CASTOR3D: linear stability studies for tokamak and stellarator configurations

E. Strumberger, S. Günter

Max Planck Institute for Plasma Physics, 85748 Garching, Germany

Introduction Three-dimensional tokamak and stellarator equilibria are in the focus of present fusion research. While stellarators are characterized by a complex 3D magnetic field topology, 3D tokamaks are devices with weakly broken axisymmetry. Reasons for the asymmetry of tokamak configurations are e.g. three-dimensional resistive wall structures, which can reduce the growth rates of external modes, and magnetic perturbation fields. The latter are applied to mitigate, or even to suppress edge localized modes (ELMs). The design of 3D fusion devices, as well as the analysis and interpretation of the corresponding plasma discharges, require appropriate numerical tools that are able to handle their geometry. The newly developed, linear stability CASTOR3D code [1] is such a tool. In the following, we present stability studies for a 3D tokamak and a quasi-axisymmetric stellarator configuration, which demonstrate the large variety of its possible applications.

CASTOR3D code The CASTOR3D code is a hybrid of the linear stability CASTOR_3DW code [2] and the resistive wall mode STARWALL code [3]. Its general 3D formulation allows to use various kinds of flux coordinates (s,v,u) ($s=\rho_{tor}$ radial, v toroidal, and u poloidal direction). There is no limitation to ideal wall structures or resistive time scales, because the extended eigenvalue problem simultaneously describes the plasma perturbation and the corresponding currents in the external conducting structures. Depending on the complexity of the conducting structures, they are either discretized by a spectral method, or by triangular finite elements. The resistivity in each triangle may be different.

Several code extentions and improvements have been made in comparison to the preceding code version described in Ref. [1]. In addition to the plasma resistivity the current code version includes parallel ion viscosity, $-\Delta_{||}\pi \approx \mu_{||}\Delta_{||}\vec{v}$, and plasma flow, $\vec{V}_0 = \frac{1}{2\pi}\Omega(s)\vec{r}_{,v}$, with v being the direction of the (quasi)-symmetry (toroidal direction in cylindrical coordinates for tokamaks, and toroidal direction in Boozer coordinates for quasi-axisymmetric stellarators). The code has been fully MPI parallelized, and the non-hermitian eigenvalue problem is solved using the parallelized SLEPc-Krylov-Schur solver [4].

Stability studies First, we consider a 3D AUG-type equilibrium with an advanced q-profile (two $q=2$ surfaces). Eight upper and eight lower magnetic perturbation (MP) coils located at the inner side of the PSLs (Passive Stabilization Loops) produce an $n_p=1$ perturbation field which causes a one-periodic corrugation, δ_N, of the flux surfaces as shown in Fig. 1. The 3D and the corresponding axisymmetric equilibrium (MP coils off) are unstable with respect to a vertical instability and coupled tearing modes. Figure 2 shows the vertical instability of the 3D equilibrium (left), and an $n = 1$-type double tearing mode (right). The growth rates of these modes as functions of the specific wall resistivity, η_w, and the toroidal rotation frequency, Ω_0, are depicted in Fig. 3.

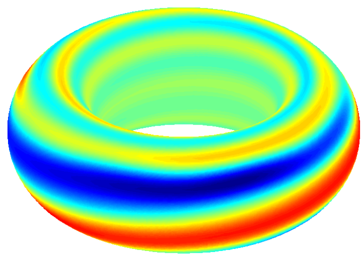
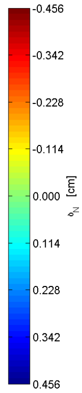



