Investigation of supersonic heat-conductivity linear waves in ablation flows

G. Varillon[1,2], J.-M. Clarisse[1], A. Couairon[2]

[1] *CEA, DAM, DIF, F-91297, Arpajon, France*

[2] *CPHT, Ecole polytechnique, CNRS, Université Paris-Saclay, 91128 Palaiseau, France*

Introduction

Ablation flows relevant to inertial confinement fusion (ICF) are classically described by gas dynamics equations with nonlinear heat conduction. Standard descriptions assume an isothermal conduction region [1, 2]. However a local analysis in terms of linear propagating waves [3, 5] reveals that temperature stratification in this region gives rise to supersonic upstream propagating waves as a consequence of nonlinear heat conduction. Such "heat conductivity" waves occur for a regime of high heat propagation and are associated with heat flux perturbation inhomogeneities that convey inhomogeneities in temperture and density perturbations. The latter may trigger radiative heat transport instabilities [4] that could destabilize the ablation flow.

In the present work, we conduct numerical computations of linear perturbations in self-similar ablation flows. We drop the restriction of a local analysis to address globally the non uniformity and unsteadyness of realistic ablation flows, taking into account exactly, by contrast with previous works reported in [3, 5], linear wave coupling and damping due to heat diffusion. The entire deflagration structure of an ablation wave is described from the fluid external surface where an incoming radiation flux and external pressure are applied, up to the forerunning shock wave front. This corresponds to the ealry stage of an ICF target implosion. We focus on self-similar ablation flows with subsonic and supersonic expansion, and possibly containing a Chapman–Jouguet point [5]. Transmission mechanisms (diffusion, advection and source term) are compared in the conduction zone, and the analysis in terms of hyperbolic waves is found to be relevant.

A local pseudo-characteristic analysis

We consider the motion of a polytropic gas subject to radiative flux and material pressure at its external boundary. Radiation transfers are considered under radiation heat conduction, and diffusive effects are dominated by radiative heat flux. The flow is considered in slab symetry in the x direction. Dimensionless equations of motion in Lagrangian coordinate m – $\mathrm{d}m = \bar{\rho}\mathrm{d}x$ –

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$$\partial_t(1/\bar{\rho}) - \partial_m \bar{u} = 0\,, \quad \partial_t \bar{u} + \partial_m \bar{p} = 0\,, \quad \partial_t(\bar{T}/(\gamma-1) + \bar{u}^2/2) + \partial_m(\bar{p}\bar{u} + \bar{\varphi}) = 0\,, \tag{1}$$

where $\bar{\varphi} = \bar{\Psi}(\rho, T, \partial_x T)$, $\bar{\Psi}$ representing the heat flux dependence on density, temperature and its gradient, with $\partial_{\partial_m T}\bar{\Psi} < 0$. This system is closed by the equation of state $\bar{p} = \bar{\rho}\bar{T}$.

The ablation wave structure extends from the flow external surface ($m = 0$) through the ablation front up to the forerunning shock wave front. Time dependent pressure and heat flux are applied at $m = 0$ while Rankine-Hugoniot jump relation are applied at the shock wave front. Solutions to (1) described continuously all peculiarities of the radiative ablative wave.

Considering first order perturbations of (1)

$$\bar{V}(m,t) = (\bar{\rho}, \bar{u}, \bar{T})^\top \to \bar{V}(m,t) + \varepsilon V(m,t) + O(\varepsilon^2),$$

with $\varepsilon \ll 1$ and $V(m,t) = (\rho, u, T)^\top$, perturbations V are governed by

$$\begin{cases} \partial_t(\frac{\rho}{\bar{\rho}}) + \partial_m(\bar{\rho}u) = 0, \\ \partial_t u + \bar{\rho}u\partial_m \bar{u} + \partial_m p - \frac{\rho}{\bar{\rho}}\partial_m \bar{p} = 0, \\ \frac{\partial_t T}{\gamma-1} + \frac{\bar{\rho}u\partial_m \bar{T}}{\gamma-1} = \bar{p}(\frac{\rho}{\bar{\rho}}\partial_m \bar{u} - \partial_m u) - p\partial_m \bar{u} + \frac{\rho}{\bar{\rho}}\partial_m \bar{\varphi} - \partial_m \varphi, \end{cases} \tag{2}$$

