Hybrid Simulations of Shocks in Weakly Collisional Systems

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Abstract

Hybrid codes have been widely used to model space physics for many years, and with growing interest in collisionless shock physics – and shock generation – we aim with this project to develop a hybrid simulation code to accurately model high energy plasma dynamics with an arbitrary degree of collisionality by exploiting recently developed pseudo-spectral methods for solving the Vlasov-Fokker-Planck equation for the ions, while adopting a fluid approach for the electrons. With this code we envisage the potential to perform a detailed numerical study of shock dynamics in the transition between the collisional and collisionless regimes, carrying out an in-depth parameter scan to identify clear and measurable signatures marking the transition between these regimes for shocks, and increasing our current predictive capability. Through insight gained from these simulations, along with modern experimental data, our currently limited understanding of shocks in the weakly collisional regime can be greatly advanced, giving us a powerful tool for modelling both laboratory and astrophysical plasmas.

Introduction

Due to their prevalence in astrophysical systems, collisionless shocks are fascinating structures to explore in the laboratory. However, producing conditions relevant to astrophysical system in the laboratory is not trivial. Of particular concern is the role of collisions, which although often not dominant, may still contribute to the dynamic fields in the plasma.

In the case of weakly collisional shock physics, the identification of clear and measurable signatures marking the transition between the collisional and collisionless regimes is of great interest and importance in furthering our understanding of collisionless shocks. This necessitates a tailor-made code to fully capture the relevant dynamics, while remaining computationally viable.

The Code

We are developing a hybrid simulation code that will accurately model high energy plasma dynamics. It will be applicable to model both laboratory and astrophysical plasmas and, as well as shock physics, could also be extended to model further phenomena such as particle acceleration, plasma fluid stability, and magnetic reconnection.

For the ions we consider a tensor expansion of the Vlasov-Fokker-Planck equation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \frac{q}{m} \mathbf{E} \cdot \frac{\partial f}{\partial \mathbf{v}} + \frac{q}{mc} \mathbf{v} \times \mathbf{B} \cdot \frac{\partial f}{\partial \mathbf{v}} = \left(\frac{\partial f}{\partial t} \right)_c \tag{1}$$

$$f(\mathbf{v}) = f_0(\mathrm{v}) + \cdot \frac{\mathbf{v}}{\mathrm{v}} \mathbf{f}_1(\mathrm{v}) + \frac{\mathbf{vv}}{\mathrm{v}^2} : f_2(\mathrm{v}) + \ldots \tag{2}$$

where, by taking the suitable angular moments, we obtain a coupled chain of equations that can be solved using a suite of numerical techniques we are free to choose.

The collisions are modelled using the Fokker-Planck collision operator, in terms of the Rosenbluth potentials:

$$\left(\frac{\delta f}{\delta t} \right)_c = \frac{\partial}{\partial \mathbf{v}} \cdot \left\langle \frac{\Delta \mathbf{v}}{\Delta t} \right\rangle f(\mathrm{v}, t) + \frac{1}{2} \sum_{i,j} \frac{\partial^2}{\partial \mathrm{v}_i \partial \mathrm{v}_k} \left[\left\langle \frac{\Delta \mathrm{v}_i \Delta \mathrm{v}_k}{\Delta t} \right\rangle f(\mathbf{v}, t) \right]. \tag{3}$$

Finally we assume an isothermal electron distribution. Then, in conjunction with an appropriate Ohmic law and the relevant Maxwell's equations, we solve the system by updating using an operator-split approach.

Results

Through simple runs of the code with only the relevant operators active, we can demonstrate that it captures the expected recognizable physical phenomena.

Drifts: In Figure 1 it is shown that the code can accurately model drifts. In this case a magnetic field and perpendicular electric field are initialized and held constant, with an initial Maxwellian isotropic ion distribution, f_0, and the drift velocity is plotted against the analytic expectation.

