Low-frequency fishbone driven by passing fast ions in Tokamak plasmas

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Introduction

The internal kink modes with dominant poloidal and toroidal wave number $m = 1$ and $n = 1$ can be strongly destabilized by both the perpendicular and tangential neutral-beam injection [1, 2]. With perpendicular injection, the driven modes can be considered as either an energetic particle mode (EPM) with mode frequency comparable to the toroidal precession frequency of the trapped energetic ion [3] or a "gap" mode with mode frequency close to the thermal ion diamagnetic frequency [4]. With tangential injection, both the high-frequency mode and the low-frequency mode have been observed [2]. For the high-frequency branch, the mode was considered as an EPM with frequency determined by energetic particle toroidal circulation frequency [5, 6]. For the low-frequency branch, the mode was modeled as a "gap" mode with the thermal ion diamagnetic frequency [7]. It is still of great interest to extend the theory of EPM to the low-frequency branch to complete the fishbone analysis in the case of tangential beam injection.

In this work, we propose a theoretical model to consider the low-frequency fishbone observed in the tangential injection as an EPM. Specially, the low-frequency mode driven by a resonant interaction between the passing beam ions and the wave with $\omega = \omega_\phi - \omega_\theta$ is studied, where ω_ϕ and ω_θ are respectively the circulation frequency in toroidal and poloidal direction of passing fast ions. With the effect of finite orbit width (FOW) of fast ions, the instability can be excited by passing fast ions. It is found that magnetic shear at the $q = 1$ radius plays an important role in the instability whereas the effect of the background plasma beta is weak. In particular, there exists a critical magnetic shear below which the beam ion beta threshold for EPM excitation is very small. For moderate or higher magnetic shear the beam ion beta threshold is about a few percent. These results are consistent with experimental observation of the low-frequency fishbone in the HL-2A tokamak [8].

Fishbone dispersion relation for passing fast ions

To evaluate the contributions of passing fast ions to the peturbed potential energy, the internal kink perturbation with dominant $n = 1$ and $m = 1$ mode numbers and a slowing down distribution function of fast ions of tangential neutral beam injection is adopted. With the generalized variational principle, we obtain the fishbone dispersion relation as[3],

$$-i\Omega/\Omega_A + \delta\hat{W}_T + \beta_{h0}\delta\bar{W}_h(\Omega) = 0, \tag{1}$$

$$\delta\hat{W}_T = 3\pi(1-q_0)\varepsilon_1^2\left[13/144 - \left(\beta_{ps} + \beta_{h0}\bar{\beta}_{ph}\right)^2\right], \tag{2}$$

$$\delta\bar{W}_h = -\frac{\pi}{2}\int_0^{r_s} dr\left(\frac{r}{r_s}\right)^2\frac{d\overline{\langle P_h\rangle}}{dr} + \frac{1}{3}\pi\frac{\overline{\langle P_h\rangle}(r_s)}{\varepsilon_1}\left(\frac{\Delta_b}{r_p}\right)_s\left[A - \frac{2F(\Omega)}{\pi s_1}\right], \tag{3}$$

$$F(\Omega) = \frac{1}{\pi}\left\{-8\Omega^{\frac{3}{2}}\left[\tan^{-1}\frac{1}{\sqrt{\Omega}} + \tanh^{-1}\frac{1}{\sqrt{\Omega}}\right] + \left(1+3\Omega^2\right)\ln\left(\frac{\Omega+1}{\Omega-1}\right) + 10\Omega\right\}, \tag{4}$$

