Curvilinear high-order Lagrangian hydrodynamic code for the laser–target interaction

Jan Nikl[1,2], M. Kuchařík[2], M. Holec[3], and S. Weber[1]

[1]*ELI-Beamlines, Institute of Physics, Academy of Sciences of the Czech Republic, 18221 Prague, Czech Republic*

[2]*Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University, 11519 Prague, Czech Republic*

[3]*Centre Lasers Intenses et Applications, Université de Bordeaux-CNRS-CEA, UMR 5107, F-33405 Talence, France*

Abstract

The laser–target interaction for high-intensities of the laser is very numerically demanding problem, since the dynamics of the process is very rapid. The classical Lagrangian hydrodynamic codes suffer from the progressing distortion of the mesh. Moreover, high resolution of the mesh is needed to capture the highly non-homogeneous interaction. The curvilinear finite elements present an alternative addressing both problems. The appropriate numerical methods have been developed over the past years. A new hydrodynamic code involving them is presented here and feasibility of the methods is shown on an example of a full simulation of the laser–target interaction.

Introduction

The laser–target interaction is highly complex problem having numerous applications. Many of them can be found for the intensities of the laser $\lesssim 10^{15}$ W/cm^2 and *ns*-pulses like the ICF relevant experiments [1] or ultra-high intensity pulses interaction with the pre-plasma originating from several orders of magnitude less intense laser pre-pulse [2]. The plasma undergoes rapid expansion and a strong shock wave is formed at the same time. Under these conditions, the Lagrangian hydrodynamics is the preferable way to simulate the plasma formation and the expansion, since the movement of the fluid is followed by the moving numerical mesh. However, there are limiting factors inherent to this kind of the codes. The problem appears when the codes are scaled from one dimension to multiple dimensions, where the mesh entangling is encountered in many cases. The approach we have chosen to address this problem are the curvilinear finite elements, which can essentially track the deformation of the mesh without the need to perform mesh regularization to a large extend.

Curvilinear high-order hydrodynamics

The plasma in this text is treated as a two-temperature fluid, where the electrons and ions are not in the thermal equilibrium as it happens in the plasma corona in the context of the laser–plasma interaction. The governing Euler equations of the hydrodynamics in the Lagrangian frame and with the terms specific to the laser plasma modelling then read:

$$\textbf{mass equation} \qquad \frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \vec{v}, \tag{1}$$

$$\textbf{momentum equation} \qquad \rho \frac{\partial \vec{v}}{\partial t} = \nabla \cdot (\sigma_e + \sigma_i), \tag{2}$$

$$\textbf{electron energy equation} \qquad \rho c_{Ve} \frac{\partial T_e}{\partial t} = \sigma_e : \nabla \vec{v} + G_{ei}(T_i - T_e) - \nabla \cdot (\vec{q}_e + \vec{S}), \tag{3}$$

$$\textbf{ion energy equation} \qquad \rho c_{Vi} \frac{\partial T_i}{\partial t} = \sigma_i : \nabla \vec{v} + G_{ie}(T_e - T_i), \tag{4}$$

where the primary quantities are ρ denoting the mass density, $\vec{v}$ velocity, T_e, T_i electron and ion temperature respectively. In addition, there are the exchange coefficients originating from the closure model. The heat fluxes $\vec{q}_e$ and the Poynting vectors $\vec{S}$ are given by the respective models of heat diffusion and laser absorption. Finally, the system is closed by the equation of state (EOS) giving the values of the electron and ion specific heat c_{Ve}, c_{Vi} and the electronic and ionic stress tensors σ_e, σ_i respectively. The presented hydrodynamic code PETE (Plasma Euler and Transport Equations) has available the EOSes from HerEOS library, which can consistently interpolate given EOS in terms of the thermodynamics [3]. However, inline evaluated EOS of ideal gas is used here for the illustrative purposes.

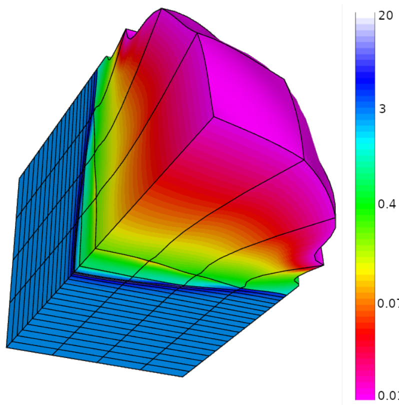
The presented hydrodynamic code PETE (Plasma Euler and Transport Equations) is based on the high-order finite elements method essentially. The hydrodynamic scheme from [4] has been implemented and extended to treat the two-temperature model of the plasma. To fully exploit the energy conservation enabled by the semi-discrete formulation, also the temporal integration is performed using the RK2-Averaged scheme proposed in [4]. However, it can be recognized that the energy conservation is violated for real EOSes, where the specific heats c_{Ve}, c_{Vi} depend on the temperature and/or density unlike of the case of the ideal gas EOS. This problem is addressed by the SSI-like correction [5], which eliminates the need to invert the EOS in fact.

