Alpha channeling by inverse nonlinear damping of ion Bernstein waves

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Theoretical studies suggest that the free energy available in the fusion alpha particle distributions in tokamak fusion reactors might be converted into radio frequency waves, which are then absorbed by electrons or ions to produce heating and current drive [1]. The schemes proposed to realize this "*alpha channeling*" concept require the convective amplification of externally launched RF waves. It occurs if the waves can produce a diffusion path in phase space, such that higher energy alpha particles in the plasma center diffuse to lower energy towards the plasma edge. In additions, the time scales for alpha channeling should be faster than the time scales involved in collisional heating of plasma electrons. It is quite challenging to find waves that can be excited in tokamak plasmas to accomplish these requirements. Alpha power channeling into RF waves absorbed by plasma ions might produce an hot-ion mode with larger fusion reactivity compared to the regime of collisional electron heating, mainly if the absorbing ions are accelerated at energies near the peak of the fusion cross-section. Moreover, the free energy of the alpha particle distributions channeled to RF waves is subtracted to the modes that can be driven unstable and deteriorate the plasma confinement. These promising features motivates further studies to realize the concept.

Here we propose a new scheme for alpha channeling, based on a scenario of second harmonic cyclotron damping of mode-converted ion Bernstein waves (IBW) on minority Tritium ions in D-H(T) tokamak plasma. This scenario has been proposed as an efficient method to improve the fusion yield expected in thermal equilibrium [2]. Despite the dilution due to the presence of the Hydrogen, which is necessary to allow the mode conversion of the fast waves, the T ions accelerated in the energy range of 50-100 keV, i.e. near the peak of DT fusion cross section, might produce higher reactivity than the ideal isotopic blend DT at thermal equilibrium, considering the same kinetic profiles. Preliminary studies suggest that such scheme might be also more efficient, in term of fusion reactivity, than the standard scheme of 2nd harmonic Tritium heating by fast waves (FW) with ideal DT isotopic composition. The T ions are accelerated by IBW at energies below the critical energy for equal collisional energy exchange with plasma electrons and ions, so that the collisional D heating might be also more efficient than expected with FW.

In this scenario, IBW nonlinear inverse Landau damping on the fusion alpha particles might be observed at Doppler-shifted half-integer resonant layer $\omega = (3/2)\Omega_\alpha + k_\parallel v_\parallel$ (Fig. 1). The kinetic theory of the nonlinear IBW damping at half-integer ion cyclotron resonance has been developed by M. Porkolab [3]. Evidences of power damping of mode-converted IBW on plasma ions at $\omega = (3/2)\Omega_D$ in D(H) plasma of ASDEX have been reported [4]. We discuss here how the inversion of population of the alpha particle distribution only in the velocity space, during its time evolution towards a slowing-down steady state, might provide the free energy for nonlinear inverse Landau damping. In this regard, our analysis differs from previous approaches to the alpha channeling.

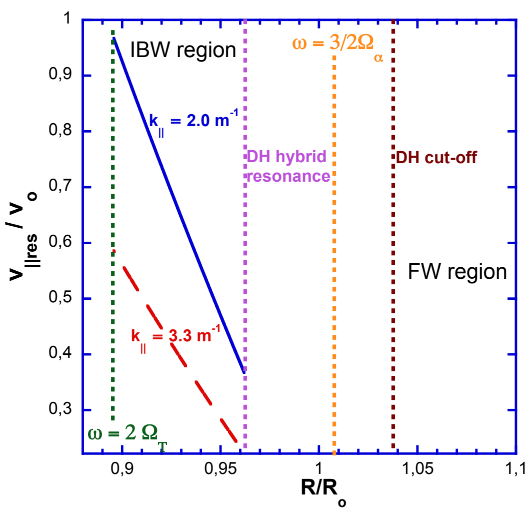

Fig. 1. Scheme of alpha power channeling in tokamak plasma with isotopic composition $n_H/n_D = 0.9$ $n_T/n_e = 0.05$ and magnetic field on axis is $B_o = 2.8\,T$. The FW are coupled from the low field side, at the operating frequency $f_o = 32\,MHz$. Resonant parallel velocities $v_{\parallel,res}$, normalized to the velocity v_o at the peak (3.5 MeV) of the alpha source energy spectrum, are shown for parallel wavenumber $k_\parallel = 3.3\,m^{-1}$ (red line) and $k_\parallel = 2.0\,m^{-1}$ (blue line). R is the major radius coordinate and R_o is the position of the magnetic axis.