where $\varphi = \rho\partial_\rho \bar{\Psi} + T\partial_T \bar{\Psi} + \partial_m T \partial_{\partial_m T}\bar{\Psi}$. In vector form, this system reads

$$\partial_t V + \bar{A}(m,t)\partial^2_{m^2} V + \bar{B}(m,t)\partial_m V + \bar{C}(m,t)V = 0\,, \tag{3}$$

where $\bar{A}$, $\bar{B}$ and $\bar{C}$ are 3×3 matrices depending on $(\bar{\rho}, \bar{u}, \bar{T})$ from (1) [5]. Matrix $\bar{A}(m,t)$ contains only a diffusive term on temperature, matrix $\bar{B}(m,t)$ is hyperbolic and matrix $\bar{C}(m,t)$ depends mainly on gradient of base-flow variables.

The reduced, purely hyperbolic, system

$$\partial_t V + \bar{B}(m,t)\partial_m V = 0 \Leftrightarrow \partial_t W + \bar{\Lambda}(m,t)\partial_m W = 0 \tag{4}$$

with $\bar{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3) = \bar{L}\bar{B}\bar{R}$ and $W = \bar{L}V$ – $\bar{L}$ and $\bar{R}$ being local left and right eigenvectors matrices – defines local characteristics as $\mathrm{d}W_i = 0$ along $\mathcal{C}_i$.

In the conduction region $\lambda_{2,3}$ correspond to quasi-isothermal right and left going acoustic waves, while λ_1 is a right going wave associated to *supersonic heat-conductivity* ruled by $\partial_\rho \bar{\Psi}$ and $\partial_T \bar{\Psi}$. In the post-shock region, right and left going isentropic acoustic waves ($\lambda_{1,3}$) and entropy wave (λ_2) are recovered [5].

System (3) can be expressed in the characteristic basis of the reduced hyperbolic problem (4)

$$\partial_t W + \bar{\mathscr{A}}\partial^2_{m^2} W + \bar{\Lambda}\partial_m W + \bar{\mathscr{C}}W = 0\,, \text{ with } \bar{\mathscr{A}} = \bar{L}\bar{A}\bar{R} \text{ and } \bar{\mathscr{C}} = \bar{L}\bar{C}\bar{R}. \tag{5}$$

This system (5) cannot be held as an evolution equation but can be analysed at any point (m,t). For each component W_i the variation rate reads

$$\delta W_i = \sum_{j=1}^{3} (-\bar{\mathscr{A}}_{ij}\partial^2_{m^2} W_j - \lambda_i \partial_m W_i - \bar{\mathscr{C}}_{ij} W_j)\,\delta t.$$

Therefore contributions from diffusion, *advection* and source terms can be compared at any given flow location and time.

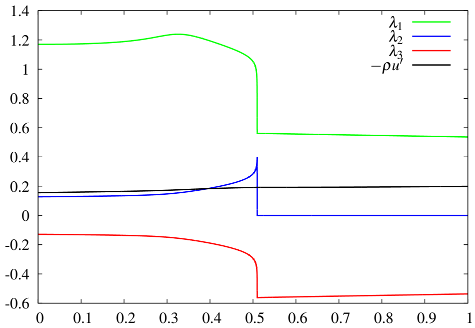

Figure 1: *Base-flow at* $t = 1$, $m_{schock} = 1$.

Application to fast expension flow

For applications we consider radiation heat conduction model $\bar{\varphi} = -\bar{\rho}^{-1}\bar{T}^{6.5}\partial_m \bar{T}$. By comparing the response of system (2) to a sine wave forerunning in W_1 applied at $m = 0$ for a sufficiently long wavelength [5], we are able to confirm the local analysis (Fig. 2).

We observe *a posteriori* that projections of solutions to (2) on pseudo-characteristic variable, W_i Fig. 2, follow the *a priori* characteristics $\mathcal{C}_i$ of the reduced hyperbolic system (4). Moreover, advection dominates over diffusion in the conduction region, and source terms are neglibible (Fig. 3).

