A reduced drift magnetic island theory of neoclassical tearing modes for low collisionality plasmas

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Neoclassical tearing mode (NTM) [1] suppression or avoidance is important for successful tokamak operation. One of the most promising NTM control techniques is to generate microwaves at the electron cyclotron frequency to drive current inside the island, and replace the missing bootstrap current that provides the island growth. This O-point EC current drive can be applied to reduce the island width, and so mitigate the degradation of confinement in fusion devices, such as ITER, and/or suppress the NTM. To calculate how much current is required for the NTM stabilisation, we need to develop further the NTM threshold physics. According to the modified Rutherford equation (MRE) [2], the time evolution of the island width w is described by $\left(\tau_R / r_s^2\right) dw/dt = \Delta' + \int \delta j_{\|} d\mathbf{x}$ with τ_R being the resistive diffusion time, and r_s denoting the position of the rational surface. The first term on the right represents the free energy available in the equilibrium current density, while the second term comes from the localised layer current density perturbation parallel to the magnetic field. The main contribution to $\delta j_{\|}$ is the sum of the bootstrap, polarisation and curvature currents. For islands wider than the ion banana orbit width ρ_{bi}, these MRE contributions are well described by the conventional neoclassical theory. However, the NTM stability threshold is near $w \sim \rho_{bi}$, where the existing theory is no longer valid. In our current work, we focus on the bootstrap and polarisation contributions to the island evolution, considering small magnetic islands w compared to the tokamak minor radius r, but allow $\rho_{bi} \sim w$ to extend the existing theory.

To find the electron and ion responses to the magnetic and corresponding electrostatic perturbations, we start with a drift-kinetic equation of the following form: $V_{\|}\nabla_{\|} f_j + \mathbf{V}_E \cdot \underline{\nabla} f_j + \mathbf{V}_b \cdot \underline{\nabla} f_j - \left(eZ_j / m_j V\right)\left[V_{\|}\nabla_{\|}\Phi + \mathbf{V}_b \cdot \underline{\nabla}\Phi\right]\partial f_j / \partial V = C\left(f_j\right)$. Here $V_{\|}$ is the component of velocity in the direction along the magnetic field lines, $\mathbf{V}_{\|} = V_{\|}\mathbf{b}$, $\mathbf{b} = \mathbf{B}/B$. $\mathbf{V}_E = \left[\mathbf{B} \times \underline{\nabla}\Phi\right]/B^2$ and $\mathbf{V}_b = -\mathbf{V}_{\|} \times \underline{\nabla}\left(V_{\|}/\omega_{cj}\right)$ are the $\mathbf{E} \times \mathbf{B}$ and total magnetic drifts, respectively; $\omega_{cj} = eZ_j B / m_j$ is the Larmor frequency with eZ_j and m_j being the particle charge and mass. Φ is the electrostatic potential, which we split into the equilibrium radial

E-field, $-\Phi'_{eqm}$, and the perturbed part, i.e. $\Phi = \Phi'_{eqm} x + \varphi$, where x is the distance from the rational surface denoted by $x \equiv \psi - \psi_S = 0$. Here we assume that the main effect of the NTM magnetic perturbation on plasma parameters is localised to the island vicinity. The electrostatic potential is to be determined self-consistently from the plasma quasi-neutrality condition. C_j represents a model form of the electron/ion momentum-conserving collision operator, which is given by Eq. (62) in [3]. Working in the island rest frame, we seek the time-independent electron/ion distribution function $f_{j=e,i}$ of the form $f_j = \left[1 - eZ_j\Phi / T_j(0)\right] f_j^M(0) + g_j$. f_j^M is a non-shifted Maxwellian distribution, evaluated at the resonant surface, while g_j is the perturbed part of the particle distribution that responds to the NTM island. Treating the system perturbatively, we expand g_j in the small ratio of island width to tokamak minor radius, $\Delta = w / \psi_S$. Applying an orbit-averaging procedure (denoted below by angle brackets) and introducing the toroidal canonical momentum instead of the poloidal magnetic flux as a radial coordinate enables us to reduce the dimension of the problem to 4D. Then we derive the streamlines, S, along which the distribution function is constant in the absence of collisions and write an equation for the leading order distribution function, $g_j^{(0)}$, as

