

Synthetic synchrotron diagnostics for runaway electrons

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The synchrotron radiation emitted by runaway electrons is an important diagnostic for studying their properties, and many tokamak experiments are equipped with cameras for detecting this radiation. Here we present the flexible synthetic-diagnostic tool SOFT (*Synchrotron-detecting Orbit Following Toolkit*) [1], which allows the study of not only the synchrotron radiation spot shape [2, 3], but also intensity variations within the spot. SOFT takes the full angular and spectral distributions of radiation into account, as well as the electron distribution function, the magnetic geometry, orbits, and the limited spectral range of the camera. The additional information gained from synthetic imaging using SOFT provides valuable insight into the runaway-electron distribution function. With Fokker-Planck simulations [4, 5], using experimentally measured plasma parameters [6], we show that SOFT is able to reproduce the main features of synchrotron radiation measurements performed at the Alcator C-Mod tokamak.

Principles of SOFT A synchrotron radiation detector can be thought of as measuring the quantity

$$\frac{dI_{ij}}{d\lambda}(\mathbf{x}_0, \lambda) = \int_S dA \int_{\mathbf{N}_{ij}} d\mathbf{n} \int d\mathbf{x} d\mathbf{p} \frac{\hat{\mathbf{n}} \cdot \mathbf{n}}{r^2} \delta\left(\frac{\mathbf{r}}{r} - \mathbf{n}\right) f(\mathbf{x}, \mathbf{p}) \frac{dP(\mathbf{x}, \mathbf{p}, \mathbf{x}_0, \lambda)}{d\lambda d\Omega}, \quad (1)$$

where I_{ij} is the brightness of pixel (i, j) , λ is the wavelength, \mathbf{x}_0 is the position of the synchrotron detector, S is the detector surface, \mathbf{N}_{ij} is the set of all lines-of-sights \mathbf{n} corresponding to pixel (i, j) , $\hat{\mathbf{n}}$ is the detector viewing direction, \mathbf{x} is the particle position and \mathbf{p} is its momentum, and $r = |\mathbf{r}| = |\mathbf{x} - \mathbf{x}_0|$ is the distance between the detector and the particle. The runaway distribution function is denoted by $f(\mathbf{x}, \mathbf{p})$, and $dP/d\lambda d\Omega(\mathbf{x}, \mathbf{p}, \mathbf{x}_0, \lambda)$ is the power received by a detector located at \mathbf{x}_0 from a particle at position \mathbf{x} with momentum \mathbf{p} at a wavelength λ .

We make a zeroth-order guiding-center approximation and introduce “trajectory coordinates” (ρ, τ, ϕ) for the guiding-center position. The coordinate ρ is the major radius at $\tau = 0$, where the guiding-center begins its orbit (chooses which orbit to consider), τ is the time that parametrizes an orbit (indicates the guiding-center’s position in the poloidal plane), and ϕ is the toroidal angle of the guiding-center at $\tau = 0$. The momentum integral is performed over the guiding-center’s momentum at the *beginning* of its orbit, i.e. at $\tau = 0$. Variation of the guiding-center’s pitch angle due to the inhomogeneity of the magnetic field is thus embedded in the orbit time τ .

Due to toroidal symmetry, and gyrotropy of the runaway distribution function, the distribution function will be independent of the toroidal and gyrophase angles. The use of trajectory coordinates allows the invocation of Liouville's theorem, which states that the value of the distribution function is constant along the guiding-center's phase-space trajectory, implying that the guiding-center distribution function is independent of the orbit time τ . As a consequence, the distribution function can be parametrized to depend only on the particle's initial radial position, its energy, and its pitch angle on the outer midplane. Together with the detector properties (position, orientation and spectral range), the radial position, energy and pitch angle are the only parameters that can influence the synchrotron radiation measurements in a given magnetic field. SOFT is free software, available in source code form from the SOFT GitHub repository [7].

