Simulation study on resistive instabilities in a small aspect ratio reversed field pinch

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Abstract

Dynamical behavior of the resistive magnetohydrodynamic (MHD) instability in a small aspect ratio reversed-field pinch (RFP) is investigated by use of the MHD simulation model. Three-dimensional full torus geometry is treated to include the effect of the aspect ratio correctly, although the toroidal section with rectangular shape is assumed for numerical simplicity. This simulation shows that helical deformation of the equilibrium state with the growth of the instability is observed in a small aspect ratio RFP. Visualization using the 3D-Graphics allows the detailed analysis of the dynamics of the instability growth process.

1. Introduction

A reversed-field pinch (RFP) plasma is the result of a self-organization process, which is produced by a dynamo effect due to magnetohydrodynamic (MHD) instabilities. In the recent progress of the RFP study, improvement of the confinement properties by the transition to a single-helical-axis state is observed in the large experimental device [1]. Also recent numerical simulations show the possibility of reproducing such single-helical-axis states. As for a low aspect ratio RFP with $A \approx 2$, formation and rotation of the helical structure such as quasi-single-hericity states have been observed in experiments [2]. Towards the understanding of this structure formation mechanism, analysis using the detailed three-dimensional MHD simulation through the comparison with the experiment has been conducted [3].

In this study, from an analogy with the spherical tokamak (ST), we focus our attention on the RFP configuration having an elongated cross section with a small aspect ratio $A < 2$, in other words, spherical RFP. The basic characteristics of the equilibrium configuration and dynamical behavior of resistive instabilities are examined by using the MHD simulation model. Regarding the spherical RFP concept, its characteristic already has been introduced briefly in the first theoritical study on the spherical tokamak [4].

2. The Numerical Model

As a numerical simulation model, a visco-resistive non-linear three-dimensional MHD model is used here. The equations in the dimensionless form are given by

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v}\cdot\nabla)\mathbf{v} + \mathbf{j}\times\mathbf{B} - R_\nu^{-1}\nabla^2\mathbf{v}\,,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times\left(\mathbf{v}\times\mathbf{B} - R_\eta^{-1}\mathbf{j}\right),$$

where $\mathbf{v}, \mathbf{B}$ and $\mathbf{j}$ are the plasma fluid velocity, the magnetic field and the current density respectively [6]. R_ν is the Reynolds number and R_η corresponds to the magnetic Reynolds number in this normalized expression. These coefficients are assumed to be isotropic and constant in time and space. This model is successful in qualitatively reproducing the recent experimental findings in the RFP [5]. The negligible-pressure, constatnt-density approximation is assumed in the model, however three-dimensional full torus geometry is treated to include the effect of the aspect ratio correctly in this study. For numerical simplicity, the torus with rectangular cross-section is assumed. The finite difference scheme for the (r, z) plane and the pseudo-spectral method for the angular (toroidal) ϕ-direction are applied to the numerical discretization. The fourth-order Runge-Kutta method is used for time integration. Boundary conditions are such that the plasma is in contact with the wall as a viscous fluid and the wall acts as a perfect conductor for the toroidal magnetic flux. Using this rectangular cross-sectional model, the effects of aspect ratio and cross-sectional elongation on the RFP dynamics can be investigated although the spherical configuration cannot be treated.

3. Numerical Results

As the initial RFP equilibrium, a non-linear force-free magnetic field model is adopted, expressed by the following poloidal current flux:

$$I(\psi) = \frac{\alpha}{\mu_0}\psi^\xi + C$$

which corresponds to the right hand side of the Grad-Shafranov equation. Here, α and C are constants determined by specifying the equilibrium plasma current I_ϕ and toroidal magnetic flux Φ respectively. ξ is a real parameter characterizing the relaxed state. The partially relaxed state model [7] is used for the RFP equilibrium state. Therefore, the profile of the ratio λ between the magnetic field and the current is given by

$$\lambda(\psi) = \mu_0\, dI/d\psi\,.$$

Namely, it is the same as Taylor's minimum energy state when $\xi = 1$, and the shift from that state is expressed when $\xi > 1$. The F-θ diagram for various ξ parameters is obtained by solving the Grad-Shafranov equation. In the present study, ξ is chosen to be 2.0 so that the λ profile is not uniform. Also we assume that the aspect ratio R/a = 2.0 and cross-sectional elongation $\kappa =$ 1.0 i.e square cross section.

Dynamical simulations of the MHD instability are carried out by superposing a few unstable modes as perturbations on the initial equilibrium state. The following perturbation model that is applied to the plasma fluid velocity in the toroidal geometry are used here:

$$\begin{pmatrix} v_{1r} \\ v_{1\phi} \\ v_{1z} \end{pmatrix} = v_1 \sum_n \begin{pmatrix} cos(n\phi) \\ 0 \\ sin(n\phi) \end{pmatrix},$$

$$v_1 = sin^2\left(\frac{r - r_1}{r_l}\pi\right) sin^2\left(\frac{z}{z_l}\pi\right)$$

Here, n is the toroidal mode number, and r_1 and r_l are the inner radius (from the major axis) and sectional width of the torus, respectively.