Figure 1: *Contour plot of δ_N on the plasma surface*

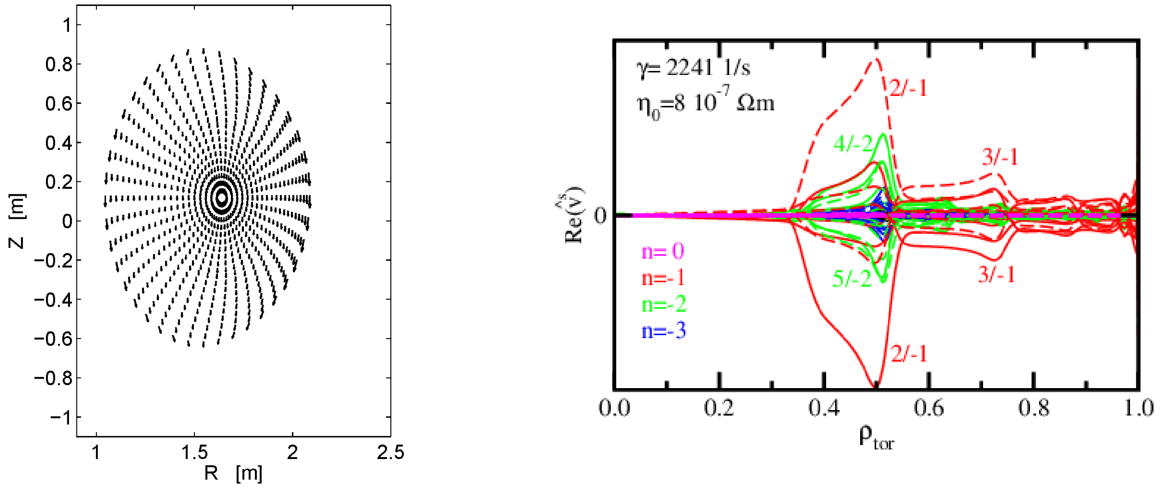


Figure 2: *(left) Mode structure of the vertical instability. (right) Real part of the eigenfunction Fourier spectrum of the radial velocity in NEMEC coordinates [1] for a $n = 1$ double tearing mode (DTM). The solid and dashed lines mark the contributions of the complex and conjugate-complex eigenfunctions, respectively. The largest contributions are marked by their poloidal and toroidal harmonics, m/n.*

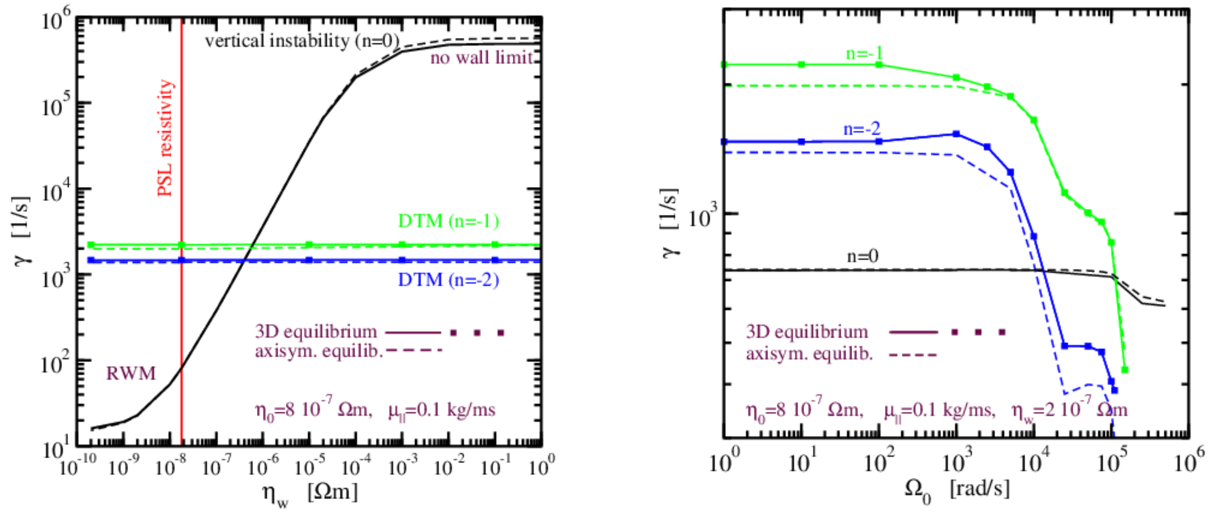
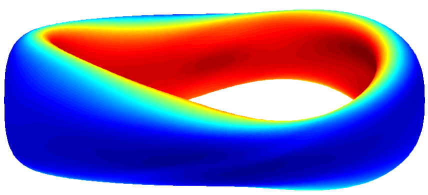


Figure 3: *(left) Growth rate as function of the specific wall resistivity, η_w. The vertical red line marks the specific resistivity of the PSLs installed in AUG. (right) Growth rates as function of the rotation frequency Ω_0 (rotation profile: $\Omega(s) = \Omega_0(1 - 0.7s^2)$.*

Although, the corrugation is relatively small around the $q = 2$-surfaces ($|\delta_N|_{\max} \approx 0.7$ cm) at least four n-harmonics couple together as shown in Fig.2 (right). In contrast to the axisymmetric equilibrium the eigenvalues of the two existing orthogonal solutions are no longer degenerated. In Fig. 2 the complex and conjugate-complex Fourier harmonics of the $n=1$ DTM have opposite signs, and the eigenvalue amounts to $\gamma = 2241$ 1/s. Not shown is the case with $\gamma = 2228$ 1/s (equal signs of the harmonics). However, only one eigenvalue exists for the vertical instability because of its $n=0$ character. While the PSLs have a stabilizing effect on the vertical mode, a differential rotation of the plasma reduces the growth rates of the $n=1$, and $n=2$-type DTMs (see Fig. 3). Due to the solution of an extendend eigenvalue problem, the CASTOR3D code handles the huge variation (five orders of magnitude) of the growth rate of the vertical instability, which reaches from the no wall limit to the resistive wall mode (RWM) time scale. In Fig. 3 the two eigenvalues of one mode type obtained for the 3D equilibrium are marked by squares and solid lines, respectively. Since the corrugation of the considered AUG-type equilibrium is small, these eigenvalues are almost the same. However, the eigenvalues of the axisymmetric and the 3D equilibria vary within 5-15%.