Parametric dependences Figure 1 illustrates how the initial pitch angle, energy and initial radial position of the particle affect the synchrotron radiation spot. The figure consists of a set of simulated synchrotron radiation images using the magnetic field equilibrium of Alcator C-Mod discharge 1140403026 at $t \sim 0.742$ s. The camera is located 21 cm below the midplane, which has a strong effect on the observed spot, and causes the comet-like shape. Figures 1a and 1b show that when the particle's initial pitch angle is increased, the synchrotron radiation spot size increases from the lower left towards the upper right corner.

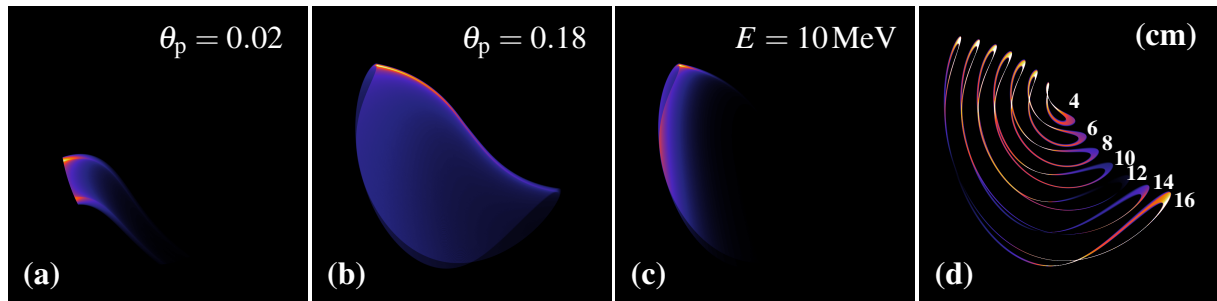


Figure 1: Illustration of the effect of four different parameters on the synchrotron radiation spot. Parts (a) and (b) illustrate the effect of the particle's pitch angle, with the energy of all particles set to $E = 30$ MeV. Part (c) shows a population of particles with the same pitch angles as in (b), but with an energy of $E = 10$ MeV. Part (d) shows contributions to the image from a few distinct radii, which shows how a radial profile would affect the image.

In comparing Fig. 1c to Fig. 1b, for which the energy of the simulated particles was changed from $E = 30$ MeV in Fig. 1b to $E = 10$ MeV in Fig. 1c, we see that the brightness gets focused in the left part of the image, corresponding to the high-field side of the tokamak. This effect arises due to the limited spectral range of the camera, which in this case is set to the range

$\lambda \in [500, 1000]$ nm. In this wavelength interval, the spectrum of the synchrotron power emitted by the particles with $E = 10$ MeV scales exponentially with magnetic field strength. Figure 1d reveals how particles at different radii contribute to the image, and in particular it indicates how a radial distribution would affect it. Particles located at radii further out add to the spot shape primarily on the left side of the image, allowing an estimate of the beam size.

Alcator C-Mod modelling On-axis temporal profiles of the electric field, plasma current and electron density were used to solve the spatially homogeneous kinetic equation for electrons in 2D momentum space using CODE [4, 5], including electric-field acceleration, collision and synchrotron-radiation reaction losses. Figure 2a shows the runaway electron distribution function for the Alcator C-Mod experimental parameters at the time $t \sim 0.742$ s, and in Fig. 2b the same distribution function weighed with the radiation emitted from each point of momentum space is shown. The latter reveals which parts of momentum-space that will dominate the synchrotron radiation image.