We evaluate the time evolution of the alpha particles distribution function $f_\alpha(\mathbf{v}, t)$ based on the Fokker-Planck equation

$$\frac{\partial f_\alpha}{\partial t} = \nabla \cdot \bar{\bar{D}} \cdot \nabla_{\mathbf{v}} f_\alpha + s_\alpha + C(f_\alpha) - \nu_l f_\alpha \quad (1),$$

where $\bar{\bar{D}}$ is the diffusion tensor in velocity space due to the IBW-α nonlinear interaction, s_α is the source of the alpha particles produced by the DT nuclear fusion reactions, C indicates the collision operator and ν_l is the rate of alpha particle losses. We neglect here the diffusion in physical space, assuming that the particle losses, due to collisional or rf-induced space diffusion and other transport phenomena, are incorporated in the scalar coefficient ν_l. Preliminary estimates suggest that the time scales involved in such processes are much longer than the scales of the time evolution in velocity space. The diffusion tensor $\bar{\bar{D}}$ should be evaluated in the framework of the kinetic theory. This requires a calculation up to the fourth order of the perturbation analysis considering non-Maxwellian alpha particle distribution functions. Our approach is based on the single-particle, single-wave force equation.

A comparison of the results obtained for majority ions in Maxwellian plasmas by the kinetic and single-particle dynamics models suggests that neglecting the collective effects might underestimate the strength of the nonlinear interaction [3]. Our modeling thus provides a conservative estimate of the α power channeling. As a result, the rf diffusion operator is

$$D(f_\alpha) = \frac{1}{\mathrm{v}_\perp}\frac{\partial}{\partial \mathrm{v}_\perp}\mathrm{v}_\perp D_{rf}\frac{\partial f_\alpha}{\partial \mathrm{v}_\perp} \quad (2)$$

where $D_{rf} = \langle \pi[\varepsilon^2 S(\mathrm{k}_\perp \mathrm{v}_\perp/\Omega_\alpha)/4]^2 \mathrm{v}_\perp^2 \Omega_\alpha^2\, \delta(\omega - k_\parallel \mathrm{v}_\parallel - 3\Omega_\alpha/2)\,\rangle_\psi$, the brackets indicate an average over a magnetic surface with label ψ, $\varepsilon = q_\alpha \mathcal{E}/(m_\alpha \Omega_\alpha \mathrm{v}_\perp)$, $\mathcal{E}$ is the amplitude of the IBW electric field and

$$S(\lambda) = -\sum_{n=-\infty}^{+\infty} \gamma_n\, \mathrm{J}_n(\lambda)\mathrm{J}_{n-3}(\lambda)(n-3)(-1)^n \quad (3),$$

with $\gamma_n = 1/[1-(n-3/2)^2]$, and J_n are the Bessel functions of the first kind of order n. The analytic expression of the diffusion coefficient after the surface average, taking into account the finite width of the nonlinear resonance in the velocity space as well as the spectral width of the launched waves, is quite complex and will be reported elsewhere. We model the energy spectrum of the alpha particle source around the peak E_o as a Gaussian $S_\alpha(E) = S_{\alpha o}\exp[-(E-E_o)^2/\Delta E^2]\{(\sqrt{\pi}/2)\Delta E[1+\mathrm{erf}\,(E_o/\Delta E)]\}^{-1}$. The source in velocity space is $s_\alpha(\mathrm{v}) = m_\alpha S_\alpha(m_\alpha \mathrm{v}^2/2)/(4\pi \mathrm{v})$. The collision operator for alpha particles in velocity coordinates (v,μ), where $\mu = \mathrm{v}_\parallel/\mathrm{v}$ is the cosine of the pitch angle, can be approximated by the following operator [5]

$$K(f_\alpha) = \frac{1}{\tau_s \mathrm{v}^3}\left\{\mathrm{v}\frac{\partial}{\partial \mathrm{v}}[(\mathrm{v}^3+\mathrm{v}_c^3)f_\alpha] + \frac{\mathrm{v}_c^3}{2}p\frac{\partial}{\partial \mu}[(1-\mu^2)]\frac{\partial f_\alpha}{\partial \mu}\right\} \quad (4)$$

where τ_s is the slowing down time, v_c is the critical velocity for equal rates of energy exchange in collisions with the backgrounds electrons and ions and $p \cong 1/\sum_i m_\alpha n_i/(m_i n_e)$. For distribution functions with inverted profiles $\partial f_\alpha/\partial \mathrm{v}_\perp > 0$ in the region $D_{rf} > 0$ where the nonlinear interaction with IBW occurs, the alpha power can be channeled into IBW power, i.e. $P_{IBW\to\alpha} < 0$, where $P_{IBW\to\alpha}$ is the power density delivered by IBW to the alpha particles

$$P_{IBW\to\alpha} = -4\pi\int_{-\infty}^{\infty} d\mathrm{v}_\parallel \int_0^\infty d\mathrm{v}_\perp\, \mathrm{v}_\perp\left(\frac{1}{2}m_\alpha \mathrm{v}^2\right) D_{rf}\frac{\partial f_\alpha}{\partial \mathrm{v}_\perp} \quad (5)$$

A fractional step algorithm [6] has been used to solve the Fokker-Planck equation, splitting the collisional and rf-induced diffusion operators and alternating the solutions of two differential equations: $\partial f_\alpha/\partial t = D(f_\alpha) + (s_\alpha - \nu_l f_\alpha)/2$, $\partial f_\alpha/\partial t = K(f_\alpha) + (s_\alpha - \nu_l f_\alpha)/2$.