In this preliminary study, spacial discretization is $(N_r, N_\phi, N_z) = (64{,}64{,}64)$, and parameters of MHD equations are $R_\nu = 10^3$ and $R_\eta = 10^5$, respectively. Fig.1 shows a change in time for the three-dimensional magnetic energy distribution expressed by 3 transparent isosurfaces. It was observed that deformation of the equilibrium state with the growth of the instability began around t ~ $5\tau_A$ as the total magnetic energy rapidly increases (Fig.1(a)). Here τ_A is the Alfven time. Afterwards the helical deformation gradually occurs as time advances (Fig.1(b)-(d)). However, this configuration does not collapse immediately after the instability grew up, and this helical state is kept at a certain time. This can be understood from the fact that oscillation is observed in time evolution of the total magnetic energy. Although it is necessary to examine the time variation of each mode energy in detail, it is considered that a mode around n~5 grows as a dominant mode as a whole. In addition, it was observed that stochastic behavior of the magnetic field line structure occurred with growth of the instability in a plasma periphery. It is required to elucidate the influence of the toroidal effect and a characteristic in the small aspect ratio configuration in comparison with the numerical results by the cylindrical approximation in past studies [6, 8].

4. Summary

The non-linear MHD simulation of the three-dimensional toroidal geometry allowed the detailed analysis of dynamics of the resistive instability. We demonstrate the simulation analysis on the unstable mode in a small aspect ratio RFP and observed the non-linear growth of instability modes. Also, the stochastic behavior of the magnetic field line structure was observed in a plasma periphery by tracing its time variation. It is necessary to survey the non-linear growth process of a dominant mode by analyzing the magnenic energy evolution of each unstable mode. Additionally, the effect of ultra low aspect ratio and the non-circularity of the cross section should be investigated to elucidate a characteristic of the MHD behavior in this configuraiton.

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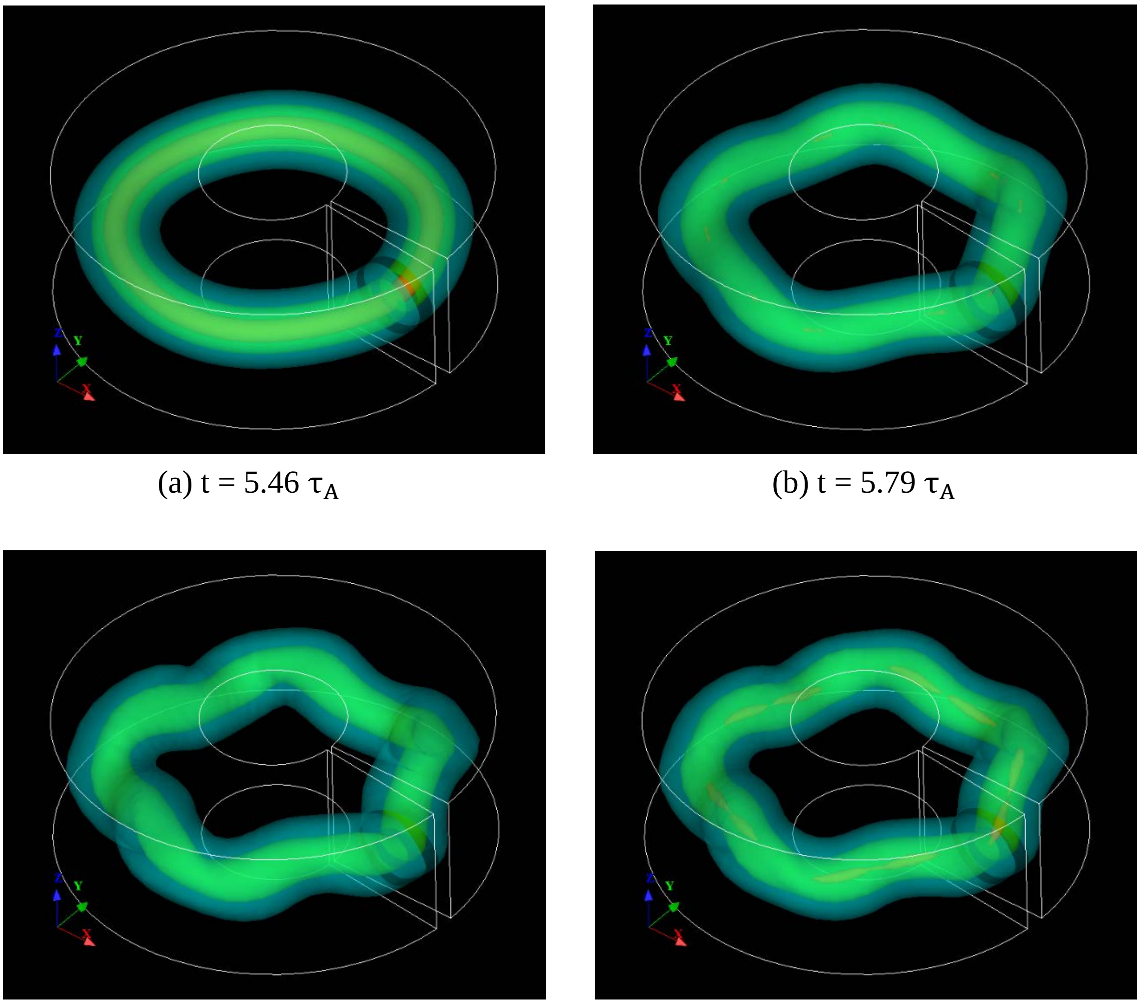

(a) $t = 5.46\ \tau_A$ (b) $t = 5.79\ \tau_A$

(c) $t = 6.12\ \tau_A$ (d) $t = 6.45\ \tau_A$

Fig.1 Time variations of the three-dimensional magnetic energy distribution expressed by 3 transparent isosurfaces.