Numerical tokamak equilibria with pressure anisotropy

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INTRODUCTION

In order to achieve fusion temperatures modern tokamaks rely on auxiliary heating methods. These heating methods deposit energy in the charged particles in a specific direction and thus generate significant pressure anisotropy in the plasma[1, 2], modifying the momentum conservation equation and ultimately affect the equilibrium and stability properties. The high energy heating used in present experiments, and also projected for the future, is enough to maintain this anisotropy contrary to the isotropisation due to collisions. Therefore is of importance to include the pressure anisotropy for studies of equilibria in experiments with auxiliary heating.

THE EQUATIONS OF EQUILIBRIUM WITH PRESSURE ANISOTROPY

The pressure anistoropy can be expressed through the quantity $\sigma = \mu_0(p_\parallel - p_\perp)/|\mathbf{B}|^2$, where $p_\parallel$ and $p_\perp$ are the elements of the CGL pressure tensor parallel and perpendicular to the magnetic field. σ can be positive or negative depending on whether the direction of the heating is parallel or perpendicular to the magnetic surfaces. Under the assumption that σ is uniform on magnetic surfaces a Generalised Grad-Shafranov equation can be obtained which along with a Bernoulli equation for the effective isotropic pressure, defined as $\overline{p} = (p_\parallel + p_\perp)/2$, describe the equilibrium. Specifically, the MHD equilibrium states of an axisymmetric magnetized plasma with pressure anisotropy are determined by the following equation written in convenient units by setting $\mu_0 = 1$

$$(1-\sigma-M_p^2)\Delta^\star\psi - \frac{1}{2}(1-\sigma-M_p^2)'|\nabla\psi|^2 + \frac{1}{2}\left(\frac{X^2}{1-\sigma-M_p^2}\right)' + R^2\overline{p}_s' + \frac{R^4}{2}\left(\frac{(1-\sigma)\rho(\Phi')^2}{1-\sigma-M_p^2}\right)' = 0 \quad (1)$$

along with the Bernoulli relation for the pressure,

$$\overline{p} = \overline{p}_s(\psi) - \rho\left[\frac{\upsilon^2}{2} - \frac{R^2(\Phi')^2}{1-\sigma-M_p^2}\right] \quad (2)$$

Here, the poloidal magnetic flux function $\psi(R,z)$ labels the magnetic surfaces; $M_p(\psi)$ is the Mach function of the poloidal velocity with respect to the poloidal Alvén velocity; $\rho(\psi)$ and $\Phi(\psi)$ are the density and the electrostatic potential; $X(\psi)$ relates to the toroidal magnetic field; for vanishing flow the surface function $\overline{p}_s(\psi)$ coincides with the static effective pressure; υ is the velocity modulus which can be expressed in terms of surface functions and R; $\Delta^\star =$

$R^2\nabla\cdot(\nabla/R^2)$; and the prime denotes a derivative with respect to ψ. In the absence of flow and anisotropy (1) reduces to the usual Grad-Shafranov equation. Derivation of (1) and (2) is provided in [3]. The surface quantities $M_p(\psi)$, $\Phi(\psi)$, $X(\psi)$, $\rho(\psi)$, $\sigma_d(\psi)$ and $\overline{p}_s(\psi)$ are free functions for each choice of which (1) is fully determined and can be solved whence the boundary condition for ψ is given.

Equation (1) can be simplified by the transformation

$$u(\psi)=\int_0^{\psi}\left[1-\sigma(f)-M^2(f)\right]^{1/2}df \tag{3}$$

which reduces (1) to

$$\Delta^{\star}u+\frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-\sigma_d-M_p^2}\right)+R^2\frac{d\overline{p}_s}{du}+\frac{R^4}{2}\frac{d}{du}\left[(1-\sigma_d)\rho\left(\frac{d\Phi}{du}\right)^2\right]=0 \tag{4}$$

Also, (2) is put in the form

$$\overline{p}=\overline{p}_s(\psi)-\rho\left[\frac{\upsilon^2}{2}-R^2(1-\sigma)\left(\frac{d\Phi}{du}\right)^2\right] \tag{5}$$