$$\left[\frac{\hat{w}}{\hat{L}_q}\hat{p}_\varphi \Theta(\lambda_c - \lambda) - \hat{\rho}_\vartheta \omega_D - \frac{\partial}{\partial \hat{p}_\varphi}\Big|_{\xi,\vartheta} \frac{1}{2}\left\langle \frac{\hat{\rho}_\vartheta}{\hat{V}_\parallel}\hat{\Phi}\right\rangle_\vartheta\right]\frac{\partial g_j^{(0)}}{\partial \xi}\Big|_{S,\vartheta,\lambda,\hat{V},\sigma} = \left\langle \frac{R^2 B}{I V_\parallel} C_j \right\rangle_\vartheta$$

with $S = \dfrac{\hat{w}}{4\hat{L}_q}\left[2\left(\hat{p}_\varphi - \dfrac{\omega_D \hat{\rho}_\vartheta \hat{L}_q}{\hat{w}}\right)^2 - \cos\xi\right]\Theta_p - \omega_D \hat{\rho}_\vartheta \hat{p}_\varphi \Theta_t - \dfrac{1}{2}\left\langle \dfrac{\hat{\rho}_\vartheta}{\hat{V}_\parallel}\hat{\Phi}\right\rangle_\vartheta$. $\Theta_{p,t}$ is the Heaviside function that corresponds to the passing and trapped regions, respectively. Here we have introduced dimensionless variables: $\hat{\rho}_{\vartheta j} = I \cdot V_{Tj} / (\omega_{cj} \cdot w)$ with $V_{Tj} = (2T_j / m_j)^{1/2}$ being the electron/ion thermal velocity, $\hat{V}_\parallel = V_\parallel / V_{Tj}$, $\hat{V} = V / V_{Tj}$, $\hat{w} = w / \psi_S$, $\hat{L}_q^{-1} = (\psi_S / q)\partial q / \partial \psi$, $L_B^{-1} = (\psi_S / B)\partial B / \partial \psi$ and $\hat{\Phi} = eZ_j\Phi / T_j(0)$. $\hat{p}_\varphi$ is $\hat{x} - \hat{\rho}_{\vartheta j}\hat{V}_\parallel$ with $\hat{x}$ being the poloidal flux function centered around the rational surface and normalised to the island half-width. ω_D is the magnetic drift frequency. $\hat{p}_\varphi$, poloidal angle ϑ (which is averaged over in the above equation) and helical angle ξ form an initial set of spatial variables. To describe velocity space, we use the pitch angle λ, the absolute value of velocity V and $\sigma = V_\parallel / |V_\parallel|$. In addition, we have taken $\partial p_\varphi / \partial \psi \simeq 1$ to leading order and have assumed that the fastest p_φ variation is in Φ and $g_j^{(0)}$.

Thus, $g_j^{(0)}$ becomes a function of $\{S, \xi, \lambda, V; \sigma\}$. The S function is used to represent the streamlines for the passing and trapped ions and electrons. In the absence of Φ, constant S contours in the (ψ, ξ) plane map out "drift" islands, which are identical to the real magnetic island structure, but shifted in the radial direction by an amount of a few poloidal Larmor radii (and slightly modified in the presence of the self-consistent Φ). This radial shift appears as $\rho_\vartheta \omega_D$ in the S definition and hence the $\sigma = +1$ shift is equal but opposite to the $\sigma = -1$ shift. If collisions are neglected, the combined effect of parallel flow, ∇B and curvature drifts would force the particle distribution function to be flattened on these constant S "drift" islands. Introducing collisions at next order determines the full form of the perturbed distribution function. Applying a perturbative approach for collisions ($\nu_j / \varepsilon\omega << 1, \nu_j$ and ω are collision and island propagation frequencies, respectively; ε is the inverse aspect ratio), we obtain $\partial g_j^{(0,0)} \big|_{S,\vartheta,\lambda,V,\sigma} / \partial\xi = 0$ to 0th order. $g_j^{(0,0)}$ is then $g_j^{(0,0)}\left(S\left(p_\varphi, \xi, \lambda, V, \sigma\right), \lambda, V; \sigma\right)$ and is obtained from the solvability constraint when we proceed to next order, introducing collisions. An annihilation operator introduced to eliminate the term in $g_j^{(0,1)}$ allows us to consider separately the regions inside and outside these S islands.

