Discrete-particle effects on Landau damping in particle-in-cell simulation

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Abstract

The discrete-particle effects in particle-in-cell (PIC) simulations can numerically enhance the thermalization of collisionless plasmas, such that they can potentially change the dynamic properties of the simulated plasma system. The simulation results show that the numerical fluctuation induced by discrete-particle effects can be remedied by taking ensemble average over many computer runs to obtain the Landau damping rate, which is consistent with the theoretical estimation. But the nonlinear phase trapping can only be recovered from the numerical noise by using a reasonable number of macro-particle number in a Debye region. Moreover, both Krook-type and head-on collision models are implemented in the PIC simulation for studying the Landau damping in collisional plasmas. The convergence of numerical results due to discrete-particle effects in PIC simulations will be discussed in the paper.

Key words: Particle-in-cell Simulation, discrete particle effects, numerical thermalization, particle collision, Landau damping

Particle-in-cell (PIC) simulations have been widely applied on the study of frontier research topics of plasma physics, e.g. plasma fusion, laser-plasma interactions, energetic particle sources and radiation sources et al. [1] The popularity of the PIC method is due to its easy implementation and parallelization with excellent scalability to simulate large plasma systems without the loss of the kinetic behaviors of plasmas. [2] Nowadays, the research issues related to collisional plasmas become more important, especially in the fusion research and laser- or microwave- produced plasmas. [3] The PIC method can be easily extended by including different particle collision models using the Monte Carlo methods for simulating collisional plasmas. [4]

Even though the PIC method has been successfully applied on the study of plasma physics, there remain some numerical issues that inhibit researchers from obtaining a low-noise and convergent simulation results. One of these challenging issues is the numerical thermalization due to the finite size particle effects in particle-in-cell simulations of a collisionless plasma. Even though the numerical thermalization time τ_R cannot always be quantitatively defined in simulations, it was first studied by J. Dawson using a one-dimensional electrostatic sheet model. [5] The simulation results showed that τ_R scales with $N_D{}^2$, where $N_D \equiv n_0\lambda_D$ is the number of particles per Debye length (denoted as λ_D) and n_0 is the corresponding number density of particles. Some following works demonstrated the scaling law is $\tau_R \propto N_D$ for two-dimensional (2D) PIC simulation, where $N_D \equiv n_0\lambda_D{}^2$, and $\tau_R \propto N_D/\ln N_D$ is expected for three-dimensional model with $N_D \equiv n_0\lambda_D{}^3$. [6] The scaling law has also been proved and explained by the analysis based on the Balescu-Lenard kinetic theory. [7]-[8] Moreover, Turner observed that τ_R is anomalously shortened to scale with N_D while the Monte-Carlo collisions is added in 1D PIC simulations. [9] The following theoretical work also demonstrated N_D-scaling of τ_R in 1D collisional PIC simulations using the Balescu-Lenard-Landau kinetic theory. [10] The additional Monte-Carlo collisions in 1D PIC simulations enhance the discrete-particle effect and recover the first-order thermal relaxation. These studies pointed out that the numerical criteria in collisional PIC simulations should be carefully re-examined, since the numerical thermalization could distort simulation results.