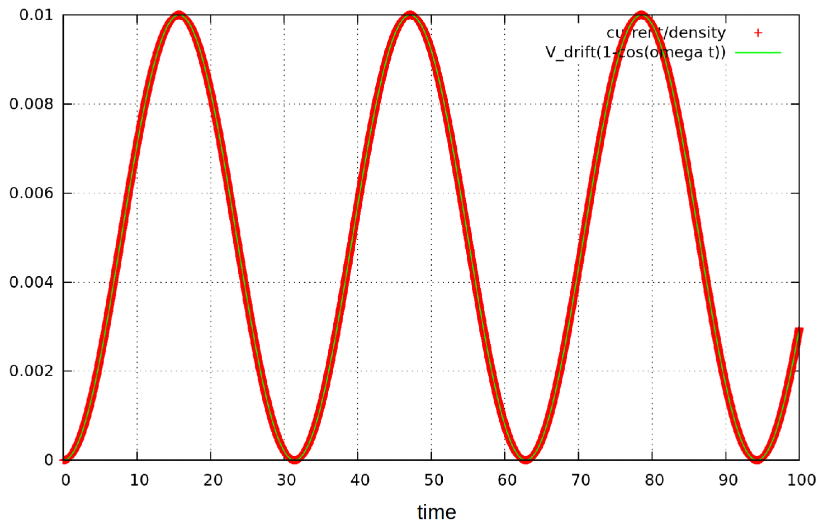

Figure 1: Evolution of f_{1z} for uniform E and B-fields. Here $E = 0.001\hat{\mathbf{x}}$ and $B = 0.2\hat{\mathbf{y}}$, such that, in our units, the $E \times B$ drift, $\mathbf{v}_d = \frac{\mathbf{E}_\perp \times \mathbf{B}}{B^2}$, is $0.005\hat{\mathbf{z}}$, with Larmor period $T = 2\pi/\omega = 10\pi$.

Isotropic Collisions: In Figure 2 we initialise f_0 as a simple Maxwellian with bump-on-tail. We then overlay the outputs at various times (in units of the characteristic relaxation time) to show that, as expected, the collisional operator causes the distribution to relax to a Maxwellian on the expected timescale, and converges stably.

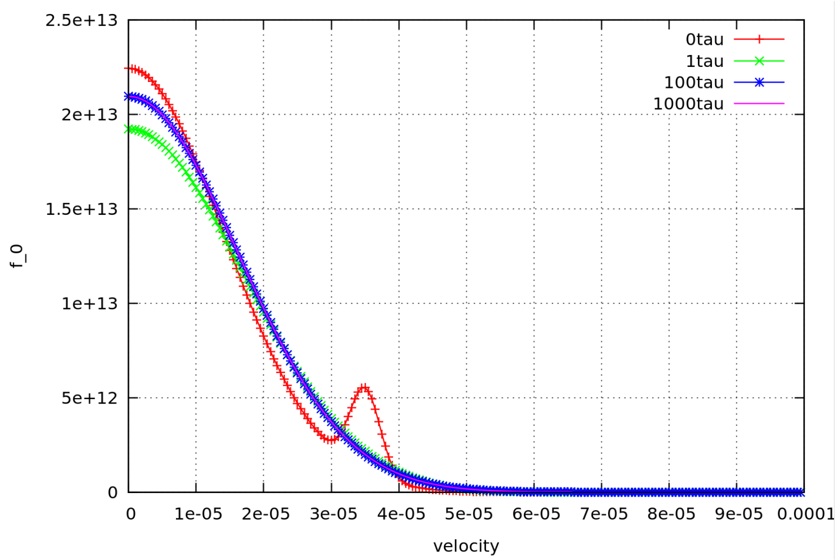

Figure 2: Relaxation of f_0 after a range of times, where the characteristic relaxation time is given by: $\tau_i = \left(\frac{4\pi\Gamma_{ii}}{3}\frac{n_i}{\sqrt{2\pi}v_t^3}\right)^{-1} = \frac{3\sqrt{m_i}(kT_i)^{3/2}}{4\sqrt{2\pi}nq^4\ln\Lambda}$

Anisotropic Collisions: In Figure 3 we initialise the distribution with a net current, with suitable f_0 and f_1, testing the accuracy of the anisotropic collision operator. We adopt a similar approach of linearised anisotropic collisions [1] which result in the current to decay exponentially. By fitting an exponential to the curve we can calculate the numerical scattering rate $\nu_s \simeq 2.67 \times 10^{-4}$, and verify it corresponds to the expected scattering rate.

In Figure 4 we test a simple particle + field solver, by simulating an ion-acoustic wave. The ions are initialised with a sinusoidal perturbation in space, and the electric field is updated using the electron pressure gradient, recovering the correct phase velocity.

