Implementation of Multiple species collision operator in Gyrokinetic code GS2

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Introduction

Considering that collisions are known to play important role in direct set of physical phenomena in tokamak plasmas, it is essential to develop reliable and robust collisional operator for multiple species when addressing the transport of impurities and momentum transport. Although impurity density is very low in tokamak plasmas, its collision frequency can be of same order as of the main ions due to the impurities large charge number. The collision operator previously implemented in the gyrokinetic code GS2 consists of a gyro-average of the exact linearized Landau test-particle operator and a model field-particle operator that conserves number, momentum, and energy while satisfying Boltzmann's H-Theorem. It also has finite larmor radius effects included in it. Each of these properties is satisfied exactly in the code, and the collisions are treated implicitly in time. However, it only accounts for like-species collisions of ions. Here we adopt the model proposed by Sugama [2] to include collisions between different ion species in GS2. In order to retain exact numerical conservation properties, we modify Sugama's model field-particle operator. Furthermore, we have implemented a recursive implementation of the Sherman-Morrison identity to facilitate computationally efficient matrix inversion for the implicit solve. This collision operator is advancement of previous self-collision operator in [3] [4].

Linearised collision operator

The collision operator for cross species collision is used from [2] which satisfies the particle, momentum and energy conservation. It satisfies the adjoint relation and boltzmann's H theorem at equal temperatures and its gradients. Test particle part has maxwellian in the background and field particle part has fields in the background. The collision operatorr is taken from [2]

$$C^L_{ab}(\delta f_a, \delta f_b) = \underbrace{C^T_{ab}(\delta f_a)}_{\textit{Test particle}} + \underbrace{C^F_{ab}(\delta f_b)}_{\textit{Field particle}} \qquad (1)$$

The linearised collision operator for any temperature is

$$C_{ab}^{T}(\delta f_a) = \mathscr{L}\delta f_a + \mathscr{D}\delta f_a \tag{2}$$

where

$$\mathscr{L} = \frac{\nu_D^{ab}(v)}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial\delta f_a}{\partial\xi} + \frac{1}{1-\xi^2}\frac{\partial^2\delta f_a}{\partial^2\phi} \tag{3}$$

is a lorentz or pitch angle scattering operator

$$\mathscr{D} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{{\nu_\parallel}^{ab}(v)}{2}v^4 f_{0a}\frac{\partial}{\partial v}\frac{\delta f_a}{f_{0a}}\right] \tag{4}$$

Field particle term has to be chosen such that it satisfies the given properties below

Properties

Collision operator conserve particles [5], momentum [6] and heat [7]. It also have to satisfies the Boltzmann's H theorem [8].

$$\int d^3v C_{ab}^{L}(\delta f_a) = 0 \tag{5}$$

$$m_a\int d^3v\mathbf{v}C_{ab}^{L}(\delta f_a) = m_b\int d^3v\mathbf{v}C_{ba}^{L}(\delta f_b) \tag{6}$$

$$m_a\int d^3v\frac{v^2}{2}C_{ab}^{L}(\delta f_a) = m_b\int d^3v\frac{v^2}{2}C_{ba}^{L}(\delta f_b) \tag{7}$$

$$T_a\int d^3v\frac{\delta f_a}{f_{0a}}C_{ab}^{L}[\delta f_a] + T_b\int d^3v\frac{\delta f_b}{f_{0b}}C_{ab}^{L}[\delta f_b] \le 0 \tag{8}$$

Numerical method to implement collison operator

GS2 is a flux tube simulation code for low frequency turbulence studies. It solves gyrokinetic equations [1]. It treats collisional and collision-less physics separately because of operator splitting of the gyrokinetics equation terms. Equation 10 calculates the distribution function from collisions. The collision operator is divided in two parts which is given in equation 13 and 14. Equation 13 and 14 solves lorentz and diffusion test and field particle pieces solves separately which conserves the moments independently.

$$\frac{\partial\delta f_{a\mathbf{k}_\perp}}{\partial t} = C_{GK}[\delta f_{a\mathbf{k}_\perp}] + \mathscr{A}[\delta f_{a\mathbf{k}_\perp}] \tag{9}$$

$$\frac{\delta f_{a\mathbf{k}_\perp}^{n+1} - \delta f_{a\mathbf{k}_\perp}^{*}}{\Delta t} = C_{GK}[\delta f_{a\mathbf{k}_\perp}^{n+1}] \tag{10}$$

$$\frac{\delta f_{a\mathbf{k}_\perp}^{*} - \delta f_{a\mathbf{k}_\perp}^{n}}{\Delta t} = \mathscr{A}[\delta f_{a\mathbf{k}_\perp}^{n}, \delta f_{a\mathbf{k}_\perp}^{*}] \tag{11}$$

$$\delta f_{a\mathbf{k}_\perp}^{n+1} = (1-\Delta t C_{GK})^{-1}\delta f_{a\mathbf{k}_\perp}^{*} \tag{12}$$

$$\delta f_{a\mathbf{k}_\perp}^{**} = (1-\Delta t(L_{ab}+\underline{U_{Lab}}))^{-1}\delta f_{a\mathbf{k}_\perp}^{*} \tag{13}$$

$$\delta f_{a\mathbf{k}_\perp}^{n+1} = (1-\Delta t(D_{ab}+\underline{U_{Dab}}+\underline{E_{ab}}))^{-1}\delta f_{a\mathbf{k}_\perp}^{**} \tag{14}$$

