The tearing instability in relativistic magnetohydrodynamics

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Abstract

Magnetic energy dissipation in relativistic plasmas is a crucial process operating in many environments typical of high-energy astrophysics, such as pulsar winds and nebulae, magnetars, and magnetized disks around black holes. In many cases such dissipation is required to be of explosive type, given that flaring activity is often observed in such objects in the form of sudden releases of gamma rays. Here we discuss the role of the aspect ratio of the reconnecting current sheet, which, as for classic and Hall magnetohydrodynamics (MHD), when sufficiently small it is known to lead to a very rapid evolution of the spontaneous tearing instability and to explosive secondary reconnection events (*super-tearing* or *plasmoid instability*). Multi-dimensional simulations of resistive, relativistic MHD are presented for various magnetizations and plasma betas, and 2D results show a quasi-universal evolution, occurring on the *ideal* (relativistic) Alfvén time.

Introduction

Relativistic plasmas are ubiquitous in the field of high-energy astrophysics, from the conditions in the early Universe, to the interiors, magnetospheres and winds of neutron stars (NSs), to the engines and outflows powering gamma-ray bursts (GRBs) or active galactic nuclei (AGNs). Short transient outbursts of energy release characterize many of these objects, and the most important examples are certainly the so-called soft gamma repeaters (SGRs) and anomalous X-ray pulsars, two manifestations of *magnetars* [16], and the gamma-ray flares observed in the Crab nebula [22, 18, 15]. In these cases the natural explanation is that efficient magnetic reconnection in a magnetically-dominated relativistic plasma is at work, though a detailed modeling is hard to achieve since the rise times of the flaring events require instabilities occurring on just a few light-crossing times of the typical size of the source.

The classical mechanism invoked to trigger reconnection in plasmas is the *tearing instability* [8], for which it is known that the linear growth rate, normalized to the (ideal) Alfvén time $\tau_a = a/c_A$ is $\gamma\tau_a \simeq 0.6 S_a^{-1/2}$, where c_A is the Alfvén speed proportional to the magnetic field strength, a is the characteristic width of the equilibrium current sheet,

$S_a = ac_A/\eta$ is the Lundquist number, usually very large, and η is the plasma resistivity. This rate is typically far too small to explain classical reconnection events (e.g. the solar flares), unless the reconnecting current sheets are very thin $a \ll L$, with L the macroscopic sheet's length on which the new normalizing quantities τ and S are re-computed [1]. In this case $\gamma\tau \simeq 0.6\,(L/a)^{3/2}S^{-1/2}$, and if during a dynamical thinning process the critical threshold $a/L \sim S^{-1/3}$ is reached, the growth rate becomes *ideal* [20], that is $\gamma\tau \simeq 0.6$ or

$$\gamma \simeq 0.6\,c_A/L, \tag{1}$$

independently on S (assuming $S \gg 1$ as usual). This has been also confirmed by numerical MHD simulations [11, 5, 23, 12] and, with some modifications, in the MHD-Hall regime [21, 19]. As a consequence, the tearing instability of an equilibrium Sweet-Parker current sheet, for which $a/L \sim S^{-1/2}$, necessarily leads to *super-tearing* reconnection events [13], since the growth rate increases with S beyond the ideal threshold. This situation is clearly paradoxical, and the only way out is to realize that Sweet-Parker stationary current sheets cannot actually form in nature [20].

The relativistic MHD tearing instability in thin current sheets

An investigation of the tearing instability in 2D thin current sheets using the relativistic MHD regime (bulk speeds approaching the speed of light, relativistically hot temperatures, extreme magnetization), as appropriate for the sources of high-energy astrophysics, has been carried out for the first time in [6]. Numerical simulations have been performed by using the ECHO code for relativistic MHD [3], adopting *implicit-explicit* IMEX Runge-Kutta techniques to integrate the stiff terms arising from the integration of the Maxwell equation for the electric field in a conducting plasma [4, 2]. The code has been recently updated to treat non-ideal dynamo and chiral effects in (general) relativistic plasmas [7].

