A Hybrid Vlasov Fokker-Planck Code for Laboratory Astrophysics Applications

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Abstract

Laser produced plasma experiments are frequently being used to investigate plasma phenomena of astrophysical relevance. The high densities and velocities generated in the laboratory can provide conditions to investigate weakly collisional or collisionless plasma physics. Numerical simulations play a vital role in this effort, providing predictions in the developmental stage, and assisting the interpretation of results. To this end, we are currently developing a new hybrid simulation code designed to explore the complex interaction between fast moving plasmas and their self-generated electromagnetic fields including an effectively arbitrary level of collisionality. This hybrid code solves the Vlasov-Fokker-Planck equation for the ions while electrons are treated as a charge neutralising fluid. This can accurately model non-Maxwellian ion distributions, covering both high frequency and hydrodynamic timescale phenomena, and can provide a reliable description for plasmas both in the collisionless and weakly collisional regimes. We will present some validation tests for the code to show that it can accurately model several key processes.

Introduction

Laboratory experiments of ever increasing power and sophistication are being regularly exploited to investigate complex plasma phenomena, such as shocks [1, 2, 3], turbulence [4, 5], magnetic reconnection [6, 7], etc. The plasmas produced in these experiments show a broad range of characteristics, and very often only show qualitative similarity with astrophysical counterparts. Making the connection between these two, arguably disparate systems, relies on the ability, not only to isolate, but to accurately model the key process under investigation. Numerical simulations are a particularly useful tool in this regard. Two methods that are commonly used are magneto-hydrodynamic simulations [8, 9], which neglect the particle kinetics, but can accurately treat radiation transport and other fluid effects, and fully electromagnetic particle in cell simulations [10, 11] , which in principle retain all the kinetic information of the particles, but are both computationally expensive and are not ideally suited to collisional systems. Hybrid-schemes that neglect electron kinetics, offer an attractive medium, typically accurate provided one is concerned primarily with length scales exceeding the electron inertial length ($\approx 5n_{18}^{-1/2}$ μm where n_{18} is the electron density in units of 10^{18} cm^{-3}), and that a fluid treatment of the electrons is adequate.

While several such codes exist, we focus specifically on the Vlasov-Fokker-Planck approach,

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} + q\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right) \cdot \frac{\partial f}{\partial \vec{p}} = \mathscr{C}_{\mathrm{FP}}, \tag{1}$$

since collisions between particles are also important in many such experiments. Here $\mathscr{C}_{\mathrm{FP}}$ are the Fokker-Planck collision terms, determined using the Rosenbluth potentials. The numerical scheme is based on the approach pioneered by Bell et al [12], which uses a spherical harmonic expansion of the particle angular distribution.

$$f(\vec{p}) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} f_l^m(p) Y_l^m(\theta, \phi), \tag{2}$$

reducing the problem to a system of coupled equations, truncated at some finite l, that can be solved using standard methods. The code is built on top of the already developed and tested OSHUN code [13], which updates an electron distribution in 2D in physical space and 3D in momentum space, together with Maxwell's equations. We instead solve for the ions, and use a fluid treatment for the electrons. For simplicity, we focus on an inertia-less isothermal electron distribution, that maintains charge neutrality at all times. Thus, using an appropriate Ohm's law, e.g.

$$\vec{E} = -\frac{\vec{v}_e}{c} \times \vec{B} - \frac{1}{n_e e} \nabla P_e + \eta \vec{J} + \ldots \tag{3}$$

it is possible to update both the ion distribution, and the magnetic field, via Faraday's law

$$\frac{\partial \vec{B}}{\partial t} = -c \nabla \times \vec{E}, \tag{4}$$

Together with Ampere's law, this closes our system of equations.

Code tests and validations

We wish to highlight a number of the key and unique aspects of the code. First we test its ability to capture simple magnetohydrodynamic waves in one dimension. In Figure 1, we present a polar plot of the Alfvén mode as well as the fast and slow

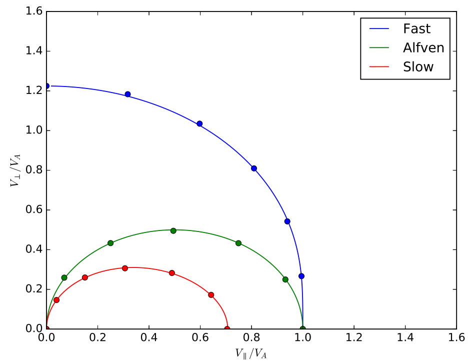

Figure 1: *Linear MHD wave speeds at different angles to the magnetic field. See text.*

magnetosonic modes, as a function of angle to the magnetic field. The dots represent the numerical phase velocities, determined from the simulations, while solid lines are the analytic theory. The simulations were performed in a plasma beta of $\beta = 0.25$, with a spatial resolution of $\Delta x/r_g = 20$, where r_g is the gyroradius of ions moving with the mean thermal velocity. For simplicity, these simulations neglected the Hall term in the Ohm's law for field updates

$$\vec{E} = -\frac{\vec{v}_i}{c} \times \vec{B} \tag{5}$$

but included it in the particle mover (since otherwise there is no net force on the ions). The resolution of the momentum space in the simulations below are all using $\Delta p = p_{\mathrm{th}}/30$.

