Transport properties of the fully ionized plasma in the first Born approximation of the linear response theory

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Abstract

The Chebyshev polynomial expansion of the Fermi distribution function is applied to the case of an arbitrary plasma degeneracy. On the base of the linear response theory the behaviour of the electrical conductivity, the heat conductivity and the thermopower of the fully ionized plasma is investigated. The applicability of the method to the partly ionized plasma is discussed.

Motivation

The dielectric screening in many-particle systems with the Coulomb interaction has the fundamental dynamical nature. This leads to computational problems in the first Born approximation (Lenard-Balescu collision integrals) for the electron-electron collisions contribution to transport properties and to the impossibility to obtain corresponding integrals in the strong collision limit. Only in cases of extremely degenerate plasma (where electron-electron collisions are suppressed) and high-temperature low-density plasma (asymptotic expansions [1, 2]) transport coefficients are obtained without difficulties. There were many attempts to simulate dynamical screening in intermediate region using static one with the adjustment of the screening length, but the adequate independent way of preliminary obtaining this length has not been found.

Therefore an elaboration of direct and controlled calculating methods for electron-electron collision integrals is of interest in the investigation of plasma transport properties.

Basic assumptions

- We consider a multicomponent plasma in the adiabatic approximation. Final results are presented for the fully ionized case.
- The first Born approximation with respect to screened interactions is used. Reasons for this are grounded in [3].

Technical Description

Within the linear response theory in the formulation of Zubarev [4], transport properties are expressed via force-force correlation functions [5, 6, 7]. The electrical conductivity, the thermopower, and the heat conductivity are related to the Onsager transport coefficients $L_{ik} = L_{ki}$

according to

$$\sigma = e^2 L_{11} \tag{1}$$

$$\alpha = (eT)^{-1} L_{12}/L_{11} \tag{2}$$

$$\lambda = T^{-1}(L_{22} - L_{12}^2 L_{11}) \tag{3}$$

where

$$L_{ik} = -\frac{(-h)^{i+k-2}}{\Omega \det(d)} \begin{vmatrix} 0 & \frac{k-1}{\beta h} N_1 - N_0 \\ \frac{i-1}{\beta h}\overline{N_1} - \overline{N_0} & d \end{vmatrix}, \tag{4}$$

$$N_n = \begin{pmatrix} N_{n0} & N_{n1} & \dots & N_{nl} \end{pmatrix}, \tag{5}$$

$$\overline{N_n} = \begin{pmatrix} N_{n0} \\ N_{n1} \\ \vdots \\ N_{nl} \end{pmatrix}, d = \begin{pmatrix} d_{00} & d_{01} & \dots & d_{0l} \\ d_{10} & d_{11} & \dots & d_{1l} \\ \vdots & \vdots & \ddots & \vdots \\ d_{l0} & d_{l1} & \dots & d_{ll} \end{pmatrix}. \tag{6}$$

In (1) - (6) Ω - the system volume, N_{mn}, d_{mn} are correlation functions for the thermodynamic equilibrium, N_e - the number of electrons, h - the enthalpy per one electron, T - temperature and $\beta = (k_B T)^{-1}$.

$$d_{mn} = d_{mn}^{ei} + d_{mn}^{ee} + d_{mn}^{ea}, \tag{7}$$

$$N_{mn} = N_e \frac{\Gamma(m+n+5/2)}{\Gamma(5/2)} \frac{I_{m+n+1/2}(\beta \mu_e^{id})}{I_{1/2}(\beta \mu_e^{id})}, \tag{8}$$

with $I_\nu(y) = \frac{1}{\Gamma(\nu+1)} \int\limits_0^\infty \frac{x^\nu dx}{e^{x-y}+1}$ - Fermi integrals, μ_e^{id}- the ideal part of the electronic chemical potential.

Correlation functions d_{mn} (for electron-ion, electron-electron and electron-atom collisions correspondingly) are evaluated using thermodynamic Green's functions. The diagram technique in the first Born approximation for the Coulomb interaction $V(q) = e^2(q^2\Omega\varepsilon_0)^{-1}$, screened due to the medium polarization, gives Lenard-Balescu collision integrals [8].

Treatment of collision integrals

- In the adiabatic limit all d_{mn}^{ei} reduce to one-dimensional Zyman-type integrals. We take later the ionic charge $Z_{ion} = 1$ and use HNC method for ionic structure factor.

- In the adiabatic limit for d_{mn}^{ea} Zyman-type integrals can be used, if we know corresponding cross-sections. They are not well-defined in the dense plasma. The polarization approximation [9, 10] becomes questionable in the dense plasma [11]. Special efforts must be done for the investigation of corresponding structure factors.

- The simultaneous account of the degeneracy and dynamical nature of electronic screening in d^{ee}_{mn} was not accomplished up to now. Recently the problem was avoided for d^{ee}_{11} with $\mu^{id}_e < 0$ (approximately $\Theta > 1$) in [3]. In the present work an implementation of the Chebyshev polynomial expansion method was used for d^{ee}_{11}, d^{ee}_{12} and d^{ee}_{22} and for arbitrary plasma parameters. The 4-polynomial expansion gives approximately 0.01 accuracy in calculations of transport coefficients.

