Average flows and stochastic islands in the magnetic field line random walk

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Particle stochastic transport and acceleration are fundamental phenomena in the universe. In particular, turbulent magnetic fields exist at all scales and determine complex transport processes of the cosmic rays. The essential role is played by the magnetic field line trajectories, which represent a constraint on particle motion. Particle transport is completely determined by the field line random walk (FLRW) at small energies when particles are tied to the magnetic lines. Particle collisions, the intrinsic drift determined by curvature or gradient of **B**, or finite Larmor radius effects determine particle departure from the field lines and modify the transport. However, the FLRW remains the fundamental process for understanding particle transport.

We present here a study of the FLRW in the frame of the 2-dimensional model that consists of a magnetic field

$$\mathbf{B}(\mathbf{x},z) = \mathbf{B}_0 + \mathbf{b}(\mathbf{x},z), \tag{1}$$

where $\mathbf{B}_0 = B_0\mathbf{e}_z$ is the mean field and $\mathbf{b}(\mathbf{x},z)$ is a stochastic magnetic field that is perpendicular on $\mathbf{B}_0$. The stochastic component depends on both the perpendicular $\mathbf{x} \equiv (x_1,x_2)$ and the parallel coordinates. Usually, B_0 is constant. This is a simplification because there is space variation of B_0. The model is extended here by introducing large scale gradients of B_0 with characteristic lengths that are large compared to the correlation distances of the stochastic component. We also consider a small average component $\mathbf{B}_{0y} = B_{0y}\mathbf{e}_y$ perpendicular on $\mathbf{B}_0$. The main magnetic field is modeled by

$$\mathbf{B}_0 = B_0 \exp\left(\frac{x_1}{L_x} + \frac{z}{L_z}\right)\mathbf{e}_z + B_{0y}\mathbf{e}_y. \tag{2}$$

We use the perpendicular correlation length $\lambda_\perp$ as unit of the distances in all direction and of L_x, L_y and λ_z, the parallel correlation length of $\mathbf{b}(\mathbf{x},z)$.

The condition $\nabla\cdot\mathbf{B}(\mathbf{x},z) = 0$ imposes an average field in the perpendicular plane in order to compensate the parallel gradient of $\mathbf{B}_0$. The 2-dimensional stochastic field $\mathbf{b}$ is determined from a scalar function $a(\mathbf{x},z)$, the magnetic potential, as $\mathbf{b}(\mathbf{x},z) = \nabla_\perp \times a(\mathbf{x},z)\mathbf{e}_z$, where $\nabla_\perp$ is the gradient in the (x_1,x_2) plane and $a(\mathbf{x},z)$ is assumed to be a homogeneous Gaussian field, with zero average and the Eulerian correlation (EC) :

$$E(\mathbf{x},z) \equiv \langle a(\mathbf{0},0)\, a(\mathbf{x},z)\rangle = A^2 \exp\left(-\frac{x_1^2}{2} - \frac{x_2^2}{2} - \frac{z^2}{2\lambda_z^2}\right). \tag{3}$$

Here $\langle ... \rangle$ is the average over the realizations of the stochastic potential a and A is its mean square value. The dimensionless equations for the magnetic field lines are

$$\frac{dx_1}{dz} = R \exp\left(-\frac{x_1}{L_x} - \frac{z}{L_z}\right) \frac{\partial a(\mathbf{x}, z)}{\partial x_2} - \frac{x_1}{L_z}, \tag{4}$$

$$\frac{dx_2}{dz} = R \exp\left(-\frac{x_1}{L_x} - \frac{z}{L_z}\right) \left(-\frac{\partial a(\mathbf{x}, z)}{\partial x_1} + B_m\right) - \frac{x_2}{L_z}, \tag{5}$$

where $R = A/(B_0\lambda_\perp)$ is the ratio of the amplitude of the stochastic magnetic field and of B_0 and $B_m = B_{0y}\lambda_\perp/A$. The last terms appear due to the z-dependence of $\mathbf{B}_0$ and they ensure the zero divergence condition of the total magnetic field. The divergence "velocity" in Eq. (4) is not zero in the presence of the gradients (finite L_x and/or L_z).

The aim of present study is to determine the effects of the gradients of the mean field (2) on FLRW.