The laser absorption is modelled by the WKB approximation in the form:

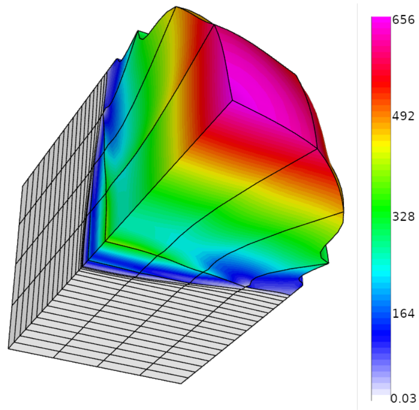
$$(\vec{n} \cdot \nabla) I_l = -\alpha_{WKB} I_l, \tag{5}$$

where I_l is the intensity of the laser related to the Poynting vector defined earlier as $\vec{S} = I_l \vec{n}$. The vector $\vec{n}$ represents the direction of the laser, which must be known a priori. The absorption coefficient α_{WKB} is calculated from the complex permittivity given in [6]. Numerically, Discontinuous Galerkin FEM method [7] is used with upwinding of the intensities.

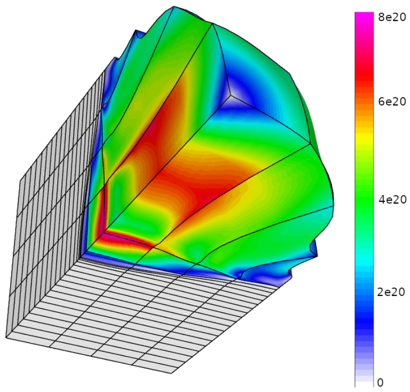
Finally, the model of the heat diffusion was implemented. The heat flux is assumed to be governed by the Fourier law as $\vec{q}_h = -\kappa \nabla T_e$. The factor κ stands for the heat conductivity



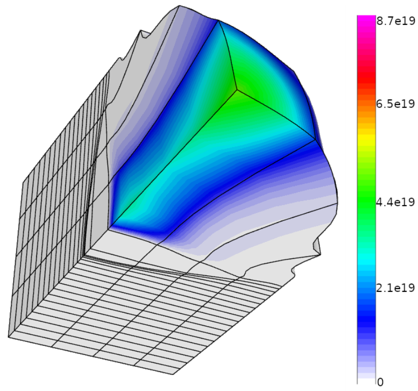
(a) Density [g/cm^3]



(b) Temperature [eV]



(c) Heat flux [erg/s/cm^2]



(d) Laser intensity [erg/s/cm^2]

Figure 1: The plasma profiles in the simulation of Aluminum ablation in the 3D Cartesian geometry at 30 ps. Quadratic thermodynamic and cubic kinematic finite elements were used.

given by the classical Spitzer formula here. From the numerical point of view, the heat diffusion problem is solved by Mixed Hybrid FEM (MHFEM) [8], where the temperature discrete space is identical with the thermodynamic one in the hydrodynamic part.

Simulation

In order to show the benefits of the presented code, a full simulation of the laser–target interaction was performed. A solid Aluminium target at the room temperature was irradiated by a constant laser pulse with the intensity of 10^{14} W/cm^2 and Gauss profile in space of the spot radius 5 μm. The flux limited heat diffusion was used with the limiting flux set to $q_{lim} = 0.05 q_{fs}$, where q_{fs} stands for the free streaming heat flux. The computational mesh in 3D Cartesian geometry with $4 \times 4 \times 20$ cells spanned over $[0,8] \times [0,8] \times [0,10]$ μm. The mesh had the geometric coefficients $1 \times 1 \times 0.96$ The quadratic thermodynamic and cubic kinematic finite elements were employed in the simulation.

The results of the simulation are plotted at 30 ps in Figure 1. It is clearly visible there that the curvilinear elements nicely track the motion of the plasma, despite the fact that the simulation resolution was enormously low.

Conclusions

The foundations of the new hydrodynamic code PETE were presented, where high-order curvilinear finite elements are applied in the Lagrangian framework. Maturity of these modern numerical methods was proved by a full simulation of the laser–target interaction. However, many physical simplifications were made like in the model of laser absorption or only the diffusive model, where the non-local transport methods may be more appropriate [9], and their numerical solution in the framework of the high-order curvilinear FEM remains the prospect of the future work.

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