Interpolations for plasma transport properties in the first Born approximation of the linear response theory

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Abstract

Closed expressions for electron-electron correlation functions and fully ionized plasma dc electrical conductivity, heat conductivity and thermopower are obtained. The approach is based on the linear response theory in the formulation of the relevant statistical operator method and takes into account both dynamical screening and arbitrary degeneracy. The expressions are constructed in the form that includes the asymptotic properties for non-degenerate [1], moderate [2, 3] and strongly degenerate [4] plasma and describe more wide density-temperature region that in [5]. The role of exchange parts in electron-electron correlation functions is discussed. The results obtained might be useful in calculating the multicomponent plasma transport properties.

Technical description

We consider a multicomponent plasma in the adiabatic approximation (masses of all ions and atoms are much greater than the electron mass). The first Born approximation with respect to screened interactions is used. Within the linear response theory in the formulation of Zubarev [6], transport properties are expressed via force-force correlation functions [7–9]. The electrical conductivity, the thermopower, and the heat conductivity are related to the Onsager transport coefficients $L_{ik} = L_{ki}$ according to

$$\sigma = e^2 L_{11} \tag{1}$$

$$\alpha = (eT)^{-1} L_{12}/L_{11} \tag{2}$$

$$\lambda = T^{-1}(L_{22} - L_{12}^2 L_{11}) \tag{3}$$

where

$$L_{ik} = -\frac{(-h)^{i+k-2}}{\Omega \det(d)} \begin{vmatrix} 0 & \frac{k-1}{\beta h} N_1 - N_0 \\ \frac{i-1}{\beta h}\overline{N_1} - \overline{N_0} & d \end{vmatrix}, \tag{4}$$

$$N_n = \begin{pmatrix} N_{n0} & N_{n1} & \dots & N_{nl} \end{pmatrix}, \tag{5}$$

$$\overline{N_n} = \begin{pmatrix} N_{n0} \\ N_{n1} \\ \vdots \\ N_{nl} \end{pmatrix}, d = \begin{pmatrix} d_{00} & d_{01} & \dots & d_{0l} \\ d_{10} & d_{11} & \dots & d_{1l} \\ \vdots & \vdots & \ddots & \vdots \\ d_{l0} & d_{l1} & \dots & d_{ll} \end{pmatrix}. \tag{6}$$

In (1) - (6) Ω - the system volume, N_{mn}, d_{mn} are correlation functions for the thermodynamic equilibrium, $N_e = \Omega n$ - the number of electrons, h - the enthalpy per one electron, T - temperature and $\beta = (k_B T)^{-1}$. In the adiabatic limit we can omit the ion flux and obtain for Eq.(6)

$$d_{mn} = d_{mn}^{ei} + d_{mn}^{ee} + d_{mn}^{ea}, \tag{7}$$

$$N_{mn} = N_e \frac{\Gamma(m+n+5/2)}{\Gamma(5/2)} \frac{I_{m+n+1/2}(\beta\mu_e^{id})}{I_{1/2}(\beta\mu_e^{id})}, \tag{8}$$

with $I_\nu(y) = \frac{1}{\Gamma(\nu+1)} \int\limits_0^\infty \frac{x^\nu dx}{e^{x-y}+1}$, μ_e^{id}- the ideal part of the electronic chemical potential.

Correlation functions d_{mn} (for electron-ion, electron-electron and electron-atom collisions) are evaluated using thermodynamic Green's functions. The lowest order of the perturbation theory (the first Born approximation) gives for the Coulomb interaction $V(q) = e^2(q^2\Omega\varepsilon_0)^{-1}$ screened due to the medium polarization Lenard-Balescu collision integrals [10].

We introduce dimensionless parameters: the coupling constant $\Gamma = (e^2\beta/4\pi\varepsilon_0)(4\pi n/3)^{1/3}$ and the degeneracy parameter $\Theta = (2m/\beta\hbar^2)(3\pi^2 n)^{-2/3} = k_B T/E_F$, where $E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$.

Interpolations for e-e collision integrals

In previous works [3, 5] the Chebyshev polynomial expansion method was successfully used for d_{11}^{ee}, d_{12}^{ee} and d_{22}^{ee} and for arbitrary plasma parameters. This gives the possibility to use closed (Γ,Θ)-dependable expressions for them in obtaining transport properties instead of long-time calculations. We describe this procedure for d_{11}^{ee}.

Interpolations should take into account the following asymptotic properties:

$$d_{11}^{ee} = d_0 \frac{\sqrt{2}}{2} \left[\left(\left(1 + 0.114\Theta^{-3/2} \right) \ln \frac{\Theta}{\Gamma} - 0.057 \right) \right] \tag{9}$$

for $\Theta \gg 1$, $\Gamma \ll 1$ [2],

$$d_{11}^{ee} = 0.531 d_0 \Theta^{1/2} \left(\ln \frac{\Theta}{\Gamma} + 1.22 \right) \tag{10}$$

for $\Theta \ll 1$, $\Gamma \ll \Theta$,

$$d_{11}^{ee} = 15.4 d_0 \Theta^2 \Gamma^{-3/2} \left(1 - (6 - 0.5 \ln \Gamma^2 \Theta) \sqrt{\frac{\Theta}{\Gamma}} \right) \tag{11}$$

for $\Theta \ll 1$, $\Gamma \gg 1$, with $d_0 = \frac{4}{3}(2\pi)^{1/2} \frac{m_e^{1/2} e^4 N_e^2 \beta^{3/2}}{\Omega(4\pi\varepsilon_0)^2}$. One of the possible interpolations with 0.1 accuracy for $\Gamma \sim \Theta \sim 0.1$ and 0.05 accuracy for $\Gamma \le 2$, $\Theta > 0.1$ and $\Theta \le 0.02$, arbitrary Γ is:

$$d_{11}^{ee} = d_0 \frac{\sqrt{2}}{2} F_1 F_2 \left(1 + 0.0345 \left(\frac{\Gamma}{\Theta} \right)^{3/2} F_3 \right)^{-1} \tag{12}$$

where

$$F_1 = \left(\Theta^2 + 0.114\Theta^{1/2} \right) \left(\Theta^2 + 0.152 \right)^{-1} \tag{13}$$

$$F_2 = \ln\frac{\Theta}{\Gamma} - 0.0570 + \frac{0.199}{0.156+\Theta} + 4\ln\left(1+0.27\sqrt{\frac{\Gamma}{\Theta}}\right) \tag{14}$$

$$F_3 = F_2\left(1+(6-0.5\ln\Gamma^2\Theta)\sqrt{\frac{\Theta}{\Gamma}}\right) \tag{15}$$

Interpolations for transport coefficients of a fully ionized plasma

Similarly, it is possible to construct a closed expression for transport coefficients of a fully ionized plasma based on asymptotic properties. Here we restrict ourselves to a graphic illustration of results. Fig.1 represents the comparison of obtained electrical conductivity with well known approximations σ^*_{0IT} and σ^*_{0ERR} for the reduced electrical conductivity $\sigma^*(\Gamma,\Theta) = \frac{m^{1/2}e^2\beta^{3/2}}{(4\pi\varepsilon_0)^2}\sigma(\Gamma,\Theta)$ made in [11,12] correspondingly.

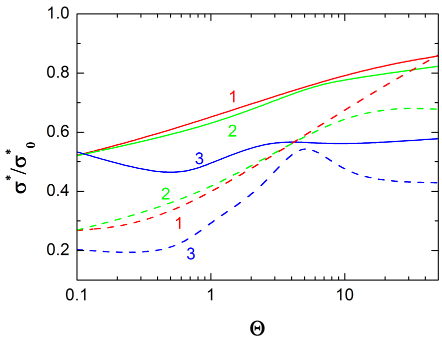

Figure 1: The σ^*/σ^*_0 ratio as the function of the degeneracy parameter. Solid lines - [11], dashed lines - [12]. 1 - Γ=0.1, 2 - Γ=1, 3 - Γ=2.

The approximation [11] made in semiempirical way for e-e scattering overestimates the values of electrical conductivity, especially in the degenerate region. The approximation [12] was constructed with accounting strong collisions and Debye-Onsager relaxation effect, and its deviations from the first Born approximation are understandable.

Fig.2 and Fig.3 illustrate the behaviour of the calculated Lorentz number $L = \frac{\lambda}{T\sigma}\left(\frac{e}{k_B}\right)^2$ and the reduced thermopower $a = \frac{\alpha e}{k_B}$ for the fully ionized plasma. The Lorentz number reveals a tendency to a Γ-dependent minimum for intermediate values of Θ. Well-known asymptotic values for the Lorentz number are 1.6183 for low-density high-temperature plasma and $\pi^2/3$ =3.2899 for the degenerate case. Note the very weak Γ-dependence of thermopower for given Θ. The corresponding asymptotic behaviour for the reduced thermopower gives 0.7053 for $\Theta \gg 1$ and $L\Theta$ for $\Theta \ll 1$. Of course, the transport coefficients obtaining for real plasma should take into account the scattering by atoms and more complex neutral components, and it is a separate problem. Parametriza-

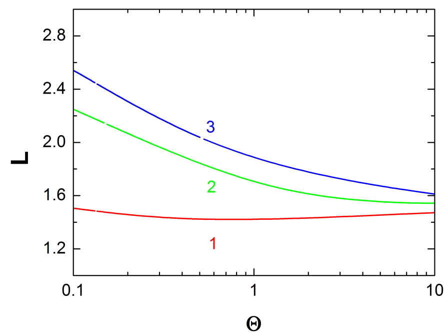

Figure 2: Lorentz number L as the function of the degeneracy parameter. 1 - Γ=0.1, 2 - Γ=1, 3 - Γ=2.

tion of d^{ee}_{mn} should greatly facilitate the overall calculations.

Exchange part of e-e correlation functions

Usually the exchange parts of Lenard-Balescu integrals for e-e correlation functions are not used for calculations because of their small value in comparison with direct ones. In addition there are some computational difficulties for the simultaneous accounting of degeneracy and dynamical screening for exchange. In the static screening approximation the leading term of exchange parts in d^{ee}_{mn} in the low-density high-temperature limit have the same order as corrections due to dynamical screening account. It is possible that for this reason the calculation of the exchange correction in the static approximation will be sufficient for arbitrary degeneration. It is obvious, that the exchange reduces e-e correlation functions and increases the conductivity, but the consequence treatment requires to obtain simultaneously exchange terms in dielectric function local field corrections.

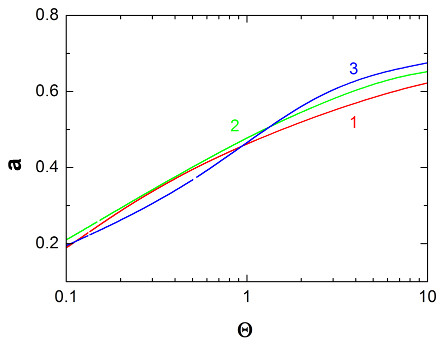

Figure 3: Reduced thermopower $a = \frac{\alpha e}{k_B}$ as the function of the degeneracy parameter. 1 - Γ=0.1, 2 - Γ=1, 3 - Γ=2.

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