We use the decorrelation trajectory method (DTM, [1], [2]) for determining the statistics of the magnetic field lines described by Eq. (4). This a semi-analytical approach based on the decorrelation trajectories (DTs), which are determined from the EC of the stochastic potential. The method was used for FLRW in the case of a constant mean magnetic field $B_0\mathbf{e}_z$ [3], [4]. The method is able to describe both the random and the quasi-coherent components of the field lines and to analyze the nonlinear affects in the FLRW.

We have shown that the nonlinear FLRW that correspond to large magnetic Kubo numbers $K_m = R\lambda_z/\lambda_\perp$ is characterized by the trapping of the magnetic lines, which generates quasi-coherent structures and decreases the diffusion. Trapping is due to the Hamiltonian structure the magnetic field line equations. The magnetic potential is conserved when $\lambda_z,\ K_m \to \infty$, and the magnetic lines wind on the contour lines of $a(\mathbf{x})$. The diffusion coefficients are zero in this limit and the process is subdiffusive. A weak variation of the magnetic filed along z transforms trapping into a local process and the FLRW becomes diffusive due to the fraction of the magnetic lines that are not trapped. Trapping determines solenoidal segments of the magnetic lines, which are uniformly distributed on each field line and are separated by segments that perform much larger displacements. The trapped magnetic field lines form localized quasi-coherent structures, which are similar to the magnetic islands since they consists of magnetic line winding around some local axis. These stochastic magnetic islands have the length L of the order of λ_z and an average transversal size ρ that is an increasing function of K_m.

The DTM is able to describe FLRW in the presence of trapping due to the main ingredient of this theory, which is the set of DT's. The equations for these trajectories have the same structure as the equations for each field line. The invariants and the topology of the real field lines are thus represented by the DT's, which show the characteristics of the decorrelation process.

In the present case, the equations for the DT's $\mathbf{X}(z)$ are

$$\begin{aligned}\frac{dX_1}{dz} &= R\,\exp\left(-\frac{X_1}{L_x}-\frac{z}{L_z}\right)\frac{\partial A^S(\mathbf{X}\exp\left(-z/L_z\right),z)}{\partial X_2}, \\ \frac{dX_2}{dz} &= R\,\exp\left(-\frac{X_1}{L_x}-\frac{z}{L_z}\right)\left(-\frac{\partial A^S(\mathbf{X}\exp\left(-z/L_z\right),z)}{\partial X_1}+B_m\right),\end{aligned} \tag{6}$$

where $A^S(\mathbf{x},z)$ is the average potential in subensembles S of the realizations of the stochastic functions that have given values of the potential $a(\mathbf{x},z)$ and of the magnetic field $\mathbf{b}(\mathbf{z},z)$ in the point $\mathbf{x}=\mathbf{0}$, $z=0$, which is taken as the "initial" condition for the field lines

$$a(\mathbf{0},0)=a^0, \quad \mathbf{b}(\mathbf{0},0)=\mathbf{b}^0. \tag{7}$$

It is obtained using conditional averages as a function of the EC [1], [3]

$$A^S(\mathbf{x},z)\equiv\langle A(\mathbf{x},z)\rangle_S=a^0E(\mathbf{x},z)-b_1^0\frac{\partial E(\mathbf{x},z)}{\partial x_1}+b_2^0\frac{\partial E(\mathbf{x},z)}{\partial x_2}. \tag{8}$$

The Lagrangian statistical averages are obtained using the DTs and their probabilities $P(\mathbf{b}^0,a^0)$ of having $\mathbf{b}^0,a^0$ at $\mathbf{x}=\mathbf{0}$ and $z=0$. For instance, the running ($z-$dependent) diffusion coefficients are

