Nonlinear Doppler reflectometry power response.

O. L. Krutkin[1,2], E. Z. Gusakov[1], S Heuraux[2], C Lechte[3]

[1] *Ioffe Institute, St Petersburg, Russia*

[2] *Institut Jean Lamour UMR 7198 CNRS-Université de Lorraine, 54011 Nancy, France*

[3]*Institute of Interfacial Process Eng. and Plasma Technology, 70569 Stuttgart, Germany*

Introduction

Turbulent transport plays a key role in limiting confinement in magnetic fusion devices. Investigation of underlying physical processes behind it requires suitable tools to study plasma turbulence. One of such tools is Doppler reflectometry (DR) diagnostic, which employs the probing of the plasma with the microwave beam in the presence of cut-off. The probing beam is tilted with respect to the magnetic surface and backscattered signal is measured. The vicinity of the probing wave turning point provides the dominant contribution to the DR signal [1]. Using the Doppler effect, the poloidal velocity of density fluctuations is determined. The Bragg resonance condition makes the measurement of fluctuation poloidal wavenumber spectrum possible by varying the probing angle.

However, such a straightforward interpretation is only possible in the linear regime of scattering, where the Born approximation over the amplitude of density fluctuations is applicable. This case was extensively studied both analytically [1] and numerically [2]. Analytical predictions have also been made for the strongly nonlinear (or saturated) case [3] when multiple forward scattering is dominant and nonlinear effects were observed in full-wave numerical modeling [4, 5].

Finally, a study of the intermediate nonlinear regime, when the amplitude of the scattered signal scales nonlinearly with density fluctuations r.m.s. was performed [6] and the criterion of transition from linear regime to nonlinear regime was derived. This analysis, however, utilized the physical optics approximation, which has limited and uncertain domain of validity due to the fact that only backscattering in plasma at cut-off region is taken into consideration.

In the present paper, the intermediate nonlinear regime of Doppler reflectometry will be considered in the framework of perturbation theory applied to the Helmholtz equation, to overcome the limitations of the physical optics.

Analytical treatment

In this work, plasma probing with the microwave beam of O-polarization is considered. Analysis is performed in slab geometry, the background density profile is assumed to be linear over radial coordinate x and uniform over the poloidal coordinate y. Density fluctuations in the analytical treatment on the other hand are assumed to be uniform over x, which corresponds to the large radial correlation length. This assumption, which resembles the physical optics approach, is made to simplify the derivation. However we will not limit the analysis to only the turning point vicinity and will consider contribution of all the plasma volume to the backscattering signal. The microwave propagating through plasma is described by the Helmholtz equation:

$$\frac{d^2}{dx^2}E(x,y)+\frac{d^2}{dy^2}E(x,y)+\left[k(x)^2-\delta k(y)^2\right]E(x,y)=0 \tag{1}$$

where $k(x)^2=\omega^2/c^2(1-n(x)/n_c)$; $\delta k(x)^2=\omega^2/c^2(\delta n(y)/n_c)$ and $n_c=m_e\omega^2/(4\pi e^2)$. To obtain the solution of this equation we will use the WKB approximation. Unperturbed solution for a single poloidal harmonic takes the form:

$$E^0(x,k_y)=f(k_y)\sqrt{\frac{k(0,k_y)}{k(x,k_y)}}\exp\left(i\int_0^{L(k_y)}k\left(x,k_y\right)dx+i\frac{\pi}{4}\right)\cos\left(\int_x^{L(k_y)}k(x,k_y)dx+\frac{\pi}{4}\right) \tag{2}$$

where $f(k_y)$ is probing beam's angular diagram. $L(k_y)$ is radial cutoff position for a given poloidal wave number and $k(x,k_y)^2=k^2(x)-k_y^2$. The first order correction to the solution - E^1, describing scattering in the Born approximation can be obtained by solving equation:

$$\begin{aligned}\frac{d^2}{dx^2}\frac{c^2}{\omega^2}E^1(x,k_y)+\frac{c^2}{\omega^2}k(x,k_y)^2E^1(x,k_y)=\sqrt{\frac{k(0,k_y)}{k(x,k_y)}}\int_{-\infty}^{\infty}\frac{\delta n(q)}{n_c}f(k_y-q)\times\\ \times\exp\left(i\int_0^{L(k_y-q)}k\left(x,k_y-q\right)dx+i\frac{\pi}{4}\right)\cos\left(\int_x^{L(k_y-q)}k(x',k_y-q)dx'+\frac{\pi}{4}\right)dq\end{aligned} \tag{3}$$

To find the amplitude of the scattered signal we utilize the reciprocity theorem [7]:

$$A_s=\frac{ie^2}{4m_e\omega^2}\omega\sqrt{P}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\delta n(y)}{n_c}E(x,y)^2dxdy \tag{4}$$

Next, by substituting formula (2) and a solution of (3) into (4) we can derive a linear response, “proportional” to δn and a quadratic one, “proportional” to δn^2. The value of δn corresponding to these two terms being equal will give us the threshold for transition from linear to nonlinear regime.

In the further derivation we will assume that the probing antenna has a narrow angular diagram centered around $K=\omega/c\sin\vartheta$, ϑ is a probing angle with respect to normal to the magnetic surface (which in our case is directed along x). To simplify the derivation, we will also assume a Gaussian spectrum of turbulence $\delta n(k_y)=\delta n_0 l_{cy}\exp\left[-k_y{}^2l_{cy}^2/8\right]$, where l_{cy} is the poloidal correlation length of fluctuations. While the expression for the quadratic scattered signal is too lengthy to write it here, in the end one can arrive at a number of nonlinearity thresholds corresponding to different parameter ranges.

For $l_{cy}^2 >> Lc/\omega$:

$$\begin{aligned} &\frac{\delta n}{n_c} >> \frac{c}{\omega L\cos\vartheta}\exp\left(\frac{-K^2 l_{cy}^2}{4}\right) \quad \sin\vartheta << \sqrt{\frac{c}{\omega L\cos\vartheta}} \quad (a) \\ &\frac{\delta n}{n_c} >> \sin^2\vartheta\exp\left(\frac{-K^2 l_{cy}^2}{4}\right) \quad \sqrt{\frac{c}{\omega L\cos\vartheta}} << \sin\vartheta \quad (b) \end{aligned} \qquad (5)$$

In the opposite case of $l_{cy}^2 << Lc/\omega$:

$$\begin{aligned} &\frac{\delta n}{n_c} >> \frac{c}{\omega l_{cy}}\sqrt{\frac{c}{\omega L\cos\vartheta}}; \quad \sin\vartheta << \sqrt{\frac{c}{\omega L\cos\vartheta}} \quad (a) \\ &\frac{\delta n}{n_c} >> \frac{c\sin\vartheta}{\omega l_{cy}}; \quad \sqrt{\frac{c}{\omega L\cos\vartheta}} << \sin\vartheta << \frac{c}{\omega l_{cy}} \quad (b) \\ &\frac{\delta n}{n_c} >> \sin^2\vartheta\exp\left(\frac{-K^2 l_{cy}^2}{4}\right); \quad \frac{c}{\omega l_{cy}} << \sin\vartheta \quad (c) \end{aligned} \qquad (6)$$

Numerical factors in these formulae were excluded, as some of them are just estimations of the scattered field, not exact values. It should be noted, that these criteria are in agreement with each other in intermediate parameter ranges, which means they properly describe all possible parameter values. Finally, using the fact that nonlinearity criterion and saturation criterion coincide in case of normal probing beam incidence [6, 8], formulae (5) can be generalized for arbitrary radial correlation length l_{cx}:

$$\begin{aligned} &\frac{\delta n}{n_c} >> \frac{c\,e^{\frac{-K^2 l_{cy}^2}{4}}}{\omega\sqrt{L l_{cx}\ln\left(L/l_{cx}\right)}\cos\vartheta} \quad \sin\vartheta << \sqrt{\frac{c}{\omega L\cos\vartheta}} \quad (a) \\ &\frac{\delta n}{n_c} >> \sin^2\vartheta\sqrt{\frac{L}{l_{cx}\ln\left(L/l_{cx}\right)}}\,e^{\frac{-K^2 l_{cy}^2}{4}} \quad \sqrt{\frac{c}{\omega L\cos\vartheta}} << \sin\vartheta \quad (b) \end{aligned} \qquad (7)$$