Note that no quadratic term as $|\nabla u|^2$ appears anymore in (4). Transformation (3) does not affect the magnetic surfaces, it just relabels them. Also, one finds for $p_{\parallel}$ and $p_{\perp}$ the relations:

$$p_{\perp}=\overline{p}-\sigma\frac{B^2}{2}\qquad\text{and}\qquad p_{\parallel}=\overline{p}+\sigma\frac{B^2}{2} \tag{6}$$

Considering now rotation parallel to the magnetic field, $\vec{\upsilon}=(M/\sqrt{\rho})\vec{B}$, where M is the Afvénic Mach number of the (total) parallel velocity which is exactly equal to the poloidal Mach number M_p, the electric field vanishes; therefore (4) becomes:

$$\Delta^{*}u+\frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-\sigma-M^2}\right)+R^2\frac{d\overline{p}_s}{du}=0 \tag{7}$$

which is identical in form with the Grad-Shafranov equation describing static equilibria, while (5) takes the form:

$$\overline{p}=\overline{p}_s-\rho\left(\frac{\upsilon^2}{2}\right) \tag{8}$$

STATIONARY ANISOTROPIC FIXED BOUNDARY EQUILIBRIUM CODE HELENA

The code HELENA, is a fixed boundary equilibrium solver [4] available on the EUROfusion Gateway and being used for modelling purposes. The code was extended for incompressible parallel flow in [5]. Here we will further extend the code in the presence of pressure anisotropy. The static Grad-Shafranov equation used in the code is written as:

$$\Delta^{*}\psi=-F\frac{dF}{d\psi}-\mu_0R^2\frac{dP}{d\psi}=-\mu_0Rj_{tor} \tag{9}$$

By observing that there is a correlation between the quantities in (7) and (9):

$$\psi\longleftrightarrow u\quad,\quad P(\psi)\longleftrightarrow\overline{p}_s(u)\quad,\quad F\frac{dF}{d\psi}\longleftrightarrow\frac{1}{2}\frac{d}{du}\left(\frac{X^2}{1-\sigma_d-M^2}\right) \tag{10}$$

the solver of the static code HELENA can be used to calculate the stationary equilibrium for plasma rotation parallel to the magnetic field and pressure anisotropy, though the output will no longer correspond to the "natural" quantities in the ψ-space. In order to preserve compatibility with the experiments and other codes the calculated by the solver quantities (now in the u-space) must be mapped to the "natural" ψ-space. For the mapping one must consider the following basic correspondence:

$$P_{\text{HELENA}} \longleftrightarrow \overline{p}_s \quad , \quad \psi_{\text{HELENA}} \longleftrightarrow u \quad , \quad F_{\text{HELENA}} \longleftrightarrow \frac{X}{\sqrt{1-\sigma-M^2}} \tag{11}$$

By applying the inverse of transformation (3) and taking into account the relations (11), we get the following expressions for the magnetic field, current density and pressure:

$$\vec{B} = \frac{F_{\text{HELENA}}}{\sqrt{1-\sigma-M^2}}\vec{\nabla}\phi - \frac{1}{\sqrt{1-\sigma-M^2}}\vec{\nabla}\phi \times \vec{\nabla}u \tag{12}$$

$$\vec{J} = \left[\frac{-1}{\sqrt{1-\sigma-M^2}}\Delta^* u + \frac{1}{2}\frac{1}{(1-\sigma-M^2)^{3/2}}\frac{d(\sigma+M^2)}{du}|\vec{\nabla}u|^2\right]\vec{\nabla}\phi + \frac{d}{du}\left(\frac{F_{\text{HELENA}}}{\sqrt{1-\sigma-M^2}}\right)\vec{\nabla}\phi \times \vec{\nabla}u \tag{13}$$

$$\overline{p} = P_{\text{HELENA}} - \frac{1}{2R^2}\frac{M^2}{1-\sigma-M^2}\left(F^2_{\text{HELENA}} + |\vec{\nabla}u|^2\right) \tag{14}$$

where the subscript HELENA refers to the computed by the code quantities.