where $\Omega_A = \tilde{\omega}_A/\left[(v_h/R_0)(s_1\Delta_b/r_s)\right]$, $\tilde{\omega}_A = v_A/\left(3^{1/2}R_0 s_1\right)$, $v_A = B_0/\sqrt{\mu_0 m_i n_i}$. $\varepsilon_1 = r_s/R_0$, r_s is the radial position of the rational surface of $q_s = 1$. $\bar{\beta}_{ph} = -\left(1/\varepsilon_1^2\right)\int_0^{r_s} dr\,(r/r_s)^2\, d\overline{\langle P_h\rangle}/dr$ with $\overline{\langle P_h\rangle} = \langle P_h\rangle(r)/\langle P_h\rangle(0)$ and $\langle P\rangle \equiv \frac{1}{2\pi}\int_0^{2\pi} d\theta P$. β_{ps} refers to [3]. $\beta_{h0} = 2\mu_0\langle P_h\rangle(0)/B_0^2$. $\Delta_b = q_s\rho_h$ is orbit width of passing fast ion, $\rho_h = v_h/\omega_c$, $v_h = \sqrt{2\varepsilon_0/m_h}$, $\omega_c = eZB_0/m_h$ is gyrofrequency. $r_p = -\left[(1/p_h)\,dp_h/dr\right]^{-1}$, $s_1 = (r_s/q_s)(dq/dr)_s$ and $A = \left(\langle P_h\rangle_{+1} - \langle P_h\rangle_{-1}\right)/\langle P_h\rangle$, where $A = 1$ for pure co-injection, $A = -1$ for pure counter-injection, and $A = 0$ for balanced injection. $\Omega = \bar{\omega}/(s_1\Delta_b/r_s)$, $\bar{\omega} = \omega/(v_h/R)$. $\delta\hat{W}_T$ is the fluid contribution including fast ion pressure. $\delta\bar{W}_h$ is recognized as the kinetic contribution of passing fast ions. In this term of Eq.(3), the first component results from anistropic contribution of fast ions, the second component relates to the effect of injection direction and the last component is non-adiabatic contribution of $p = -1$ term which contains F function of Eq.(4). Based on Eq. (1), the theory of EPM is demonstrated that the mode frequency and the growth rate are treated in a non-perturbative way instead of assuming a diamagnetic frequency for the mode[7].

Results

We investigate Eq.(1) numerically for the fishbone on HL-2A excited by passing particles. We consider the typical HL-2A parameters: $B_0 = 1.3$ T of toroidal magnetic field magnitude, $R_0 = 1.6$ m of major radius, $a = 0.4$ m of minor radius, $n_e = 1.3\times10^{13}$ cm^{-3} of plasmas density, safety factor profile $q = q_0 + 2(r/a)^2$, $\Delta = 0.25a$ of fast ion radial profile width of $p_h = p_{h0}\exp\left(-r^2/\Delta^2\right)$ of fast ion pressure profile, $A = 1$ for pure co-injection, NBI injection energy $\varepsilon_0 = 44.142$ keV, $\Delta_b = 0.083a$ of fast ion drift orbit width, $v_h/v_A = 0.37$ of normalized fast ion injection speed.

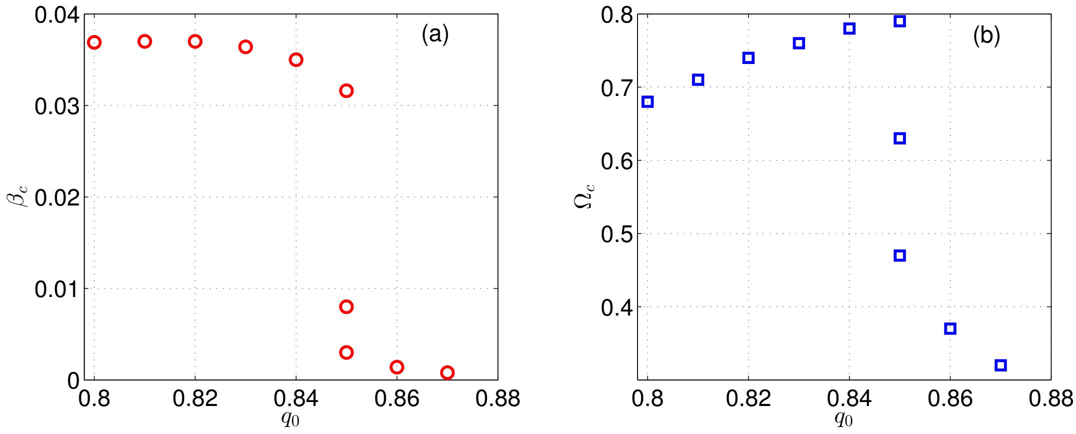

Figure 1: (a) The threshold β_{th} of β_{h0} versus q_0, (b) The threshold Ω_{th} of Ω_r versus q_0 for $\beta_{ps} = 0$, $\Delta = 0.25a$ of fast ion radial profile width.

Table 1: Dependence of critical beta and mode frequency on poloidal thermal plasma beta for $A = 1, q_0 = 0.85$.

