Transport theory of phase space zonal structures

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Abstract

A set of equations is derived that describes the transport of particles and energy in a thermonuclear plasma on the energy confinement timescale. The equations thus derived allow to study collisional and turbulent transport self-consistently retaining the effect of magnetic field geometry without assuming any scale separation between fluctuations and the reference state. In a previous article [1], transport equations holding on the reference state lengthscale have been derived using the moment approach introduced in [2]. Furthermore it has been shown how this approach is not suitable for the description of smaller length-scales. In this work, this analysis is extended to micro- and meso-scales adopting the framework of phase space zonal structure theory [3, 9]. Previous results are recovered in the long wavelength limit and, in the general case, transport equations in the phase space for particles and energy are obtained that correctly take into account meso-scale structures.

Introduction

Describing the evolution of macroscopic plasma profiles on long time scales requires to treat, on the same footing, transport induced by Coulomb collisions and by turbulent fluctuations. A first principle approach is recommended and several theories have been proposed, see e.g. Ref. [4]. Following this framework, in a previous article [1], we have derived a set of equations governing particle and energy transport on the energy confinement time using standard first order gyrokinetic theory. The result is consistent with Ref. [4] and the relevant transport equations reduce to the ones found in that work if we introduce appropriate spatiotemporal averages consistently with the separation of scales between plasma reference state and fluctuating quantities assumed therein. The derivation of a transport theory that does not rely on this assumption, allowing, thus, to describe profile corrugations on the spatiotemporal scales self-consistently generated by the plasma, is the scope of this work.

Fundamental equations

In this work we study transport processes in strongly magnetized plasmas and, therefore, for each species, the particle distribution function can be written as the sum of a reference distribution function F_0 and a small perturbation δf, where the characteristic (macroscopic) lengthscale of variation of F_0, i.e. L, is such that $\delta f/F \sim \rho/L \sim \delta \ll 1$ and ρ is the Larmor radius. We further assume the so-called drift ordering, i.e. $cE/B_0 v_{th} \sim \mathscr{O}(\delta)$, where v_{th} is the particle thermal speed, and other symbols are standard. Electromagnetic fields are written as the sum of reference fields, self-consistently determined within the reference state varying on the equilibrium lengthscale L, and of fluctuations. We assume axisymmetry of the reference state and, therefore, without loss of generality, the reference magnetic field can be written as $\boldsymbol{B}_0 = F\boldsymbol{\nabla}\phi + \boldsymbol{\nabla}\phi \times \boldsymbol{\nabla}\psi$. Following Ref. [5], we adopt the gyrokinetic ordering for fluctuating quantities, i.e. $|\partial_t|/|\Omega| \sim |\delta B/B_0| \sim \boldsymbol{\nabla}_\parallel/\boldsymbol{\nabla}_\perp \sim \boldsymbol{k}_\parallel/\boldsymbol{k}_\perp \sim \mathscr{O}(\delta)$, where Ω is the particle cyclotron frequency in the reference state magnetic field. Consistently with the scope of this work, we formally define the leading order plasma response to zonal structures, i.e. the component of the distribution function undamped by collisionless processes, see e.g. Ref. [3, 9], in term of its adiabatic and non-adiabatic components: $\delta f_z = e^{-\boldsymbol{\rho}\cdot\boldsymbol{\nabla}}\delta\bar{G}_z + \frac{e}{m}\delta\phi_{0,0}\frac{\partial\bar{F}_0}{\partial\mathscr{E}}$ where $\mathscr{E} = v^2/2$ is the energy per unit mass, μ is the magnetic moment adiabatic invariant $\mu = v_\perp^2/(2B_0)$ and the $0,0$ subscript to $\delta\phi$, i.e. the electrostatic potential, denotes the $m = n = 0$ component with m and n being respectively the poloidal and toroidal mode numbers of the fluctuation. We also assume that the equilibrium guiding center distribution is isotropic, that $\partial_\mu \bar{F}_0 = 0$, and that the usual low-β tokamak ordering applies. The (leading order) non-adiabatic gyrocenter plasma response to zonal structures $\delta\bar{G}_z$, is obtained solving the first order nonlinear gyrokinetic equation [5, 6]:

$$\left(\partial_t + v_\parallel \nabla_\parallel + \boldsymbol{v}_d \cdot \boldsymbol{\nabla}\right)\delta\bar{G}_z = -\frac{e}{m}\frac{\partial\bar{F}_0}{\partial\mathscr{E}}\frac{\partial}{\partial t}\left\langle\delta\psi_{gc}\right\rangle_z - \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}\Big|_z , \tag{1}$$

where:

$$\left\langle\delta\psi_{gc}\right\rangle_z = \hat{I}_0\left(\delta\phi_{0,0} - \frac{v_\parallel}{c}\delta A_{\parallel 0,0}\right) + \frac{m}{e}\mu\hat{I}_1\delta B_{\parallel 0,0}, \tag{2}$$

the last term of Eq. (1) is composed only by the product of fluctuations with opposite toroidal mode number, $\hat{I}_n(x) \equiv (2/x)^n J_n(x)$ [7], $J_n(x)$ are the Bessel functions, $\lambda^2 \equiv 2(\mu B_0/\Omega^2)k_\perp^2$ and the definition of $\hat{I}_n$ acting on a generic function $g(\boldsymbol{r}) = \int d\boldsymbol{k}\hat{g}(\boldsymbol{k})\exp(i\boldsymbol{k}\cdot\boldsymbol{r})$ is the following $\hat{I}_n g(\boldsymbol{r}) \equiv \int d\boldsymbol{k} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\hat{I}_n(\lambda)\hat{g}(\boldsymbol{k})$. This equation states that zonal structures are driven by zonal fields, i.e. fields with $n = m = 0$, and by nonlinear coupling between the gyro-center response and the perpendicular gradient of the fluctuating fields that generate terms with the same property.

Transport equations

Up to the leading order in δ, the linear part of the particle free streaming operator can be written, see Ref. [8], as:

$$\partial_t + v_\parallel\nabla_\parallel + v_\parallel\nabla_\parallel\left(\frac{Fv_\parallel}{\Omega}\right)\frac{\partial}{\partial\psi} . \tag{3}$$

We introduce the (drift/banana center) response $\delta\bar{g}_z$, such that $\delta\bar{G}_z = e^{-iQ_z}\delta\bar{g}_z$, and impose:

$$i\nabla_{\parallel}Q_z = i\nabla_{\parallel}\left(\frac{Fv_{\parallel}}{\Omega}\right)\frac{k_z}{d\psi/dr}\;; \tag{4}$$

where $k_z \equiv (-i\partial_r)$ in order to simplify the nonlinear gyrokinetic equation. Integrating Eq. (4) we obtain the following expression:

$$Q_z = F(\psi)\left[\frac{v_{\parallel}}{\Omega} - \overline{\left(\frac{v_{\parallel}}{\Omega}\right)}\right]\frac{k_z}{d\psi/dr}\,, \tag{5}$$

where we have introduced the "bounce" average along unperturbed particle orbits: $\overline{[\ldots]} \equiv \tau_b^{-1}\oint d\ell/v_{\parallel}[\ldots]$ and τ_b is the time required for particles to complete an (integrable) close poloidal orbit in the reference magnetic field. We can therefore rewrite Eq. (1) as

$$\left(\partial_t + v_{\parallel}\nabla_{\parallel}\right)\delta\bar{g}_z = e^{iQ_z}\left(-\frac{e}{m}\frac{\partial\bar{F}_0}{\partial\mathscr{E}}\frac{\partial}{\partial t}\left\langle\delta\psi_{gc}\right\rangle_z - \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}\right)\,. \tag{6}$$