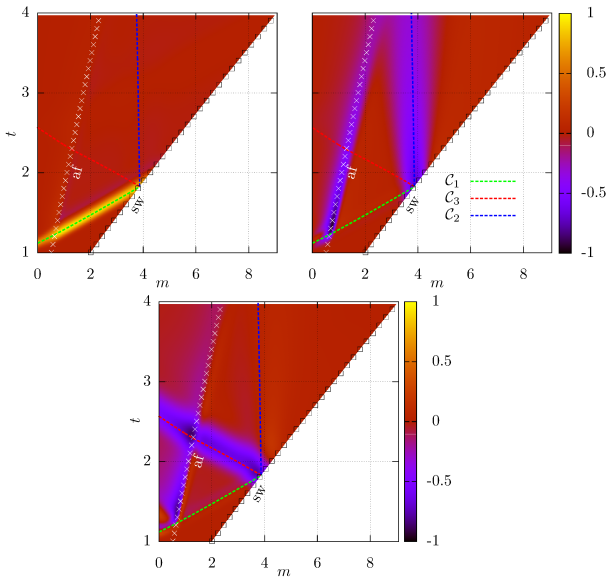

Figure 2: W_1 *(left)*, W_2 *(right)* *and* W_3 *(bottom)*.

These observations highlight the existence of a *supersonic heat-conductivity* propagating wave in the conduction zone and its ability to transmit perturbations from the flow external boundary to the ablation front at supersonic speed. As a consequence, a density perturbation is transmitted to the ablation front by this *heat-conductivity* wave. Analysis of responses to heat-flux perturbations applied at the external boundary on a supersonic expansion flow shows that advection is still dominant over diffusion in the conduction region, and thus could transmit density perturbation along *heat-conductivity* characteristic $\mathcal{C}_1$. However, density component of the wave in this particular flow is too small to be significant.

An external pressure perturbation forced at the surface is almost entirely converted to quasi-isothermal acoustic waves. As the expansion is supersonic, perturbation never cross the Chapmann–Jouguet point and cannot reach the ablation front, except a small part that is advected as superspnic the *heat-conductivity* wave.

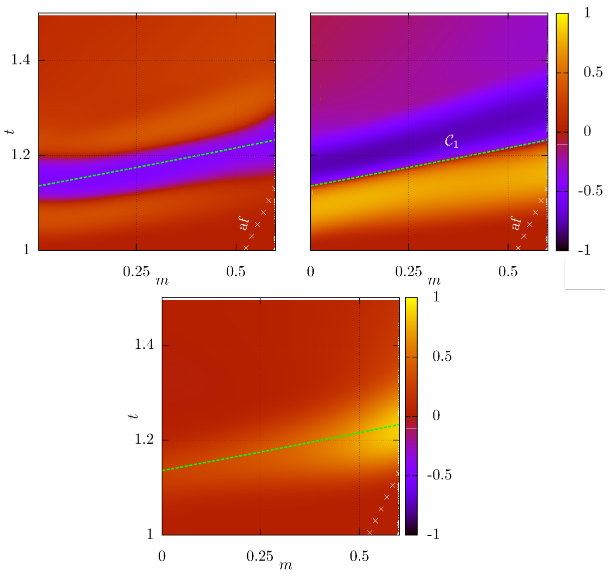

Figure 3: *Conduction region - contributions to δW_1 from diffusion (left), advection (right) and source terms $\times 10$(bottom) normalized maximum of advection contribution.*

Conclusion

The present work confirms the existence of a *supersonic heat-conductivity* linear hyperbolic waves that can transmit density perturbations from an ablation flow external boundary up to the ablation front. Such *heat-conductivity* waves arise from heat conductivity dependance on density and temperature and flow non-uniformities.

Perturbations of sufficiently long wavelength follow the characteristic trajectories predicted by the reduction of the general equations (2) to its hyperbolic part (4). Quantitative analysis of advection, diffusion and source terms shows that advection mechanism is dominant even in the conduction region. Therefore density perturbations can be effectively advected along *supersonic heat-conductivity* waves up to the ablation front and could possibly trigger radiative heat transport instability [4]. However, application to a supersonic expansion flow with external heat-flux or pressure perturbations shows that the density contribution to the *heat-conductivity* mode is actually negligible.

Further works should consider transverse perturbations and shorter longitudinal wavelength.

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