Conclusion and Future work

We have demonstrated the ability of our code to solve some essential processes for 1D kinetic simulations. We are currently implementing a more complete field solver, and with appropriate modification, the code will enable detailed studies of shock dynamics in the transition between the collisional and collisionless regimes to be preformed. Many ongoing experiments designed to explore shock physics in laboratory plasma conditions [2, 3, 4] will fall in a parameter range ideally suited to this numerical approach. We have shown that the code can accurately model drifts, handle collisions, reproduce basic fluid theory results, and anticipate will be able model

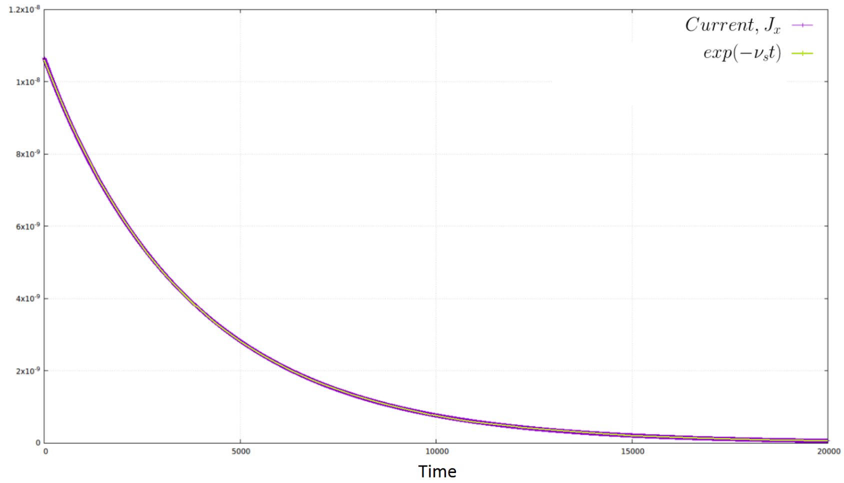

Figure 3: Plot of the current against time due to the implementation of the anisotropic collision operator, to model the effects of coulomb collisions, showing exponential decay of the initial current.

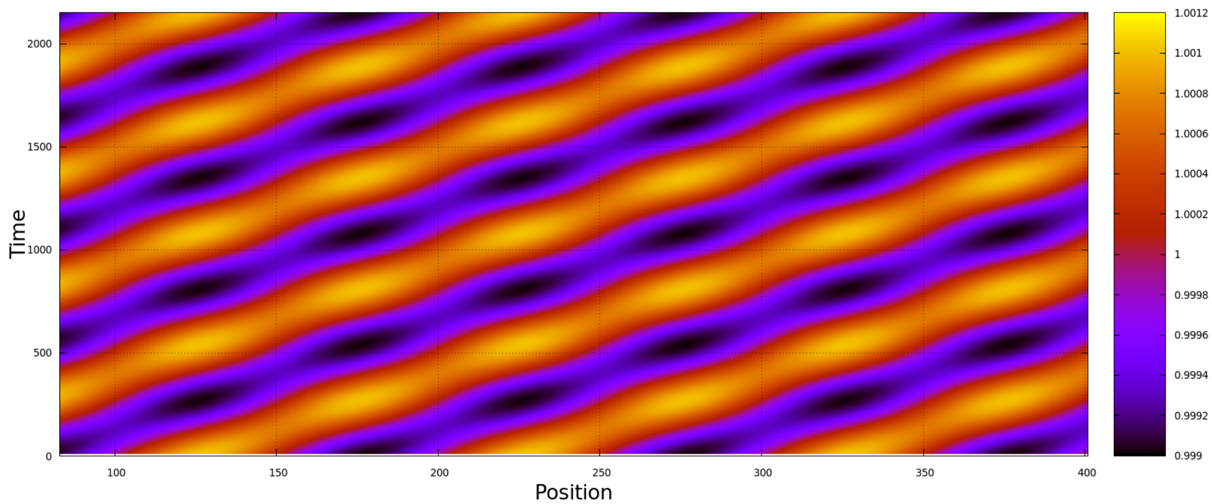

Figure 4: Output from 1D simulation (at a specific velocity), of the normalised density, showing the propagation of an ion-acoustic wave, with periodic boundary conditions. Time is displayed on the y-axis in units of the ion gyrofrequency

shock structures in the trans-collisional regime [5] in the near future. Through insight gained from these simulations, along with experimental data, our current understanding of collisionless shocks can be greatly advanced.

References

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