The first equation, with boundary conditions $f_\alpha \rightarrow 0$ for $v_\perp \rightarrow \infty$ and for $|v_\parallel| \rightarrow \infty$ and $\partial f_\alpha / \partial v_\perp = 0$ for $v_\perp = 0$, has been solved by the Crank-Nicolson method on a rectangular mesh in the cylindrical velocity space of coordinates $(v_\parallel, v_\perp)$. An analytic solution has been obtained for the second equation, based on expansion in series of Legendre polynomials P_n $f_\alpha = \sum_n g_{\alpha n}(v, t) P_n(\mu)$, and solving the equation obtained for $g_{\alpha n}$. The numerical code ALFs (**Al**pha **F**okker-Planck **s**olver), written in FORTRAN90 to implement the algorithm, is still in debugging and testing phase. Preliminary results have been obtained considering an alpha particle source at $E_o = 0.2\ MeV$, RF and plasma parameters of Tab I and II (Fig. 2). During the time evolution towards the steady state up to 28% of the α power is delivered to IBW. At the steady state, though the inversion of population still occurs, 5% IBW power damping on the α particles is obtained.

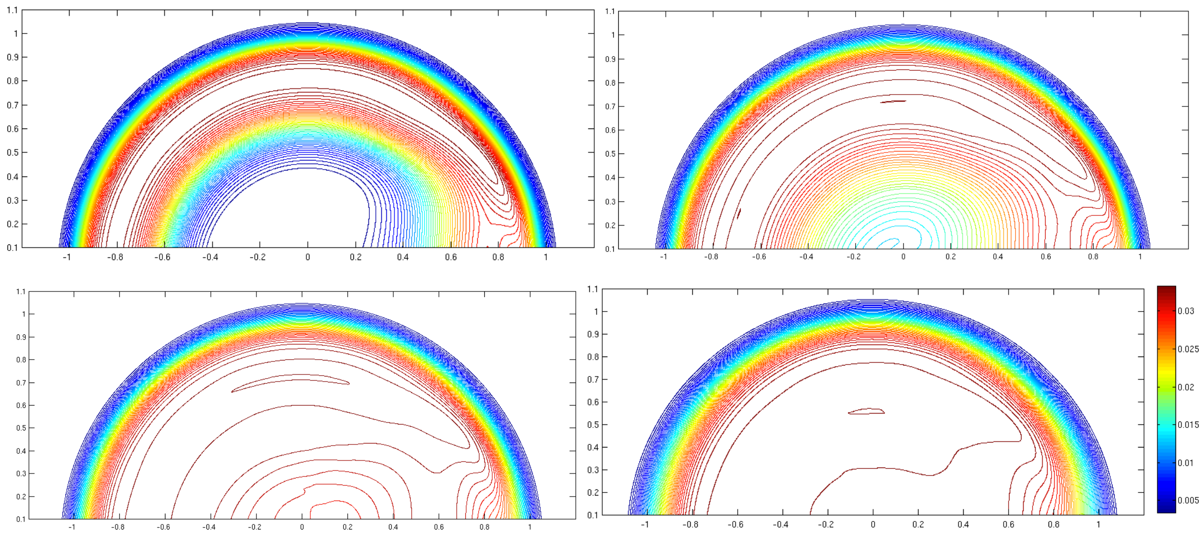

Fig. 2. Contour plots of alpha particle distribution function $f_\alpha(v_\parallel, v_\perp)$ at time 0.12 τ_s (up left), 0.16 τ_s (up right), 0.20 τ_s (down left) and 0.28 τ_s (down right). The velocities (the abscissa is $v_\parallel$) are in units of $v_o = \sqrt{2E_o/m_\alpha}$ and f_α is in units $S_{\alpha o} v_o^{-3} v_s^{-1}$ with values indicated by the common color bar. Inversion of population occurs also at the steady state, for $t \geq 0.28\ \tau_s$. The RF and plasma data used are indicated in Tab. I and Tab. II.

f_o	$k_\parallel$	$k_\perp$	$\mathcal{E}$	R_{res}	$\Delta\varphi$	$\Delta\theta$
$32\ MHz$	$-0.08\ cm^{-1}$	$5.0\ cm^{-1}$	$12\ stV\ cm^{-1}$	$276\ cm$	$0.2\ rad$	$0.78\ rad$

Tab. I. IBW parameters. Here R_{res} is the position of the resonance, $\Delta\varphi$ and $\Delta\theta$ are the toroidal and poloidal width of the area illuminated by the wave packet, $k_\perp$ is the perpendicular wave number.

B_o	$n_{e,res}$	$T_{e,res}$	$T_{i,res}$	R_o	D / H / T (%)
$2.8\ T$	$0.7\ 10^{14}\ cm^{-3}$	$5.0\ keV$	$7.5\ keV$	$300\ cm$	45 – 30 - 25

Tab. II. Plasma parameters

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