β_{ps}	β_{th1}	Ω_{th1}	β_{th2}	Ω_{th2}	β_{th3}	Ω_{th3}
0.0	0.30%	0.47	0.80%	0.63	3.16%	0.79
0.1	0.23%	0.42	1.49%	0.71	2.16%	0.76
0.2	0.17%	0.36				
0.3	0.12%	0.30				
0.4	Unstable for all β_{h0}, no marginal stability is found					
0.5	Unstable for all β_{h0}, no marginal stability is found					

The threshold β_{th} of fast ion beta β_{h0} and the corresponding real frequency Ω_{th} as a function of q_0 for marginal stability is calculated and is shown in Figure 1. Note that for $q_0 = 0.85$, there exist 3 thresholds for fast ion beta, i.e. $\beta_{th1} = 0.30\%$, $\beta_{th2} = 0.80\%$ and $\beta_{th3} = 3.16\%$. For the case of $\beta_{th1} = 0.30\%$, $\delta\bar{W}_h$ appears negative in a region of Ω_r. This indicates a mode can be driven by the transit resonant interaction between passing fast ions and the wave. The mode is unstable for $\beta_{th1} < \beta_{h0} < \beta_{th2}$ and also for $\beta_{h0} > \beta_{th3}$. For other q_0 values either higher than or less than $q_0 = 0.85$, there exists only one critical beta. The mode is unstable when the fast ion beta exceeds this threshold. Furthermore, for moderate values of fast ion beta ($< 3\%$), the mode can only be unstable for $q_0 \geq 0.85$ or magnetic shear $s_1 \leq 0.30$. Again, as q_0 or s_1 crosses the critical value $q_c = 0.85$ or $s_c = 0.3$, the thresholds of marginal stability converts basically.

Table 1 lists the dependence of critical beta β_c and mode frequency Ω_c on poloidal thermal plasma beta β_{ps} for $q_0 = 0.85$. As is shown, when $\beta_{ps} = 0.10$ changes into $\beta_{ps} = 0.20$ indicating the variable enlarges by 100%, the value of critical beta falls by less than 22%. When $q_0 = 0.85$ increases to $q_0 = 0.87$ implying the variable alters by less than 2.4%, the value of critical beta reduces by more than 70%. As a result, β_{ps} weakly affects the destabilization of the mode concerned compared to the effect of q_0. When β_{ps} is large enough that is $\beta_{ps} \geq 0.4$, there is no marginal stability found and unstable for all β_{h0} since the MHD instability is introduced.

With use of the principle of power balance for neutral beam injection of HL-2A tokamak, the central fast ion beta is finally computed as $\beta_{h0} = 0.04$. For HL-2A, we set $s_1 = 0.3$, $r_s = 0.27a$, $A = 1$ for the beam being tangentially co-injected into the tokamak, $\beta_{ps} = 0.15$ for thermal plasmas. In this case, there are three thresholds for marginal stability as mentioned above. One of the threshold obtained is $\beta_{th} = 0.035$ and the corresponding mode frequency is $f_{th} = 14.9$ kHz. For $\beta_{h0} > 0.035$, the unstable mode is mainly driven by fluid term of fast ion response whereas the mode frequency still results from wave particle resonance, which implies the MHD-induced mode is observed in the HL-2A experiment since the fast ion beta $\beta_{h0} = 0.04$ is estimated for HL-2A plasma. With $\beta_{h0} = 0.04$, we obtain the mode frequency $f = 15.2$ kHz and the mode growth rate $\gamma/\omega = 0.5\%$ in the plasma frame. The rotation frequency in experiment is about 15 kHz[8]. The frequency of the mode we get is $f = 30.2$ kHz in laboratory frame, which is close to the observed one, i.e. 30 kHz.

Conclusions

In summary, we have shown that a new low-frequency internal kink mode can be excited through a transit resonate interaction between passing energetic particles and the mode at $\omega = \omega_\phi - \omega_\theta$. The marginal instability of the mode is sensitive to magnetic shear s_1 but not sensitive to poloidal thermal plasma beta β_{ps}. With applying this theoretical model to the fishbone instability on HL-2A, according to our reasonable estimation of the central beam ion beta $\beta_{h0} = 0.06$ for the device, we identify such an instability observed in the expriment is a high-frequency branch mode induced by MHD instability.

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