Next, we consider a two-period, quasi-axisymmetric stellarator configuration with large boostrap current, aspect ratio $A=3.7$, plasma beta $<\beta>= 2.95\%$, plasma current $I_p = 8.89$ MA, and $q=1.79$ at the plasma boundary. Figure 4 shows the contour of the magnetic field strengh, B, on the plasma surface. It is $B(s,v,u) \approx B(s,u)$ with v and u being the toroidal and poloidal angles in Boozer coordinates.



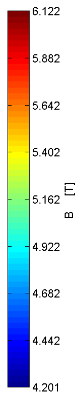


Figure 4: *Contour plot of B on the plasma surface.*

This two-periodic stellarator configuration is unstable with respect to ideal, external kink modes of the odd ($n=1,3,5,\ldots$), and even ($n=0,2,4,\ldots$) mode families as shown in Fig. 5.

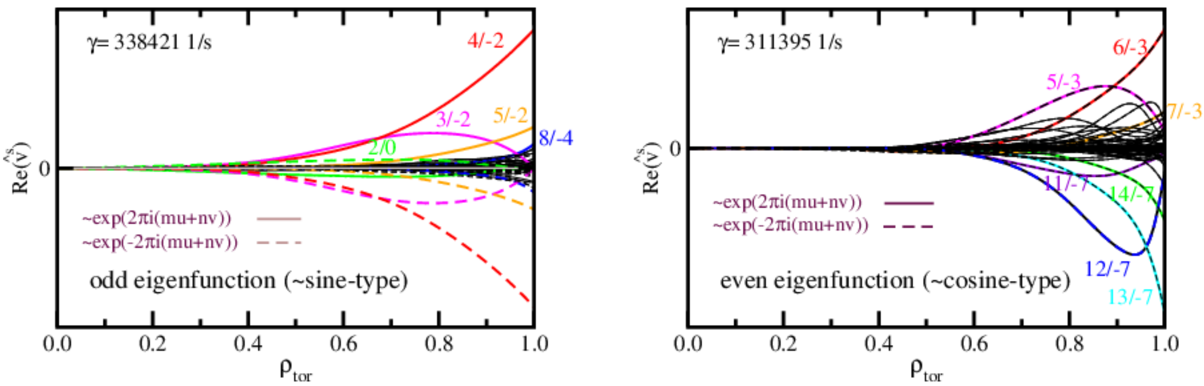


Figure 5: *Real part of the eigenfunction Fourier spectrum of the radial velocity of a sine-type mode structure of the even mode family (left), and a cosine-type mode structure of the odd mode family (right). The solid and dashed lines denote the contributions of the complex and conjugate-complex eigenfunctions, respectively. The largest contributions are marked by their poloidal and toroidal harmonics, m/n.*

Due to the stellarator symmetry of this quasi-axisymmetric configuration, the two eigenvalues of a mode-type belong to a sine- and a cosine-type eigenfunction, respectively. That is, the harmonics of the complex and conjugate-complex harmonics are equal, but with opposite sign in case of the sine-type eigenfunction (see Fig. 5). Taking into account 6 n-harmonics and 13 m-harmonics per n, the solutions yielded several eigenvalues for each mode family. Figure 6 shows the growth rates as function of the parallel viscosity coefficient, $\mu_{||}$, for some modes of the odd mode family. The parallel viscosity has a stabilizing effect on these modes. However, the effect on the $n = 1$ mode type with even eigenfunction seems to be largest. Here, the mode type of the most unstable mode changes with increasing viscosity. Furthermore, effects of plasma resistivity, plasma flow, and resistive wall structures on the stability properties of stellarators can be studied in detail with the CASTOR3D code.

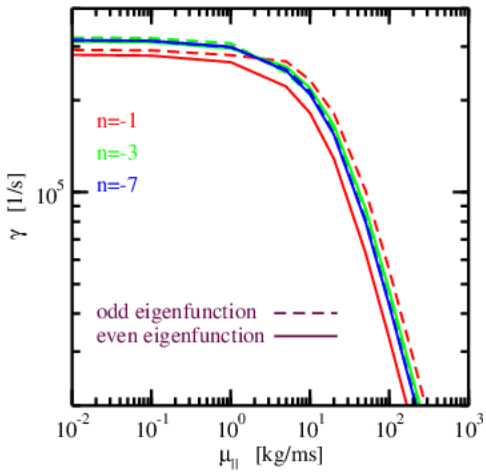


Figure 6: *Growth rates as function of $\mu_{||}$ for modes of the odd mode family.*

Summary and outlook Numerous modifications and extensions of the CASTOR_3DW and STARWALL code parts led to a synergistic effect. The possible applications of the CASTOR3D code exceed easily the possibilities of both of them. The code has a number of significant advantages. It allows: (i) to choose between various kinds of flux coordinates, (ii) to perform ideal and resistive stability studies for 3D equilibria, (iii) to take plasma inertia and resistive walls simultaneously into account, (iv) to study the effects of plasma rotation and viscosity on the stability properties, (v) to investigate vertical instabilities, and (vi) to deal with coils and multiply-connected wall structures. The MPI parallelization of the code allows an efficient solution of large eigenvalue problems. The implementation of diamagnetic drift and anisotropic heat conductivity terms is in progress.

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