Results and discussion

The first non-zero electron-electron collision integral is d^{ee}_{11}. In the high-temperature low-density limit $d^{ee}_{11}/d^{ei}_{00} = \sqrt{2}$, in the high-degeneracy limit and in the Lorentz plasma $d^{ee}_{11}/d^{ei}_{00} = 0$. There exists a d^{ee}_{11}/d^{ei}_{00} ratio maximum at intermediate values of Θ. For a more dense plasma the maximum becomes higher and its location moves to lesser Θ. This means the significant contribution of electron-electron scattering even in the strong degenerate high density plasma.

In order to ensure the smooth transition between degenerate and non-degenerate states without the knowledge of d^{ee}_{mn} in the intermediate region of Θ, some authors ([12, 13]) postulated the coefficient R_{ee} - the ratio of the real plasma electrical conductivity to those of the Lorentz plasma. Recently attempts were made to calculate d^{ee}_{11} and R_{ee} with Debye screening [14]. On the Figure 1 the corresponding results are compared with the present ones. For $\Gamma\Theta = 5$ the value $\Theta = 1.1$ is the boundary of the fully ionized plasma stability.

From (2), (3) results for the thermopower and the heat conductivity can be obtained. Here we submit high-degeneracy limits for transport coefficients. Representing $d^{ei}_{00}(T=0) = \int\limits_0^{E_F} d(U_{ei}(E))$, $a = \frac{\alpha e}{k_B}$, $L = \frac{\lambda}{T\sigma}(\frac{e}{k_B})^2$, $d_c = \frac{3}{\pi^2}\frac{d^{ee}_{11}}{d^{ei}_{00}}$ we obtain

$$a = \frac{\pi^2}{3\beta E_F}\left(1 + 1.25\frac{1 - \frac{4}{15}\frac{E_F U'_{ei}(E_F)}{d^{ei}_{00}}}{1+d_c}\right), \qquad (9)$$

$$L = \frac{\pi^2}{3}\frac{k_B^2}{e^2}(1+d_c)^{-1} \qquad (10)$$

If $\Theta \ll 1$ and $\Gamma \gg 1$, $d_c \approx 3\frac{\Theta^{3/2}}{r_S}$. For metallic densities (r_S from 1 to 6) $d_c \approx 0.1$ for Θ from 0.1 to 0.3.

Values d^{ei}_{mn} and d^{ee}_{mn} depend only on Θ and Γ and may be expressed in closed form and used in multicomponent plasma calculations.

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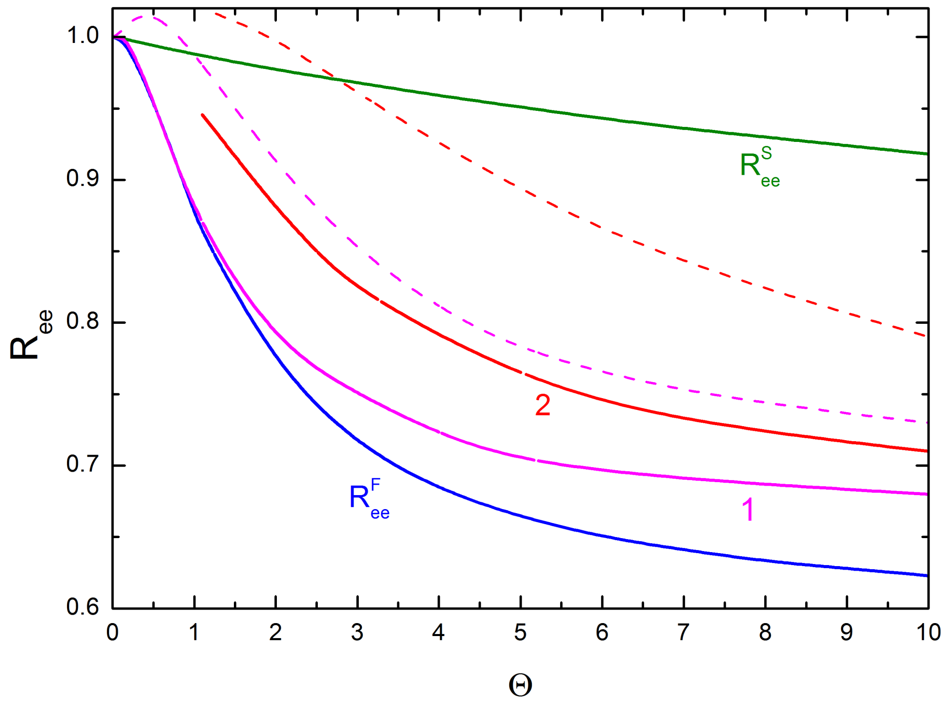

Figure 1: R_{ee}-factor in different approximations. R_{ee}^S - from [12], R_{ee}^F - from [13], 1 - $r_s = 1.842$, 2 - $r_s = 9.21$. Solid lines - present results, dashed lines - Debye screening [14].

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