$$D_i(z)=R\iint da^0 \mathbf{db}^0 P(\mathbf{b}^0,a^0)b_i^0X_i^S(z). \tag{9}$$

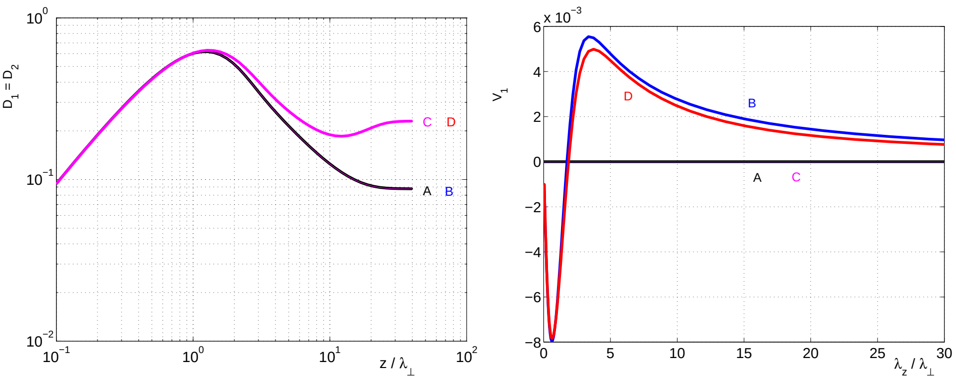

Figure 1: Effects of the main field gradients on FLRW for $B_m=0$: the running diffusion coefficients (left panel) and the flow velocity (right panel).

The effects of the gradients can be deduced from the typical results presented in the Figures 1 and 2 for $B_m=0$ and $B_m=0.2$, respectively. The black lines correspond to the basic model with constant main field B_0 (Case A with $L_x\to\infty$, $L_z\to\infty$), the blue curves are for $B_0(x)$ (Case b with $L_x=20$, $L_z\to\infty$), the magenta lines for $B_0(z)$ (Case C with $L_x\to\infty$, $L_z=20$) and the red lines for $B_0(x,z)$ (Case D with $L_x=20$, $L_z=20$). The other parameters are $R=1$ and $\lambda_z/\lambda_\perp=10$, which corresponds to $K_m=10$ (nonlinear regime).

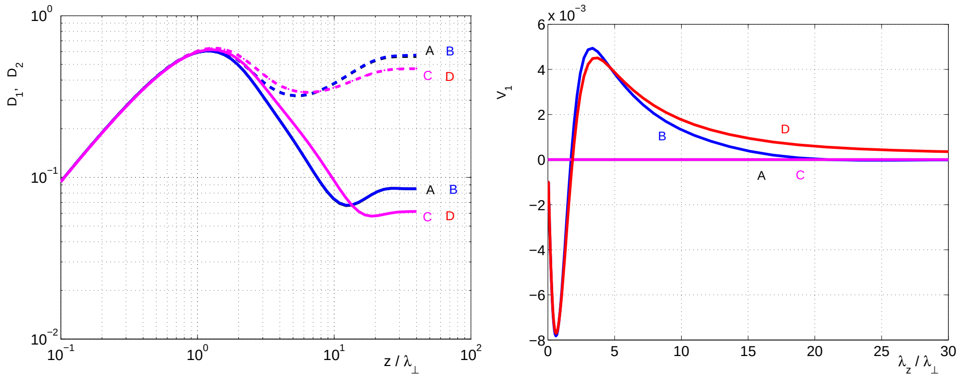

Figure 2: Same as in Figure 1, but for $B_m = 0.2$

One can see in Figure 1 (left panel) that the running diffusion coefficients $D_1(z) = D_2(z)$ are the same for the cases $(A, \ B)$ and $(C, \ D)$, which demonstrates that a small gradient along x_1 does not influence the diffusion of the field lines. On the contrary, the presence of the parallel gradient (in the cases C, D) determines a significant increase of the diffusion.

Magnetic line flow (an average "velocity" V_1) is generated by the perpendicular gradient of $\mathbf{B}_0$, as seen in Figure 1 (right panel), where V_1 is represented as function of the parallel correlation length. The sign of V_1 is negative in the quasilinear regime (that corresponds to $\lambda_z/\lambda_\perp < 1$ in the figure) and positive (along the gradient) in the nonlinear turbulence. The parallel gradient determines the decrease of V_1.

The average magnetic field $B_m \mathbf{e}_y$ makes the FLRW anisotropic by strongly increasing the diffusion coefficient along its direction and decreasing the perpendicular diffusion (Figure 2, left panel). The influence of the parallel gradient of $\mathbf{B}_0$ on the diffusion process persists, but with opposite effect: both D_1 and D_2 are decreased. The flow of the magnetic field lines is still present (Figure 2, right panel), but with smaller V_1 and with a much stronger decay due to the parallel gradient of $\mathbf{B}_0$. A synergistic effect of the gradients and of the average magnetic field B_m appears.

In conclusion, the parallel and the perpendicular gradients of the mean magnetic field have different effects on the FLRW in the nonlinear regime: significant modifications of the diffusion and, respectively, generation of magnetic line flows.

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References

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