The underlined matrices in equation 13 and 14 are dense matrices which can be written as an outer product of vectors. The L_{ab} and D_{ab} are the tridiagonal matrices. The inversion of these matrices can be done using sherman morrision formula [4] which can be cheaper to solve than other numerical schemes. The sherman morrison method is already given in [4] to solve the self collision operator and recursive sherman sherman morrision is given in 1 to solve for cross species collsion which can also be used to solve for arbitrary number of outer products. A_0 in equation 15 is tridiagonal matrices which is equivalent to L_{ab} and D_{ab}, $\mathbf{u}$ and $\mathbf{v}$ are the two parts of each field particle pieces U_{Lab}, U_{Dab} and E_{ab}. The recursive sherman morrison is used to solve equation 18. Every $\mathbf{u}_i$ and $\mathbf{v}_i$ is used to invert its corresponding A_{i+1} after inverting it $n+1$ times, which is the number of outer products, we obtained the solution of equation 15

$$M\mathbf{x} = \mathbf{b} \Rightarrow M = A_0 + \sum_{i=0}^{n} \mathbf{u}_i \otimes \mathbf{v}_i \tag{15}$$

$$\mathbf{x} = M^{-1}\mathbf{b} \tag{16}$$

$$A_{i+1} = A_i + \mathbf{u}_i \otimes \mathbf{v}_i \tag{17}$$

$$\mathbf{x} = (A_n + \mathbf{u}_n \otimes \mathbf{v}_n)^{-1}\,\mathbf{b} \tag{18}$$

$$\mathbf{x} = A_n^{-1}\mathbf{b} - \left[\frac{\mathbf{v}_i \cdot A_n^{-1}\mathbf{b}}{1+\mathbf{v}_i \cdot A_n^{-1}\mathbf{u_i}}\right] A_n^{-1}\mathbf{u}_n \tag{19}$$

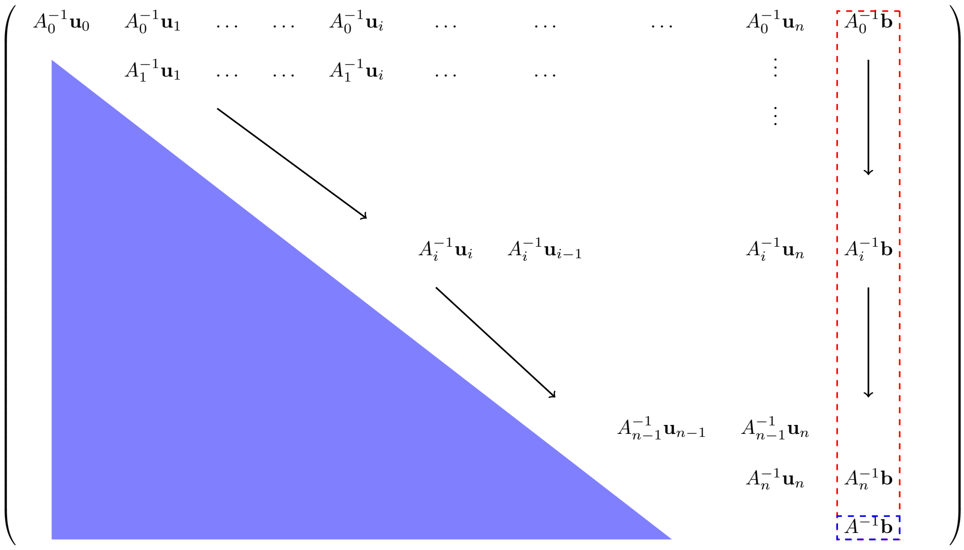

Figure 1: Flowchart of Recursive Sherman morrison formula to solve multiple species collisions

Conclusion

So far, Gyroaveraged multi-species collision operator is implemented in GS2. Recursive sherman morrision algorithm was obtained to implement in GS2's collision operator. Sherman morrision formula is used for field particle terms of collision operator. Field particle term of collision operator was obtained which satisfies the conservation of particle, momentum and energy between cross species collision. It also satisfies the boltzmann's H-theorem. Test cases for multiple species will be done in future.

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