lead to induction of the poloidal current $I_w$ in the wall and a small change in $B_0$:

$$\mu_0 I_w = 2\pi R \delta B_0 . \tag{12}$$

If the wall reacts as a magnetic flux conserver, equations (28), (30) and (41) in [6] give us

$$\delta B_0^2 = \frac{V_{pl}}{V_{pl}+V_g}\delta(2\mu_0\bar{p} - B_J^2) = \kappa^2\delta(2\mu_0\bar{p} - B_J^2) , \tag{13}$$

where the overhead bar denotes the averaging over the plasma volume $V_{pl}$, and $V_g$ is the volume of the plasma-wall toroidal region. Then (10) yields

$$\delta p_{m0} = \kappa^2 \delta\bar{p} \tag{14}$$

irrespective of $\delta B_J$. This should be compared with incongruous

$$\delta p_{m0}^{fil} = \kappa^2 \delta B_J^2 /(2\mu_0) , \tag{15}$$

which is the consequence of (10) at $\delta B_0 = 0$. The superscript *fil* indicates that such must be $\delta p_{m0}$ in the models/codes with a wall replaced by the toroidal filaments and $dB_0 / dt = 0$, so that only the poloidal field is considered varying while $\partial\mathbf{B}/\partial t \neq 0$ during the plasma evolution.

To calculate $\delta p_{m1}$, we use the integral result from [6] for

$$F_r \equiv \int_{wall}(\mathbf{j}\times\mathbf{B})\cdot\mathbf{e}_r d\tau = \oint_{wall}\mathbf{f}_w\cdot\mathbf{e}_r dS_w = \oint_{wall}\mathbf{n}_w\cdot\mathbf{e}_r\delta p_m dS_w = 0.5 S_w \delta(\varepsilon_w p_{m0} + p_{m1}) . \tag{16}$$

Here $\mathbf{e}_r = \nabla r$ is the unit vector along the major radius, $d\tau = dS_w d_w$ is the volume element, $S_w \equiv (2\pi)^2 R b_w$ is the full lateral area of the wall, and $dS_w = 2\pi r b_w d\theta$ is its element. The last term in (5) does not give a contribution in $F_r$ because of the assumed up-down symmetry. The final equality here is obtained with substitution (9) and $\mathbf{n}_w\cdot\mathbf{e}_r = \cos\theta$ for a circular wall.

According to equation (44) from [6],

$$F_r = 0.5 S_w \varepsilon_w \kappa^2 \delta\left\{\bar{p} + \frac{B_J^2}{\mu_0}\left[\ln\frac{b_w}{b} + \frac{\ell_i}{2}\right]\right\}, \tag{17}$$

where $\ell_i \equiv \overline{B_\theta^2} / B_J^2$ is the internal inductance per unit length of the plasma column, $B_\theta$ is the poloidal field so that $B_J \equiv B_\theta(b)$. This combined with (14) and (16) gives us

$$\delta p_{m1} = \varepsilon_w \kappa^2 \delta \frac{B_J^2}{\mu_0}\left[\ln\frac{b_w}{b} + \frac{\ell_i}{2}\right] \tag{18}$$

whereas the ‘filamentary’ or ‘fixed $B_0$’ consequence of (11) will be

$$\delta p_{m1}^{fil} = \varepsilon_w \kappa^2 \delta(\Lambda_w B_J^2)/(2\mu_0)\,. \tag{19}$$

**4. Discussion**. Expressions (14) and (18) covering both TQ and CQ are immensely different from (15) and (19) describing $\delta p_{m0}$ and $\delta p_{m1}$ in the models that eventually disregard the poloidal current induced in the wall, such as the used in the DINA code [5], for example. In such models, only the toroidal current is allowed in the wall, while $B_0$ (entering (8), (10) and (11) here) is treated as a time-independent constant so that always $\delta B_0 = 0$. Then (15) and (19) yield $\delta p_{m0}^{fil} = 0$ and (because of $\Lambda_w$) $\delta p_{m1}^{fil} \neq 0$ during TQ at $\delta B_J = 0$, in contrast to predictions of our equations (14) and (18): $\delta p_{m0} = \kappa^2 \delta \overline{p}$ and $\delta p_{m1} = 0$.

The difference is also substantial for the CQ at $\delta \overline{p} = 0$. From (14) we have $\delta p_{m0} = 0$ during CQ, while (15), the consequence of (10) at $\delta B_0 = 0$, gives large $\delta p_{m0}^{fil} = \kappa^2 \delta B_J^2/(2\mu_0)$.

The derived relations show that TQs must lead to $\delta p_{m0} < 0$ (while $\delta p_{m1} = 0$ at $\delta B_J = 0$). Then $F_r = 0.5\varepsilon_w S_w \delta p_{m0}$, which confirms the conclusions of [5] and [6] that large forces on the wall can develop during TQs. This fact deserves notion as contradicting to the commonly adopted view [3] that high electromagnetic loads on the tokamak structures can appear at rapid quench of the plasma current, while the TQ-produced forces have been traditionally ignored.

**5. Conclusion**. When the wall is modelled by a set of toroidal filaments, as in the DINA code [5], $B_0$ enters the disruption task as a constant. Then only $\Lambda_w$ in (8) will be the parameter varying during TQ at $\delta B_J = 0$, and (6) will give us $\mathbf{f}_w = \mathbf{n}_w \delta p_{m1}^{fil} \cos\theta$, see also (15) and (19). This explains the force pattern on the first plot in Fig. 9 in [5]. Actually, as it follows from (14) and (18), the distribution of the disruption force on the first wall just after TQ must be essentially different: $\mathbf{f}_w = \mathbf{n}_w \kappa^2 \delta \overline{p}$ with negative $\delta \overline{p}$. This is the reason why the DINA must be overestimating $F_r$ in [5] by factor of 2 (estimated for a circular plasma), as demonstrated in [6].


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