In the study, 1D electrostatic (ES) PIC simulations are performed to study the discrete particle effects on the damping of plasma waves (Landau damping). Ions are assumed to act as the immobile neutral background in simulations. A small amplitude propagating ES wave is initially excited in a plasma at the thermal equilibrium state. The initial plasma distribution in phase space is shown in Fig. 1. The simulations are performed by varying the particle number in a Debye length N_D, and the time evolution of the electric field profiles at $N_D = 10^3$ and 10^5, respectively, are shown in Fig. 2. Figure 2(a) clearly shows that the plasma oscillations are completely overwhelmed by the numerical noise induced by the discrete particle effects within a plasma period at $N_D = 10^3$. If we take 100 runs ($N_{run} = 100$) for the case with $N_D = 10^3$ and plot the time evolution of the averaged electric field profile (as shown in Fig. 2(b)), we can almost recover the result for $N_D = 10^5$ (Fig. 2(c)). The observation hints that the discrete particle effects can not only induce numerical thermalization in a plasma system with the plasma distribution which deviates from the Maxwellian distribution, but also induce random noise in the plasma system at thermal equilibrium if N_D is smaller than some critical value. The parametric study reveals that the numerical noise can be considerably reduced by properly choosing N_D and N_{run} (for taking average of field quantities) to satisfy the empirical criteria $N_D \times N_{run} \geq 10^6$. With the reduction of numerical noise induced by the discrete particle effects, the linear Landau damping rate thus can be determined by the PIC simulation (as shown in Fig. 3(a)) and the results agree well with the theoretical analysis (shown in Fig. 3(b)). But the nonlinear behavior of the plasma can only be correctly demonstrated by the simulation results by choosing a large N_D, i.e. the discrete particle effects distort the simulation results which can not be recovered by averaging method in the nonlinear stage.

If the plasma wave is excited in a collisional plasma, previous study showed that the Landau damping rate is determined by the summation of the damping rate in the collisionless plasma and the collision rate. [11] In the study, we consider two collision models, i.e. head-on collision and Krook type collision, in the 1D ES PIC simulation. The parameters N_D is 10^5 and N_{run} is 10 to reduce the numerical noise induced by the discrete particle effects. The results of the PIC simulation with head-on collision show good agreement with the theoretical analysis to the collision frequency $0.1\omega_p^{-1}$ (see Fig. 4). But the simulation results with Krook type collision deviate from the theoretical analysis at the collision frequency greater than $0.01\omega_p^{-1}$. The preliminary results echoes Turner's conclusions [9], i.e. the addition of collisional effects into the PIC simulation can enhance the complexity of its numerical convergence. The application of the Krook type collision on the PIC simulation of collisional Landau damping is a typical example.

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Acknowledgement

This is a report of work sponsored by Grant No. MOST104-2112-M-008-013-MY3 from the Ministry of Science and Technology, Taiwan. The authors would like to acknowledge the National Center for High-Performance Computing in Taiwan for providing computing resources to facilitate this research.