The linear analysis of the instability in the case of a two-dimensional Harris-type sheet shows that the behaviour is expected to be exactly the same as the classical one, provided that the Alfvén speed is defined as

$$c_A = \frac{B_0}{\sqrt{4\pi(\rho_0 + 4p_0/c^2) + B_0^2/c^2}} = (1/\sigma_0 + 2\beta_0 + 1)^{-1/2}\,c, \tag{2}$$

where we have assumed the equation of state for an ideal gas with adiabatic index $4/3$ and where the quantities indicated with the label 0 are the equilibrium ones far from the sheet itself. Notice that when the magnetization is very high, namely $\sigma_0 \equiv B_0^2/4\pi\rho_0 c^2 \gg 1$, and the plasma beta is small, $\beta_0 \equiv 8\pi p/B_0^2 \ll 1$, the Alfvén speed tends to the speed of light. In this case hydrodynamical terms can be neglected and the analysis can be reduced to that appropriate for the so-called *force-free degenerate electrodynamics* [9]. For different values of σ_0 and β_0 we were thus able to retrieve the classical results, both analytically and numerically. Hence, the most unstable tearing mode has growth rate and wavenumber

$$\gamma \simeq 0.6 S_a^{-1/2} c_A/a, \quad k \simeq 1.4 S_a^{-1/4}/a, \tag{3}$$

while assuming the critical (inverse) aspect ratio $a/L = S^{-1/3}$ (we used the value $S = Lc_A/\eta = 10^6$ in our resistive relativistic MHD simulations, so $a = 0.01L$), we found

$$\gamma \simeq 0.6 c_A/L, \quad k \simeq 1.4 S^{-1/6}/a, \tag{4}$$

that for values $\sigma_0 = \beta_0 = 1$ typical of the relativistic MHD regime, that means $c_A = c/2$, gives $\gamma = 0.3\,c/L$, and $k \simeq 0.14\,/a = 14\,/L$ for the chosen Lunquist number. Notice that the rise time is indeed as short as a fraction of the light crossing time of the sheet macroscopic length, as requested to explain the flaring events of high-energy astrophysics.

Nonlinear evolution and secondary plasmoid instabilities

In the present section we summarize the results of the nonlinear phase of the instability for $S = 10^6$ and $a = S^{-1/3}L = 0.01$ presented in [6], to which the reader is referred for figures, further details, and additional references. When the equilibrium current sheet is perturbed by many modes (10 in our simulations), we clearly see the linear growth of the most unstable tearing modes, say one or two of them, until mode coupling sets in and the magnetic islands and X-points typical of the tearing instability starts to interact one with another and merge. Since periodical conditions are applied along the sheet direction and plasmoids cannot escape, the final scenario is invariably that of a single X-point, where most of the field is reconnected, and of a single, large plasmoid with complex sub-structures. We have tested several combinations of the free parameters σ_0 and β_0, up to $c_A = 0.98\,c$ (extreme relativistc case), and verified that not only the linear phase simply scales with the Alfvénic time L/c_A, but surprisingly also the beginning of the nonlinear phase and the saturation, finding a sort of universal behaviour.

The production of plasmoids by secondary tearing events occurs on time scales decreasing with the plasmoid's size, and this should happen when the *local* aspect ratio L/a becomes the critical one in terms of the local S, as observed for a variety of Lundquist numbers in classical MHD simulation [5]. The final stage is reached when super-Alfvénic relativistic jets ($v \simeq c_A \simeq c$) form from the X-point and feed the largest plasmoid. The configuration around the X-point is highly reminiscent of the Petschek scenario for steady reconnection, with slow shocks around the X-point and a magnetic dissipation rate (measured as the inflow velocity divided by the Alfvén speed) roughly going as $(\ln S)^{-1}$, as predicted for relativistic and magnetically dominated plasmas [14]. When the jet leaves the narrow funnel defined by the slow shocks and enters the large plasmoid, a fast magnetosonic shock forms, heating the plasma downstream. This is the site for efficient particle acceleration to ultrarelativistic velocities, as always expected in explosive reconnection events [15]. Unfortunately we cannot follow this physics with our purely MHD code, but we plan to study this in details in the future by using particle-in-cell (PIC) techniques [17].

Additional dynamics is instead expected for 3D resistive MHD simulations, as it would be required for a more realistic modeling of reconnection scenarios. In the classical limit, the tearing instability is known to be strongly modified by the interplay with purely 3D kink-like modes and Kelvin-Helmoltz instabilities [10], so it should be interesting to investigate the corresponding relativistic case.

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