To validate the collisional part of the code the ion heat flux

$$\vec{q} = -\kappa_{\parallel}\nabla_{\parallel}(k_{\mathrm{b}}T) - \kappa_{\perp}\nabla_{\perp}(k_{\mathrm{b}}T) + \kappa_{\wedge}\frac{1}{B}\vec{B} \times \nabla_{\perp}(k_{\mathrm{b}}T) \tag{6}$$

was investigated using simplified one dimensional simulations, and the asymptotic thermal conductivities in each direction were determined. In this example, the Hall term is simplified too because the characteristic length is much longer than the ion inertial length. The electrons' effects are ignored here because the electron-ion collisional time τ_{ei} is much longer than the ion-ion collisional time τ_i, which is also the precondition in Braginskii's work[15]. The coefficients κ are validated from weakly magnetized plasma $\omega_i\tau_{ii} \ll 1$ to strongly magnetized case $\omega_i\tau_{ii} \gg 1$. The results are plotted in Figure 2, and compared to the theoretical estimates from [15].

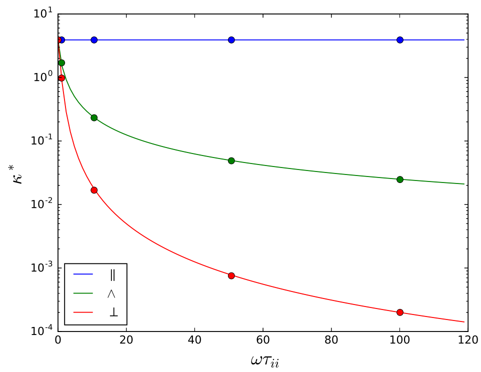

Figure 2: *Ion thermal conductivities. See text*

Finally we present an example in multi-dimensions, investigating the well-known MHD vortex problem first suggested by Orszag & Tang [14]. In Figure 3, the velocity fields and the corresponding magnetic fields at different time are compared in the upper and bottom panels.

The simulation box size $L(=64)$ is comparable to the mean free path here $\lambda_{mfp}(=66)$. There is a strong magnetic field B_z out of the plane which sets the ion gyroradius $r_g(\simeq 0.08) < \Delta x(= 0.5)$. The Ohm's law applied here is more complicated because the Hall term and dissipation are included:

$$\vec{E} = -\vec{v}_i \times \vec{B} + \frac{1}{n_i}\vec{J} \times \vec{B} - \frac{1}{m_i n_e c^2}\nabla P_e + \eta\vec{J}, \tag{7}$$

Conclusions

We have presented some basic results of a new Hybrid code. While the focus in this report has been on validating it with respect to MHD fluid theory results, the kinetic nature of the code per-

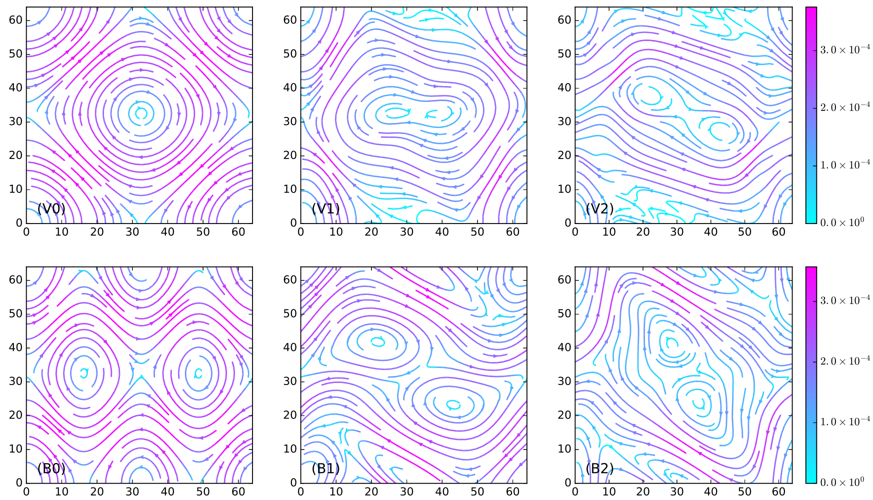

Figure 3: *Velocity (upper panels) and magnetic (lower panels) stream plots for the Orszag-Tang vortex problem. Fields shown at* $\omega_{pi}t = 0,\ 6\times10^4,\ 10^5$*, and clearly show the cascading to smaller wavelengths.*

mits it to be extended to higher frequency phenomena, and to non-Maxwellian plasma systems. Its ability to accurately capture both collisional and collisionless processes makes it a valuable predictive tool for many plasma experiments, in particular laboratory-astrophysics experiments, where both collisional and collisionless processes can be occurring simultaneously.

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