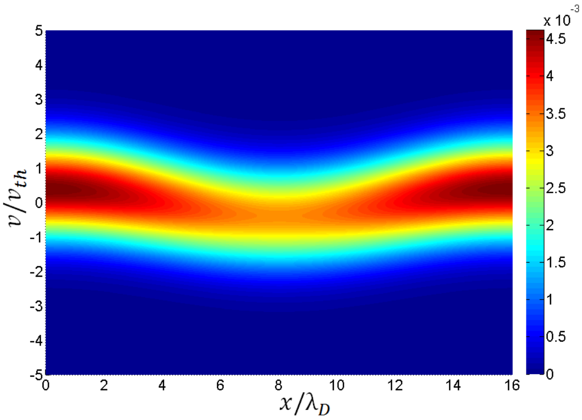

Fig. 1

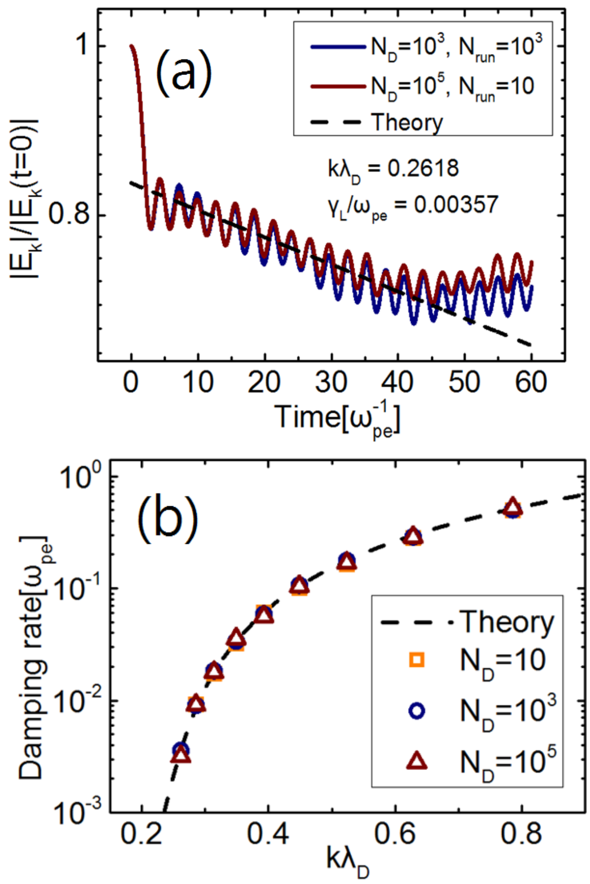

Fig. 3

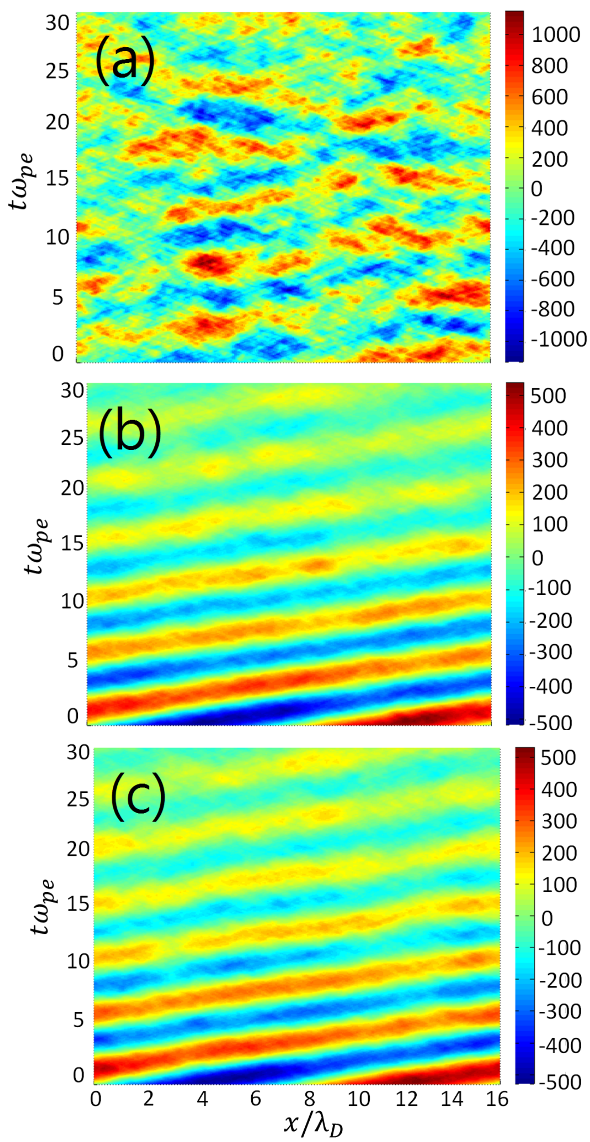

Fig. 2

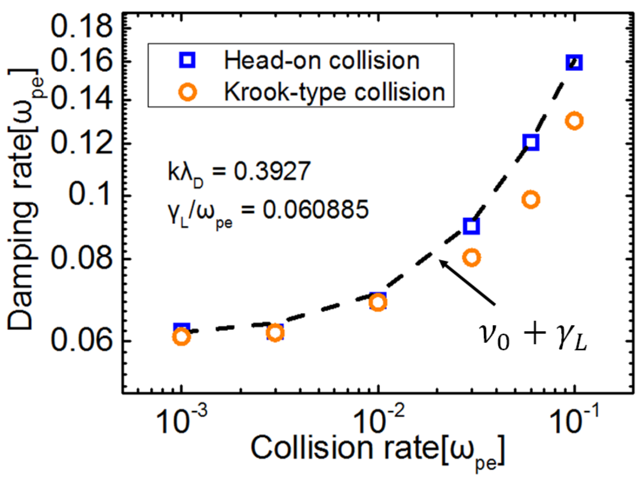

Fig. 4