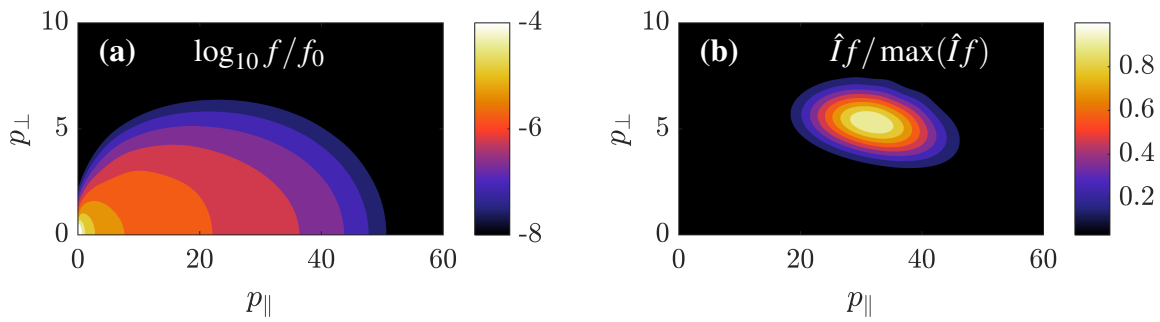


Figure 2: The momentum-space distribution function obtained by CODE, that was used in modelling Fig. 3a, is shown in part (a). In part (b), the distribution function has been weighed with \hat{I} , the amount of synchrotron radiation in the wavelength interval $[500, 1000]$ nm detected. The resulting contour plot reveals which parts of momentum space we expect to get the most significant contributions from, for this particular population of runaways. The quantity $f_0 = f(p = 0)$.

The synthetic synchrotron image obtained from SOFT using the distribution function and parameters just described is presented in Fig. 3b. Despite using a flat radial profile of runaways for the distribution function ($df/d\rho$), and with parameters valid only on the magnetic axis, the agreement of Fig. 3b with the experimentally obtained camera image Fig. 3a is remarkable. The characteristic comet shape of the synchrotron radiation spot is the combined consequence of the low placement of the camera (i.e., 21 cm below the midplane), and a relatively small dominating pitch angle (around $\theta_p = 0.15$ rad) as suggested by Fig. 2b. At the energies possessed by electrons of the population – around $E = 15$ MeV – more radiation is seen coming from the high-field side of the tokamak.

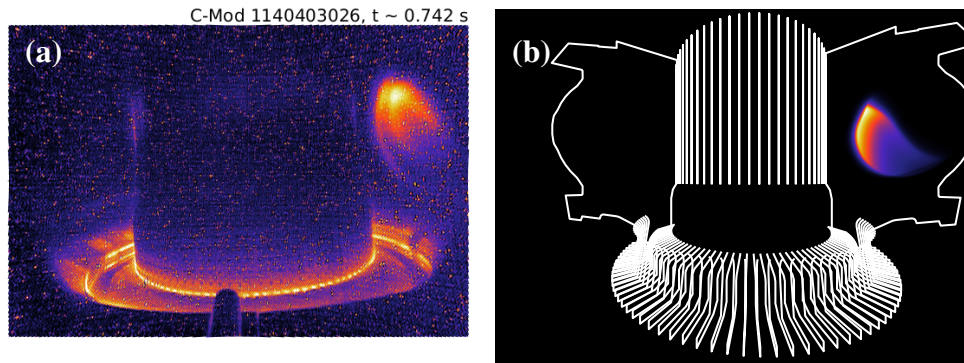


Figure 3: (a) A camera view inside C-Mod captures the spatial pattern of visible synchrotron emission during shot 1140403026 at time $t \sim 0.742$ s. This image has been corrected for camera lens distortion, and a perceptually-uniform colormap is applied to highlight details while conserving camera intensity. (b) Synchrotron image obtained with SOFT using the distribution of runaway electrons shown in Fig. 2.

Synchrotron images can be considered a weighted superposition of images resulting from various mono-energetic and mono-pitch populations of runaways, with the distribution of runaways with a given energy and pitch angle acting as the weight function. This permits study of the effect of mono-energetic and mono-pitch distributions on the synchrotron spot, and conclusions from these studies to be applied to more complex cases involving runaways that are continuously distributed in momentum-space. However, one should note that the brightness of each of the constituent images from mono-energy/mono-pitch populations depends sensitively on the energy and pitch angle considered. This has the result – as illustrated in Fig. 2b – that particles with a certain energy and pitch angle may appear to dominate the image, but in the distribution of runaways they only make up a tiny fraction of the total. While mono-energetic and mono-pitch images to some extent can match observed synchrotron images, they can not be taken as literal descriptions of the runaway distribution function.

References

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