Here, formula (7a) is in agreement with the one obtained with the help of the physical optics model [6]. However, the physical optics model does not reproduce the other formulae obtained, in particular, expression 5b, which corresponds to the most realistic situation of the DR experiment. Generalized version of (6) can also be obtained by taking into account the consistency between (5) and (6).

Numerical modeling

To validate the obtained theoretical results and highlight the differences with the physical optics description, a full-wave numerical modelling with the IPF-FD3D code [5] was performed. To generate random turbulence, a Gaussian spectrum with random phases added to it was used. Each random turbulence was normalized to have r.m.s. of 1 and then scaled to a certain fraction of n_c.

The amplitude of the scattered signal and power dependence $n_i=ln(P_{i+1}/P_i)/\ ln(a_{i+1}/a_i)$ for two calculations can be seen at figures 1 and 2. Green, orange and brown vertical lines correspond to the nonlinear transition values provided by (7b), the phys. optics and the saturation criterion [8] respectively.

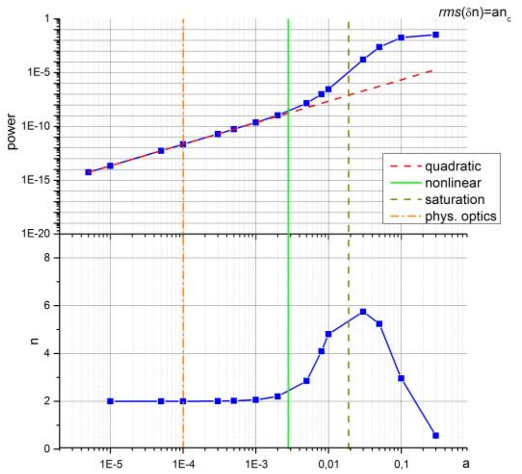

Fig 1. Calculation results for f=30GHz, L=20cm, l_{cx}=1.3 cm, l_{cy}=1.3. cm, ϑ=40°.

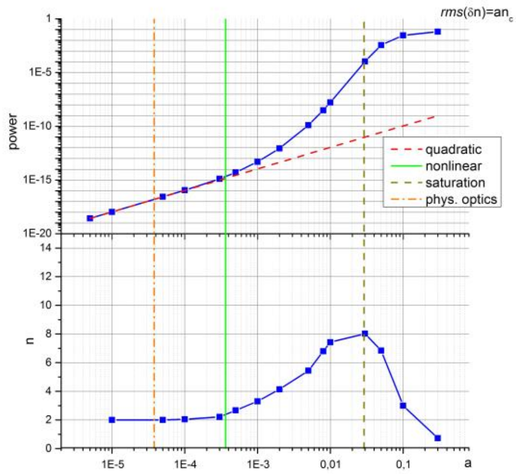

Fig 2. Calculation results for f=30GHz, L=20cm, l_{cx}=1.7 cm, l_{cy}=1.7. cm, ϑ=30°.

As it can be seen from above figures, new nonlinearity threshold describes the actual nonlinear transition better than the criterion provided by physical optics [6]. However, in the case when nonlinear criterion suggested exceeds saturation threshold calculation results were not in agreement with theory. This discrepancy is currently under investigation.

Conclusions

By applying the perturbation method to the Helmholtz equation, the new threshold for transition to nonlinear regime was suggested without relying on physical optics. It was validated numerically for a wide range of parameters, however, there are still some inconsistencies to investigate.

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