The profile of the Mach number and the pressure anisotropy parameter can be chosen peaked on-axis or off-axis and vanishing at the boundary. There are three independent parameters (M_0, n, m) and (σ_0, k, ℓ) that control their shape respectively:

$$M^2 = M_0^2(\psi^n - \psi_0^n)^m \qquad \text{and} \qquad \sigma = \sigma_0(\psi^k - \psi_0^k)^\ell$$

where M_0 and σ_0 is the maximum value on axis; ψ_0 is the value of the poloidal magnetic flux on the boundary.

CALCULATED EQUILIBRIUM WITH PRESSURE ANISOTROPY

Let us first note that inspection of Eq. (12) and Eqs. (6) imply that the pressure anisotropy acts paramagnetically for $\sigma > 0$ and diamagnetically for $\sigma < 0$, while the impact of parallel flow is paramagnetic. In the following we will present some preliminary results from the code for static equilibrium. The boundary as well as the input profiles of P' and FF' are obtained from a scenario of a 15MA ITER equilibrium based on Ref. [6].

In (Fig. 1) the $p_\parallel$-profile is plotted for the isotropic case and two cases of anisotropy. The plots suggest that the anisotropy affects greatly the profile and this effect depends on the sign of σ_0. If $\sigma_0 > 0$ then $p_\parallel$-profile flattens close to the magnetic axis and exhibits a region with strong gradient while for $\sigma_0 < 0$ the region of high pressure gradient is close to the magnetic axis and the flattening appears close to the boundary. The reverse is expected for the $p_\perp$. The anisotropy allows for modification in the two pressure profiles independently (Eq. (6). One of them can have a large gradient close to the magnetic axis while the other close to the boundary at the same time, unlike rotation which modifies the two profiles the same way.

In (Fig. 2) J_ϕ is plotted for the isotropic and two cases of anisotropy on the $z = 0$ midplane. There, it is evident that a significant change in the profile happens in the region where the anisotropy varies strongly with r. This means that it is the spatial change of the anisotropy that affects stronger the current density profile than the maximum absolute value of anisotropy itself.

These results indicate that the anisotropy affects significantly some equilibrium quantities. Therefore, it can be used in shaping the equilibrium profiles. This modification of the profiles can lead to compatible ones with those observed in discharges with transport barriers. Thus, pressure anisotropy may change the stability and transport properties of magnetically confined plasmas in connection with the formation of transport barriers.

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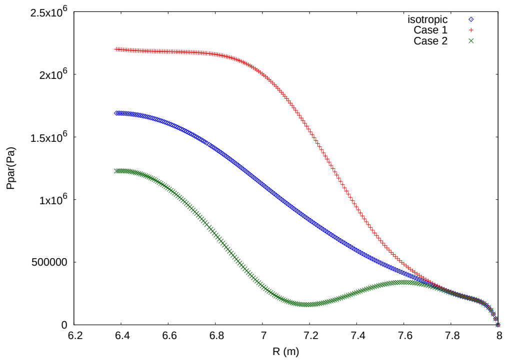

Figure 1: *Parallel pressure profiles for isotropic and 2 anisotropic cases. (Case 1: $\sigma_0 = 0.05$, $k = 2$, $\ell = 6$ and Case 2: $\sigma_0 = -0.05$, $k = 2$, $\ell = 6$).*

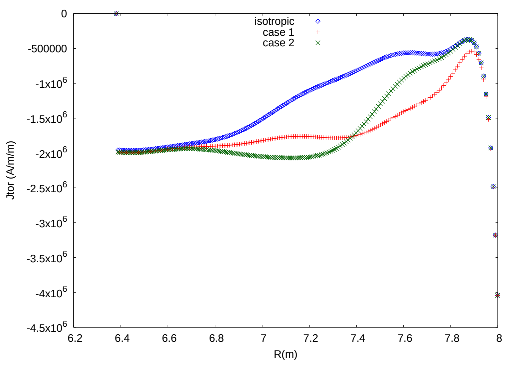

Figure 2: *Toroidal current density profiles for the isotropic and 2 anisotropic cases. (Case 1: $\sigma_0 = 0.03$, $k = 2$, $\ell = 3$ and Case 2: $\sigma_0 = 0.03$, $k = 2$, $\ell = 6$).*