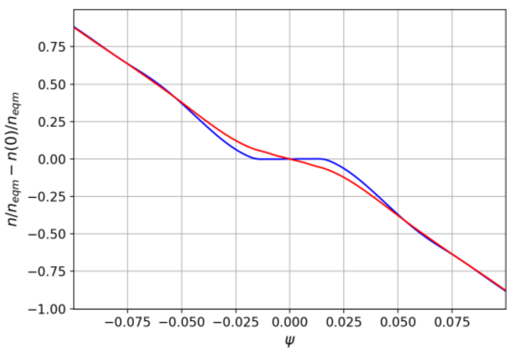

Fig. 1. *Radial ion density profile for small* $\rho_{\vartheta i} / w = 0.08$ *(blue curve) and large* $\rho_{\vartheta i} / w = 0.65$ *(red curve) across the island O-point. The density flattening is almost complete for small* $\rho_{\vartheta i} / w$ *but is replaced by a substantial gradient for* $\rho_{\vartheta i} / w \sim 1$. n_{eqm} *is the equilibrium density, i.e. in the absence of the NTM island.*

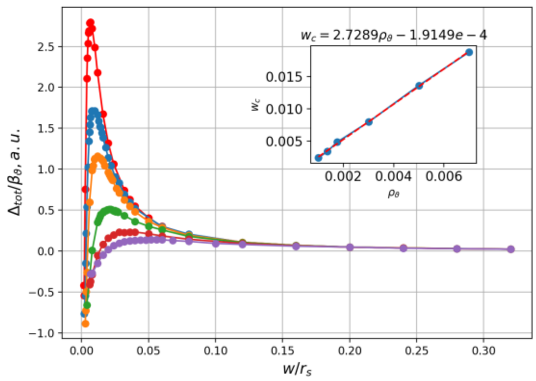

Fig. 2. *The total parallel current contribution to the island time evolution* Δ_{tot}, *normalised to* β_ϑ, *vs.* w *for different* $\rho_{\vartheta i}$ *values: (1.00, 1.35, 1.75, 3.00, 5.00, 7.00)* $\cdot 10^{-3} r_S$. *The subplot shows* w_c *as a function of* $\rho_{\vartheta i}$.

As can be seen from the definitions of S and ω_D, the total ion/electron perturbed distribution is a superposition of two drift island branches. One drift island is located in the $V_\parallel > 0$ region, while the other is equal but shifted in the opposite direction, i.e. $V_\parallel < 0$. The real magnetic island is localised between them. As the regions of distribution profile flattening are in opposite directions for $\sigma = \pm 1$, summation over σ provides a substantial gradient inside the NTM island when $w \sim \rho_{\vartheta i}$. For $\rho_{\vartheta i} / w << 1$, the density profile is flattened inside the magnetic island, as the radial shift is kept relatively small. These results are shown in Fig.1 and are referred to as finite orbit width effects. For the electrons, this radial shift $\sim \rho_{\vartheta e} / w$ is a factor $\left(m_e / m_i\right)^{1/2}$ smaller than for the ions, and the rapid electron parallel flow always tends to flatten their radial profile across the magnetic island. However, as the plasma is quasi-neutral, an electrostatic potential forms to ensure the same gradient is supported by the electrons. It has been confirmed that our solution for the potential converges and satisfies the quasi-neutrality condition. To calculate the neoclassical current contribution to the NTM, we project the cosine component of Ampere's law integrated across the island, which is written as $\int_{\mathbb{R}} d\psi \oint j_{\|} \cos\xi d\xi = \Delta_{tot} \rho_{\vartheta i} w^2 / \beta_{\vartheta}$ [3] and in general can be interpreted as an equation for $\Delta_{tot} = \Delta_{bs} + \Delta_{pol}$ [2]. Δ_{tot} as a function of w is shown in Fig.2 for different values of $\rho_{\vartheta i}$. In the limit of large island widths, Δ_{tot} decreases with w as a power function. This functional behaviour for larger w is expected from existing analytical approaches [3, 4]. In contrast, for small w the neoclassical current contribution to the MRE is reduced as the radial shift of the S islands, estimated as $\rho_{\vartheta i} / w$, becomes significant for small island widths. The fact that Δ_{tot} becomes negative for the smallest islands, $w < w_c$, corresponds to the NTM magnetic island self-healing [5]. Tokamak experimental data supports a stabilising effect for $w <$ or $\sim \rho_{bi}$ [6]. Defining the critical island size w_c as a root of $\Delta_{tot}\left(w\right) = 0$ and plotting w_c against $\rho_{\vartheta i}$, we find an approximation $w_c \approx 2.73 \rho_{\vartheta i}$, which is equivalent to $8.63 \rho_{bi}$ for $\varepsilon = 0.1$. A marginal island width proportional to ρ_{bi} has been observed on NSTX and DIII-D [6].

In summary, these new results extend the existing neoclassical theory of tearing modes and are important for constructing the NTM control modules for ITER and next generation power plants. A better understanding of the ion polarisation current contribution requires knowledge of the island propagation frequency and hence a more detailed treatment of the dissipation layer, and will be the subject of further investigation.

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