The requirement for the phase space zonal structure to be long lived, i.e. that it annihilates the linear part of the free streaming operator, imposes that $\nabla_{\parallel}\delta\bar{g}_z = 0$. The pullback operator Q_z does not depend on the ϕ coordinate and, therefore, we obtain that $\delta\bar{g}_z$ must be toroidally symmetric and characterized by $m = 0$. Therefore the equation governing the evolution of $\delta\bar{g}_z$ is:

$$\partial_t\delta\bar{g}_z = \overline{\left[e^{iQ_z}\left(-\frac{e}{m}\frac{\partial\bar{F}_0}{\partial\mathscr{E}}\frac{\partial}{\partial t}\left\langle\delta\psi_{gc}\right\rangle_z - \frac{c}{B_0}\boldsymbol{b}\times\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\cdot\boldsymbol{\nabla}\delta\bar{G}\right)\right]}. \tag{7}$$

It can be shown that the following expression holds for any velocity function:

$$\langle\langle f\rangle_v\rangle_{\psi} = \frac{4\pi^2}{V'}\sum_{v_{\parallel}/|v_{\parallel}|=\pm}\int d\mu d\mathscr{E}\,\tau_b\overline{f_{n=0}} \tag{8}$$

where we have indicated the flux surface average with $\langle\ldots\rangle_{\psi}$, and the integral in the velocity space with $\langle\ldots\rangle_v$. This result shows that the flux surface average of a velocity integral depends only on the bounce averaged response of the $n = 0$ toroidal Fourier harmonic at the leading order in the asymptotic expansion. In the presence of fluctuations in the gyro-center particle distribution, the drift/banana-center non-adiabatic particle response yields the following form of the phase space zonal structure [3, 9]:

$$\overline{\langle\delta f_z\rangle} = \overline{\left(e^{-iQ_z}\hat{I}_0\right)}\delta\bar{g}_z + \frac{e}{m}\delta\phi_{0,0}\frac{\partial\bar{F}_0}{\partial\mathscr{E}}. \tag{9}$$

Acting on this expression by ∂_t, substituting Eq. (7) and integrating in velocity space (see Ref. [8] for the detailed calculation) we obtain:

$$\begin{aligned}\partial_t\langle\langle\delta f_z\rangle_v\rangle_{\psi} = {} & \frac{e}{m}\partial_t\delta\phi_{0,0}\left\langle\left[1-\left(e^{-iQ_z}\hat{I}_0\right)\overline{\left(e^{iQ_z}\hat{I}_0\right)}\right]\frac{\partial\bar{F}_0}{\partial\mathscr{E}}\right\rangle_v \\ & -\frac{1}{V'}\frac{\partial}{\partial\psi}\left\langle\left\langle V'\left(e^{-iQ_z}\hat{I}_0\right)\overline{\left[ce^{iQ_z}R^2\boldsymbol{\nabla}\phi\cdot\boldsymbol{\nabla}\left\langle\delta\psi_{gc}\right\rangle\delta\bar{G}\right]}\right\rangle_v\right\rangle_{\psi}.\end{aligned} \tag{10}$$

This is the gyrokinetic extension of Eq. (15) in Ref. [1], and is valid for corrugations of the reference state characterized by a lengthscale up to the particle Larmor radius, provided that the well-known gyrokinetic ordering is preserved. As anticipated above, collisional transport is suppressed here but could be readily restored; e.g., by adopting a suitable gyro-averaged collision operator [10]. The third term on the RHS is the long time scale effect (not related to collisionless processes) of turbulent transport. Mesoscales in the density profile are spontaneously produced by this term. Taking the long wavelength limit of this expression, i.e. $\left(e^{iQ_z}\hat{I}_0\right) \to 1$, we recover the transport equations discussed in Ref. [8]. The expression for the energy transport is obtained with the same procedure, which yields heat fluxes that are weighted by $mv^2/2$.

Conclusions

In this work, we have derived transport equations valid on the energy confinement time scale adopting the framework of phase spaces zonal structures theory [3, 9]. The governing equations allow to describe multiple spatiotemporal scales generated by turbulent mode-mode couplings eventually invalidating the hypothesis of scale separation between reference state and fluctuations, see e.g. Ref. [4]. Furthermore, we have shown that the relevant transport equations in the long wavelength limit produce fluctuation induced fluxes consistent with Ref. [1]. These results allow to extend the concept of plasma reference state to self-consistently include spatiotemporal meso-scales produced by phase space zonal structures [11].

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